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VOLUME 4

1916

EDITED BY

ALFRED N. GOLDSMITH, Ph. D.

PUBLISHED EVERY TWO MONTHS BY

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ONE HUNDRED AND ELEVEN BROADWAY
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The Membership of the Standing Committees of The Institute for 1916 will be announced in an early issue of THE PROCEEDINGS OF THE INSTITUTE OF RADIO ENGINEERS.

PROCEEDINGS OF THE SECTIONS OF THE INSTITUTE OF RADIO ENGINEERS

WASHINGTON SECTION

On the evening of October 24, 1915, a meeting of the Washington Section of the Institute was held at the Public Library. Two papers were presented. Mr. George Clark presented a paper on "The Location of Radio Stations by Means of a Direction Finder." Mr. Louis Cohen presented a paper on "The Strength of Received Signals."

On the evening of November 27, 1915, a meeting of the Washington Section was held. A paper on "Experiments at the Naval Radio Station, Darien, Canal Zone" was delivered by Dr. Louis W. Austin.

On the evening of December 29, 1915, a meeting of the Washington Section was held. A paper on "Shore to Ship Ranges from Arc and Alternator Stations" was delivered by Mr. Leonard F. Fuller. This was followed by a discussion in which Messrs. Greaves, Clark, Kolster, Pannill, Hogan, Stone, Bullard and Hooper participated. The attendance was thirty.

BOSTON SECTION

On the evening of October 28, 1915, a meeting of the Boston Section of the Institute was held at the Cruft High Tension Laboratory, Harvard University. Professor George W. Pierce (Vice-President of the Institute) read a paper on "A Mercury Arc Oscillator and Some Measurements of Condenser Resistance." The attendance was sixty-two.

On the evening of November 24, 1915, a meeting of the Boston Section was held at the Cruft High Tension Laboratory, Harvard University. Two papers were delivered. These were "The Impedances, Angular Velocities, and Frequencies of Oscillating Current Circuits" by Professor A. E. Kennelly, and "Amplitude Relations of Coupled Circuits" by Dr. E. L. Chaffee. The first of these papers was discussed by Professor Arthur G. Webster. The second was discussed by Professor A. G. Webster, Professor J. E. Ives, Professor W. G. Cady, and Mr. Hidetsugu Yagu. The attendance at this meeting was fifty-nine.

On the evening of December 20, 1915, a meeting of the Boston Section was held at the Cruft High Tension Laboratory, Harvard University. Professor J. C. Hubbard presented a paper on "The Effect of Distributed Capacity in Inductance Coils." This paper was discussed by Professors George W. Pierce, W. G. Cady, J. E. Ives, and A. E. Kennelly. At the same meeting there were exhibits of radio apparatus by a number of companies and individuals. The following companies were represented: General Radio Company, W. J. Murdock Company, George S. Saunders and Company, the Clapp-Eastham Company, and the American Radio and Research Corporation. The private exhibitors were the following. Professor A. E. Kennelly showed a "Paul" 800-cycle telephone current generator, a "Drysdale" alternating current potentiometer, and a "Duddell" oscillograph. Messrs. Fulton Cutting and Washington showed a small transmitting set using "Chaffee" gaps operated on 500 volts D. C. Dr. E. L. Chaffee showed a Tesla coil operated by two of his gaps together with a vacuum tube for showing the spark characteristics. Professor George W. Pierce, in addition to a number of instruments, showed his mercury arc oscillator acting as a generator and also as a receiver. In the latter case the signals from Arlington were recorded on tape. Mr. H. E. Rawson showed a wave meter, and a receiving set with which reception from the South San Francisco station was demonstrated. The attendance was one hundred and forty-three.

SEATTLE SECTION

On the evening of September 4, 1915, a meeting of the Seattle Section of the Institute was held, Mr. Robert H. Marriott presiding. The organization of the Seattle Section was discussed by Messrs. Marriott, Cooper, Alexis Paysse (Secretary-Treasurer of the Section), Wolf, and Milligan. The attendance was twenty-six.

On the evening of October 6, 1915, a meeting of the Seattle Section was held in the rooms of the Seattle Chamber of Commerce. Professor Osborne of the University of Washington offered the use of rooms at the University to the Seattle Section. This offer was formally accepted, with thanks. A paper on "Radio Development in the United States from 1899 to 1915" was presented by Mr. Robert H. Marriott. The paper was illustrated by slides. Mr. Roy E. Thompson discussed the paper; after which there was a discussion by a number of the members on allied topics. The attendance was forty-two.

On the evening of November 6, 1915, a meeting of the Seattle Section was held at Denny Hall, University of Washington. Two papers were read by the Chairman, Mr. Marriott. These were "The Effectiveness of the Ground Antenna in Long Distance Reception" by Mr. Robert B. Woolverton and "The Use of Multiphase Radio Transmitters" by Mr. William C. Woodland. A discussion followed. Mr. Robert H. Marriott then outlined his plan for the investigation of "radio shadows," and the carrying out of this work was systematically begun. The attendance was thirty-seven.

THE USE OF MULTI-PHASE RADIO TRANSMITTERS*

By

William C. Woodland

(Engineer, Packard Electric Company)

The production of radio frequency current from currents of commercial frequency is a matter of great interest and usefulness. The fact that a pure sine wave is not necessary or even desirable for radio purposes makes possible a great many ways of dividing up audio frequency current so as to duplicate the results obtained by radio frequency generators.

For example, it is possible to divide up audio frequency 3-phase currents into any number of intermediate phases, all of which have equal wave peaks occurring successively. By placing a separate radio transmitter in each phase, it would seem possible to operate at almost any desired frequency.

Several advantages of multiphase current over single phase current may be pointed out.

1. In an 8-phase, 60-cycle equipment the tone of the spark would be equivalent to that given by a 480-cycle generator. The condensers, however, would operate on 60-cycle current, which would allow the use of much larger capacity and a lower voltage for the same total amount of energy. The large condensers discharging at a comparatively low voltage would give short thick sparks of low resistance, resulting in an improved efficiency. The lower voltage would increase the life of both the condensers and the transformers.

2. The reliability of the equipment would be increased; because if a transformer or condenser should break down, the message might be finished on the other phases with no more trouble than a slight weakening in power; whereas on a single phase equipment, the operator would be unable even to advise his correspondent of the nature of the difficulty.

3. There would be some advantage in having to break only 58 per cent. of the corresponding single phase current at the key.

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* Presented before The Institute of Radio Engineers, New York, November 3, 1915.

4. Spare transformers and condensers could be carried at less expense than with single phase equipment.

5. I will show a little later that it is possible to operate multiphase equipments with very high power-factors and leading current so that the generator voltage does not fall on depressing the key.

6. When operating directly on 60-cycle, 3-phase current, the efficiency is certainly very much improved.

My attention was first called to the use of multiphase current for radio transmission in the early part of 1912, when it occurred to me that by dividing up a 3-phase, 60-cycle current into a sufficient number of intermediate phases, the results of the higher frequency generators might be duplicated using only commercial current.

I did not, at that time have in mind anything further than placing a separate radio transmitter of the fixed gap type in each phase, depending on it to discharge at the peak of the wave independently of the other sets.

This plan did not meet with success on account of the fact that the maximum point of the wave is not sufficiently definite to secure the phases against interference with each other. I found also that other investigators had carried the work up to this point, but that all systems had been rendered more or less inoperative because of the interference mentioned above.

All of these difficulties were overcome by the use of a rotary spark gap in each phase which had for its purpose the definite localizing of the point of discharge.

Eight (8) phase, five (5) phase, four (4) phase and three (3) phase equipments of this description have been built with entire success.

In the remainder of this article, I shall describe more minutely the 3-phase, 120-cycle, 3-kilowatt set shown in Figures 1 and 2. The three transformers are each of 1 kilowatt rating, 120 cycles, 63.5 volts primary, and 7,500 volts secondary, and are star connected on both sides. The open ends of the star primary go direct to the generator and the other ends are closed on each other thru the relay key when operating.

The condenser set consists of 9 banks, 3 condensers in series on each of the 3 phases, each bank being of 0.035 microfarad capacity, the effective capacity per phase being 0.0166 microfarad. The condenser phases are also connected in star relation to each other.

The open ends of the condenser star go direct to the 3-phase

spark gap, by means of which they are successively discharged thru the helix.

While testing for phase order and rotation, the neutral of the condenser set is connected to the transformer neutral and each phase operated separately; but after the testing is done, the

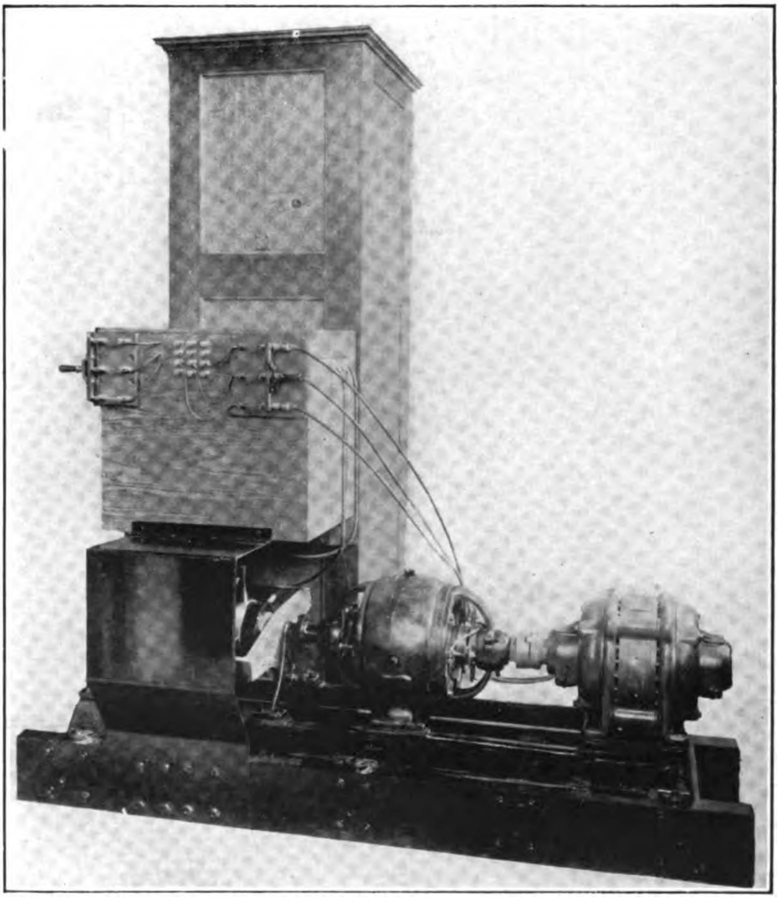


FIGURE 1—Three-Phase Transmitter

results are improved by removing the neutral connection. The shifting of the neutral serves to limit the short circuit current of the transformers and makes operation possible with much less inductance than is common.

The point above mentioned is worthy of a fuller explanation.

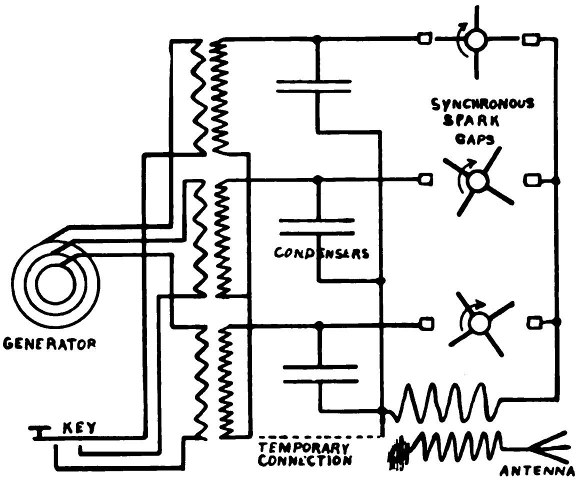


FIGURE 2—Diagram of Connections for Three-Phase Transmission

In most radio transmitters operating on spark systems, the discharge of the condenser acts as a momentary short circuit on the transformer, and means have to be provided to prevent a power arc from following the oscillatory discharge. On single phase equipments, this is usually accomplished by using a limiting inductance, which has to be capable of limiting the entire transformer output in order to be effective. On the multi-phase system, however, a much smaller inductance can be used, since the short circuit current is limited partly by the inductance of the active transformer and partly by the shifting of the neutral of the other two. The shifting of the neutral has a beneficial effect since it serves to increase the voltage on the transformer discharging next in order.

A transformer with series inductance short circuited on single phase supply draws more than double the current that it would if short circuited on a 3-phase star connection. The best results have been secured with a power factor of 90 per cent. leading; that is, the amount of inductance is less than that required for 120-cycle resonance.

The rotary spark gap which is direct driven from the generator shaft is shown in Figure 3. Considerable range of input, efficiency, and quality of tone is secured by shifting the point at which the discharge takes place. It has been found best to operate on the falling side of the wave, well over the peak, since this gives the highest efficiency combined with the best tone.

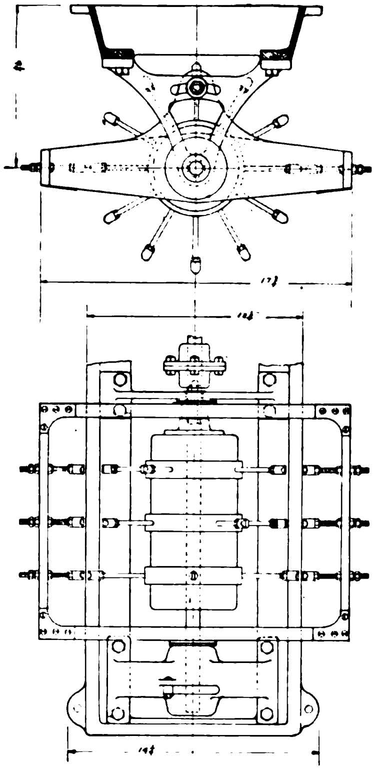


Figure 3—Three-Phase Rotary Spark Gap

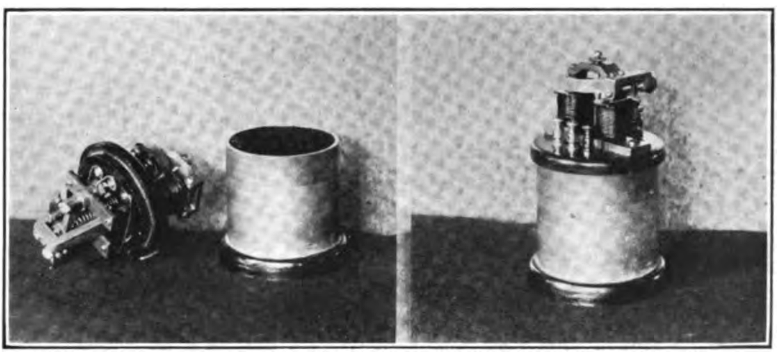


Figure 4—Oil Break Relay Key

Advancing the point of discharge only increases the input without effecting the output on the aerial.

Some form of relay key is desirable since there are a minimum of 2 primary circuits to be closed simultaneously. The oil break form shown in Figure 4 has been found very satisfactory for this purpose. Complete efficiency tests are not available at this time, but the set above described has radiated 14 amperes on a 2,000-meter wave-length without exceeding its rating of 3 kilowatts.

September, 1915.

SUMMARY: **The advantages of multi-phase over single-phase spark transmitters are given; some of which are higher tone, lower condenser voltages, greater reliability (because of spare phases in case of breakdown of one phase), and smaller current broken at key.**

A 3-phase, 120-cycle, 3-kilowatt set is then described in detail.

CAPACITIES*

BY

FRITZ LOWENSTEIN

As the seat of energy of an electrical field is in the space outside of the charged bodies we will consider the shape and concentration of the field only, but not that of the body itself. This distinction is necessary because capacities are usually attributed to the bodies charged, whereas the energy is excluded from that space which is occupied by the body. Considering the space between two charged bodies as the only seat of energy, the expression "charged body" is best replaced by "terminal surface" of the field.

Comparing geometrically similar elements of two geometrically similar fields, the elementary capacities are proportional to lineal dimensions. (See Figure 1.)

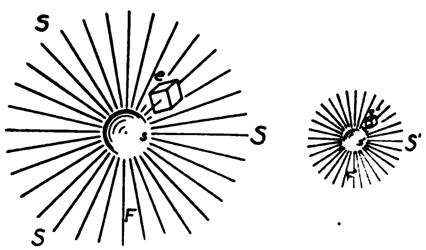


FIGURE 1

Extending this law over the entire field by the integrating process, we find that geometrically similar fields have capacities proportional to the lineal dimensions of the terminal surfaces. It is to be expected, therefore, that capacities expressed in dimensions of terminal surfaces should be of lineal dimensions.

That the capacity is by no means a function of the volume of the field or of the terminal body may be easily seen from Figure 2 where a field element is increased to double the volume by adding

* Presented before The Institute of Radio Engineers, New York, December 1, 1915.

volume in the direction of the field lines and in a direction perpendicular to the lines. In the first case the capacity has been decreased whereas in the latter case increased, altho in both cases the volumetric increase is the same.

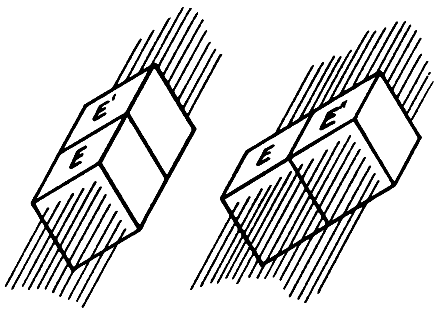


FIGURE 2

It is seen, therefore, that instead of being dependent on the volume, the capacity is rather a function of lineal dimension and therefore the maximum lineal dimension predominates.

An interesting example of this predominating lineal dimension or "maximum reach" is given by the composite capacity of two wires joining at one end under various angles, as shown in Figure 3.

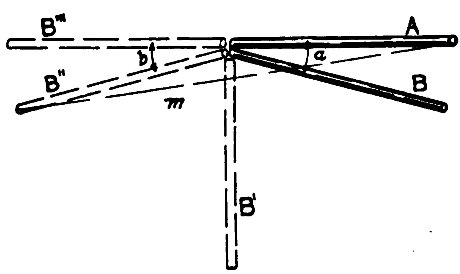


FIGURE 3

When the angle is small the composite capacity is practically the same as that of the single wire, since the addition of the second wire has not increased the maximum reach. If the second wire B be joined to A at an angle of 180 degrees, which means in straight continuation of wire A the total capacity has

oubled, as the maximum reach now is twice that of the single wire. We notice also that by deviating wire B slightly from the traight continuation of wire A, the maximum reach of the system is not materially altered, from which one may correctly conclude that turning the wire B thru an appreciable angle b does not materially change the capacity of the system. On the other hand a great change of maximum reach is produced by variations of the angle when the two wires are approximately perpendicular, and in fact the capacity of the total structure is most sensitive to changes of angle between the two elements at about 90 degrees.

In Figure 4, I have given a table of capacities per centimeter of the greater lineal dimension of the different configurations.

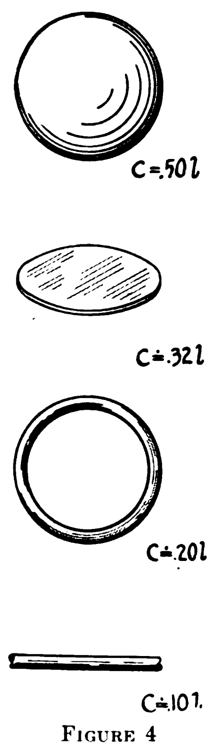


FIGURE 4

In Figure 5 the wire AB is assumed to be moved by the variable abscissae x, thereby generating a conducting sheet S.

It is instructive to follow the variation of the capacity C_x.

At $x=0$ the capacity is that of the wire C_{ab}; as long as x is small the capacity is practically constant because the width of the sheet is small compared to the length AB and a change of x does not involve a change of the predominating lineal dimension; however, as x increases and finally becomes greater than AB, it assumes the part of the predominating dimension, and, indeed, the graph shows the capacity then to be proportional to x.

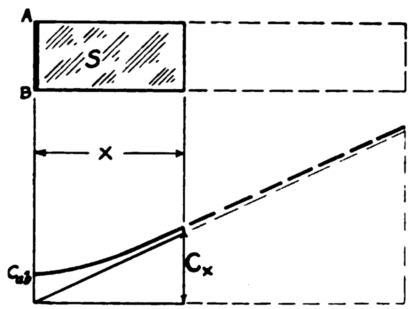


FIGURE 5

Comparing the capacities of a sphere and of a wire, it is found that the capacity of the sphere is only three or four times as great as the capacity of the wire in spite of the million times greater volume.

I have spoken of the capacities of a wire and of other bodies instead of the capacity of the field simply because I do not wish to distract attention from the familiar conceptions. Let me analyze the field shown in Figure 6, having two concentric spheres as terminal surfaces, and defining as "volumetric energy density" the energy contained in one cubic centimeter. As the energy of a field element is made up of the product of potential along the lines of force within that element and of the number of lines traversing it, the energy of a cubic centimeter of electric field is proportional to the square of the field density. Since the field density diminishes as the square of the distance from the center of field, the volumetric energy density diminishes with the fourth power of the distance from the center. The diagram to the left in Figure 6 shows the decrease of volumetric energy density.

Of greater interest than the volumetric energy density is the lineal energy density, which may be defined as the energy contained

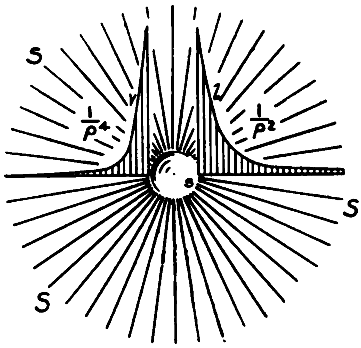


FIGURE 6

in a spherical layer of one centimeter radial thickness; and as the volume of such layer increases with the square of the distance from the center, the law follows from this fact, and from the volumetric energy density law that the lineal energy density decreases inversely as the square of the distance from the center. Such dependence is graphically shown to the right in Figure 6. The shaded surface below this curve represents the total energy of the field and it is easily seen therefrom that the maximum energy of the field is concentrated near the smaller of the two spheres.

I have taken a simple case of a field with spherical terminal surfaces to show that the concentration of energy lies near the smaller terminal surface. Similar considerations can be applied when substituting for this field radiating three-dimensionally, a field of bi-dimensional radiation (as that occurring in the case of long cylindrical terminal surfaces); where, as in this instance, the bulk of the energy of the field is to be found near the smaller one of the two terminal surfaces.

In Figure 7, I have shown a field with concentric termiral surfaces (either spherical or cylindrical), and have increased the scope of the field by reducing the size of the smaller terminal surface without, however, changing either the total number of field lines or the larger terminal surface. As the lineal energy density is very great near the smaller terminal surface, such addition of the field at that point must have materially increased the energy of the field and the change in capacity to be expected should be considerable. In fact, a considerable change in capacity of a sphere is obtained by a change of its diameter.

If, in Figure 7 the larger terminal surface alone is changed,

even materially, the total energy of the field will be increased very slightly only; due to the fact, as we have seen, that the energy density near the larger terminal surface is very small. Such a small change in energy corresponds to only a small change in the capacity of the field, from which we conclude:

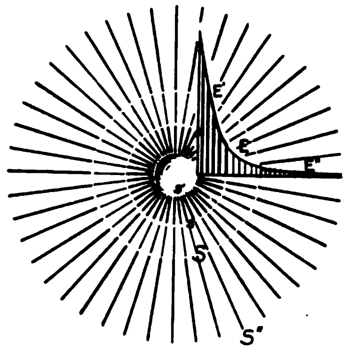


FIGURE 7

In a field having two terminal surfaces of greatly different size, a change of the smaller surface produces a great change in capacity, whereas a change of the larger terminal surface affects the capacity of the field only very slightly. The capacity of a field is, therefore, almost entirely determined by the shape of the smaller terminal surface.

That is why we may with correctness speak of the capacity of a sphere, or any other body, without mentioning the size and shape of the other terminal surface, as long as the assumption is correct that such other terminal surface is of greatly larger dimensions.

It may not be amiss to call your attention to the fact that the increase of field energy as illustrated in Figure 7 is accompanied by a decrease in capacity. This relation may easily be deduced from physical considerations, as well as from consideration of the mathematical expression for the capacity

$$C = \frac{\phi^2}{32\,\pi^2 W} \qquad \text{where } \phi = \text{total field lines}, \quad W = \text{energy},$$

wherein the capacity is expressed as a property of the field alone. I am tempted to introduce here the reciprocal value of capacity and apply to it the term "stiffness of the field," as an increase of energy would be followed by an increase of stiffness. I am,

however, loath to mar any additional insight which may be gained from these explanations by deviation from so familiar a term as capacity.

For a better conception of the slight change of capacity caused by a considerable increase of the larger terminal surface, I refer to Figure 7, where the difference of capacity is only 1 per cent in spite of the diameter of the larger terminal surface being increased 100 per cent. It appears, therefore, that that part of the capacity of an antenna which is due to the flat top is not materially changed by its height above ground.

While considering the capacity of a flat top antenna to ground, it must have occurred to many engineers, as it did to me, that the statement to be found in many text books on electrostatics is rather misleading: "That the free capacity of a body considered alone in space must not be confounded with the capacity the body may have against another body considered as a plate condenser." This statement is quite erroneous. As the strength and direction in any point of a field is of single and definite value, only one electric field can exist in a given space at a given moment, and, therefore, only one value of capacity. It is incorrect, therefore, to distinguish between free capacity and condenser capacity. This clarifying statement is deemed advisable, or at least permissible, in view of the quoted errors.

By speaking of the capacity of the field instead of that of the body, no such erroneous thought is possible, and it is clear that by free capacity of a body is meant the capacity of the field whose smaller terminal surface is the given body and whose larger terminal surface is one of vastly greater dimensions. It is not essential that this greater terminal surface be located at infinite distance, because of the fact that even if construed as of ten times the lineal dimensions of the small surface the change caused by removing it to an infinite distance would result in a change in capacity of not more than one-tenth of 1 per cent.

At a time when I had not realized the singly determined value of a field capacity, I considered a comparison between free and plate capacity as shown in Figure 8, wherein to an upper disc (of which the free capacity is $\frac{2}{\pi}r$), was added another lower disc, thereby forming a plate condenser. The problem arose in my mind to determine the distance of separation of the two plates so that the plate capacity would equal the free capacity of the single disc. From the well-known formulas for the disc

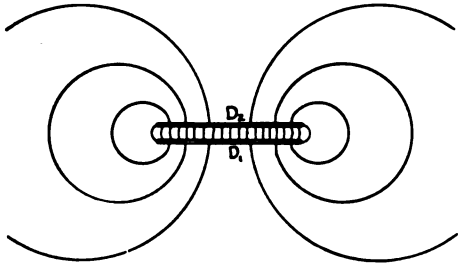


FIGURE 8

capacity and plate capacity, it would appear that the two were equal at a distance equal to $d=\frac{\pi}{8}r$, and I must confess that I had quite a struggle to decide whether in speaking of the capacity of the upper plate I would not have to add the two capacities. While such a mistake need hardly be called to the attention of the majority of engineers, I do not hesitate to make mention of it for the benefit of even the few students who might gain therefrom.

The advent of the aeroplane has opened another field, for radio communication. Whereas in the static field of an antenna, one terminal surface is artificial and the other provided by the surrounding ground, both terminal surfaces in an aeroplane outfit have to be artificial and are, therefore, open to design. The question arises in such a radio oscillator as to how much may be gained in energy for each single charge by increasing that one of the two terminal surfaces which consists of a dropped wire. The arrangement is shown in Figure 9. It is evident that

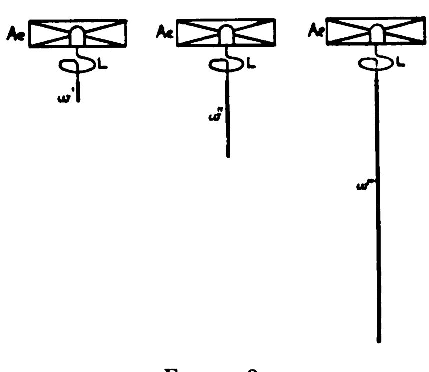


FIGURE 9

as long as the dropped wire is of smaller dimensions than the electrostatic counterpoise provided on the aeroplane, an increase in length of such dropped wire will materially increase the capacity of the field and, therefore, the energy per charge (as we may conclude by analogy from Figure 7). As soon, however, as the dropped wire is materially longer than the conductor on the aeroplane it assumes the role of the larger terminal surface of the field, and any further increase of its length will not materially contribute to an increase of electrostatic capacity nor of the energy per unit charge.

Figure 10 shows the function of the volumetric and lineal energy density in a field whose smaller terminal surface is a long cylinder. Such a field, radiating bi-dimensionally only, shows an energy concentration not so accentuated as that found in the

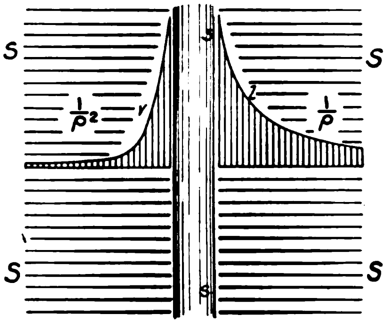


FIGURE 10

tri-dimensionally radiating field; but considering the larger terminal surface of a diameter ten times that of the smaller surface, the capacity would only be changed 1 per cent by increasing the larger terminal surface infinitely.

In all cases, therefore, where the larger terminal surface does not come closer at any point than (say) ten times the corresponding dimension of the smaller terminal surface, we need not be concerned with the actual shape of the larger terminal surface when we determine the seat of energy, the capacity and the configuration of the field lines emanating from the smaller surface. It will be seen, therefore, that from the flat top of an antenna, lines emanate almost symmetrically both upwards and downwards as though the larger terminal surface were one

surrounding the antenna symmetrically on all sides, in spite of the fact that the ground is located entirely at the bottom of the antenna. This is clearly illustrated in Figure 11.

By integrating the lineal energy density of a three-dimensionally radiating field between the radius of the smaller sphere

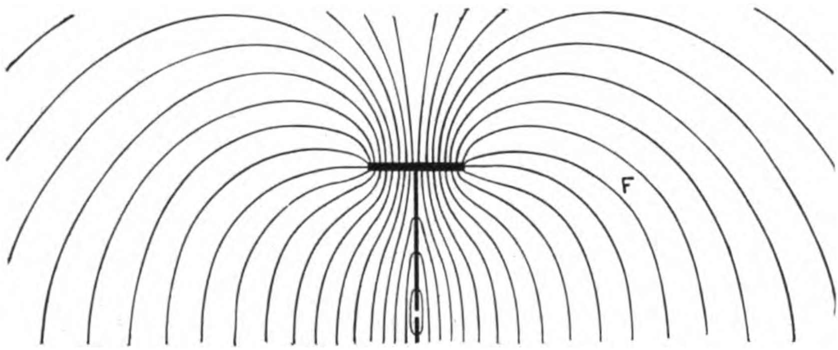


FIGURE 11

and that of the larger sphere, we can find the energy of such a field; whereby the capacity is determined. The lineal energy density follows the law of $\frac{1}{r^2}$, and its integral is proportional to $\frac{1}{r}$; and consequently the capacity of the field varies as r.

We have deduced, therefore, the capacity of a sphere from properties of the field alone, considering the sphere as a terminal surface only.

In deducing similarly the capacity of the wire from properties of the field alone, we have to start with the bi-dimensionally radiating field the lineal energy density of which follows the law $\frac{1}{r}$ as we have seen. The integral of such function is of logarithmic nature, as indeed is the capacity of the wire.

I wish to call your attention to the fact that in a sphere segments of the same projected axial length contribute equally to the capacity of the sphere, as shown in Figure 12.

If a charge were made to enter a sphere and traverse the sphere in the direction of a diameter, the sphere as a conductor would behave like a straight piece of wire of uniform lineal capacity. This fact was first recognized, to my knowledge, by

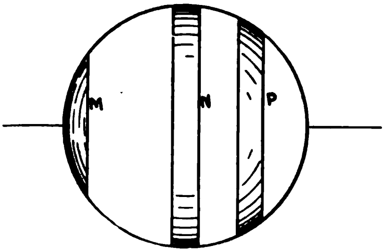


FIGURE 12

Mr. Nikola Tesla, and I expect to come back to the behavior of a sphere as a conductor of radio frequency currents at some later date.

The study of capacities of composite bodies is most instructive and conducive to a clear conception of capacity. Let, as in Figure 13, a number of small spheres of radius be so arranged as to cover completely the surface of the larger sphere, the radius R of which be 100. If each one of the

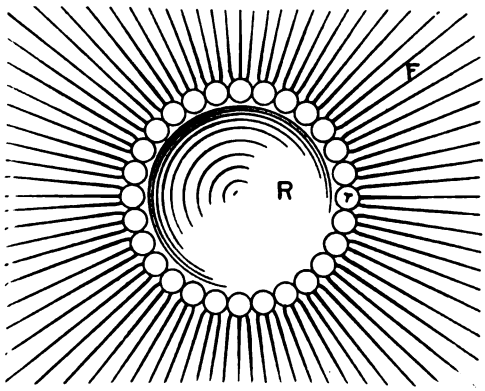


FIGURE 13

31,400 smaller spheres could be counted at its full value of capacity, the capacity of the composite body would be 31,400; as a matter of fact, however, it is not more than radius R of the larger sphere, that is 100. Indeed, the configuration of the electric field F could not have changed materially by the arrangement of the small spheres, and the capacity clearly presents itself as a property of the configuration of the field lying outside of the enveloping surface of the composite structure.

Capacity may play a part in the conduction of electricity thru liquids and gases. Let us assume a series of spheres in lineal arrangement as shown on Figure 14.

As long as the distance between the spheres is great compared to the diameter of the spheres, each sphere will retain its full capacity as given by its radius. By decreasing the distance

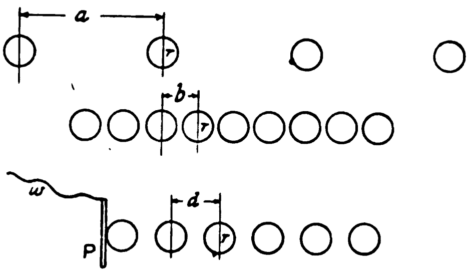


FIGURE 14

between spheres the individual capacities of the spheres decrease, because of the negative capacity coefficients. If such approximation be carried to the point of contact between the spheres, the capacity of each individual sphere would be reduced to approximately $\frac{1}{\varepsilon}$ of the original capacity. If such a row of spheres were conceived as freely movable, so as to enable each sphere to make contact with a plate *P*, which is kept charged to a certain potential, then the charges carried away by the spheres after contact with the plate would be proportional to the full capacity of each sphere as long as the spheres are far apart, and would be only $\frac{1}{\varepsilon} = \frac{1}{2.718}$*th* part of such maximum charge when the spheres are in contact. As we assumed the plate *P* to be maintained at a certain potential by an outside source of electricity, the convection current represented by the departing charges of the spheres would vary approximately in a ratio of 2.71 to 1.

In the passage of electricity thru an electrolyte, the molecular conductivity has been found to be the same for all electrolytes, and varying only with the concentration of the solution; the molecular conductivity being approximately 2.5 times as great in the very dilute solution as in the concentrated solution.

I wish to call your attention to the striking similarity between the ratio of conductivity experimentally determined in elec-

trolytes of small and large concentration and the ratio of conductivity of the row of spheres where the spheres are far apart or close together. I do not pretend at this moment that a

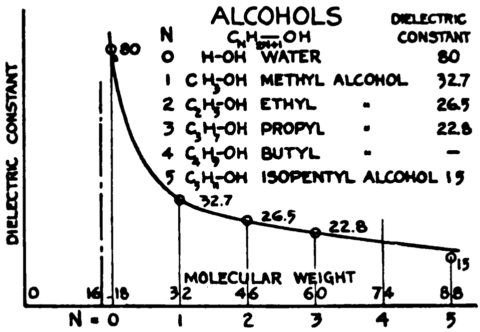


Figure 15

plausible modification of the theory of conduction thru electrolytes and gases can be based on such a coincidence; and in fact, assumptions would have to be made. For example, a lineal arrangement of the ions in the direction of the static field impressed on the electrolyte or on the gas must be assumed.

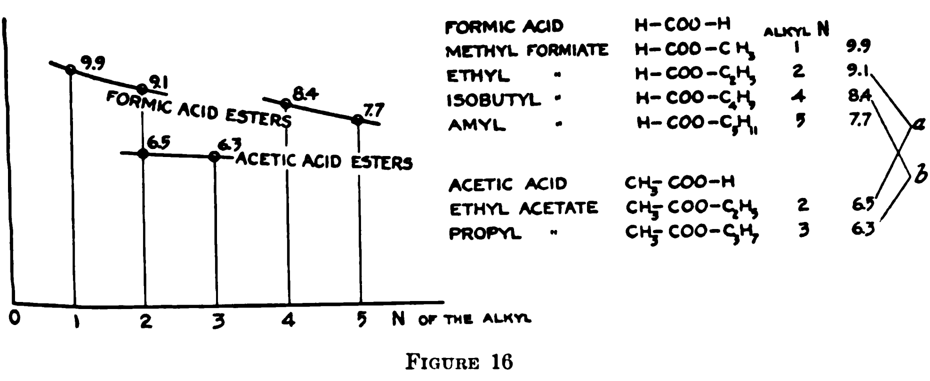


Figure 16

But the fact that such ratio in the case of the spheres is deduced from geometrical considerations alone, coupled with the fact that in electrolytes the same ratio follows from purely geometrical considerations, is sufficient to warrant further thought. I do not hesitate to bring this interesting coincidence to your knowledge, with the hope that other physicists may carry on investigations

in the same direction. I have said that the molecular conductivity of electrolytes arose from geometrical considerations only, and I think it advisable to call your attention to the foundation of such a statement. While it is true that the conductivity of different electrolytes varies considerably, it has been found that the molecular conductivity is the same for all electrolytes. The similar behavior, of the same number of molecules, independently of the weight of the molecule, therefore reduces the phenomenon to a purely geometric basis.

SUMMARY: Considering that electrostatic energy is actually in the space surrounding a charged body, the latter is called a "terminal surface." It is shown that capacity is predominantly a function of the ***maximum*** lineal dimension of the terminal surface. The volumetric and lineal energy densities in the field are defined and studied in a number of cases. It is proven that the capacity between two terminal surfaces is greatly affected by changing the lineal dimensions of the ***smaller*** terminal surface, but not so for changes of the larger. Certain current errors in connection with "mutual capacity" are considered.

The practical applications to a radio antenna and to aeroplane counterpoises are given.

When a charge traverses a sphere, entering parallel to a diameter, the sphere behaves as a conductor of uniform lineal capacity.

Applications of the theoretical considerations are also given in connection with the conductivity of concentrated and dilute electrolytes.

DISCUSSION

John L. Hogan, Jr.: The computation of antenna capacity has been a problem of great interest to radio engineers for a good many years, and many methods have been proposed. As a rule, the mathematical solution of the problem is not only very complicated, but is likely to lead to results which are so far from accurate that the labor of the computation is not repaid. Measurement methods almost always involve the erection of the antenna itself or of a substantially accurate copy of it, and so are impracticable for many uses.

In design of station apparatus, whether for sending or receiving, it is convenient to be able to determine antenna capacity quickly and easily, and for ordinary purposes an accuracy of approximately 5 per cent is all that is required. In fact, even rougher approximations than this will often satisfy the requirements, since possible variations can usually be compensated for in the selection of instruments. I have not seen published any simple method of computing antenna capacity which can be solved quickly and easily, and yet which will give results within a reasonably close value of the actual measurements; however, there is such a method in practical use.

For a number of years I have been collecting data which interconnects the geometric and electrical constants of a great many practical radio telegraph aerial systems, including those of both ship and shore stations. From these I have been able to secure an approximate relation between the area of the aerial system and its effective capacity. For flap top antennas of the usual form, the capacity is almost directly proportional to the area plus a constant, and amounts to something like 0.00024 microfarads per thousand square feet plus 0.0004. The linear relation, of course, is not sufficiently accurate for close calculation, but may be used in securing a first approximation of capacity of medium sized antennas. For more accurate results the capacity C, and area A, are related by the following expression

$$C = p\,A^{q}$$

where p and q are constants depending on the type of antenna. An expression of this same form is useful for pre-determination of capacity of umbrella antennas; for this purpose the exponent q varies with the number of wires in the umbrella and the area is measured as the surface of the cone which has the rib wires as its elements. It is not ordinarily necessary to take into

account the height of the antenna from the ground, for as a rule this distance is too great to affect the result to any marked extent. Closeness of grounded towers, etc., may have to be compensated for, and if a large fan instead of a single wire is used as a lead-in, its part of the capacity must also be added.

If I had realized that the paper and discussion tonight would approach this matter of the practical pre-determination of antenna capacities, I would have prepared some further quantitative data in that connection. I hope that at some future time I may present to the Institute complete information as to this method of computing capacities.

A NULL METHOD OF MEASURING ENERGY CONSUMPTION IN A COMPLEX CIRCUIT*

By

Alfred S. Kuhn

(Engineer, Fritz Lowenstein Company)

In April, 1914, the problem arose of measuring the power consumed in a chamber in which an electrochemical action took place. The chamber was comparatively complex, having glass and gas parts; and the chemical action would have been seriously affected had terminals been introduced into the apparatus for determining the constants of the different parts. It was also determined that the properties of the device varied considerably, but to an unknown extent, with slight changes of applied voltage and of gas pressure. Furthermore, as part of the energy supplied had to pass thru the glass, the use of direct currents was precluded.

Attempts to build a wattmeter showed that much additional apparatus would be required to carry out the measurements by the wattmeter method. When the problem rose again about a year later, the following arrangement was adopted.

Across the secondary of the power transformer, two circuits were placed in parallel. One circuit contains the reaction chamber in series with a coil. The other circuit consists of a variable capacity (a variable inductance sometimes), a variable resistance, and a coil identical with the coil in the first circuit. To each of the aforesaid coils is coupled a secondary coil, the two secondary coils being identical. The coupling between the primary and the secondary of one circuit is identical with that of the other. The arrangement of circuits is shown in the illustration.

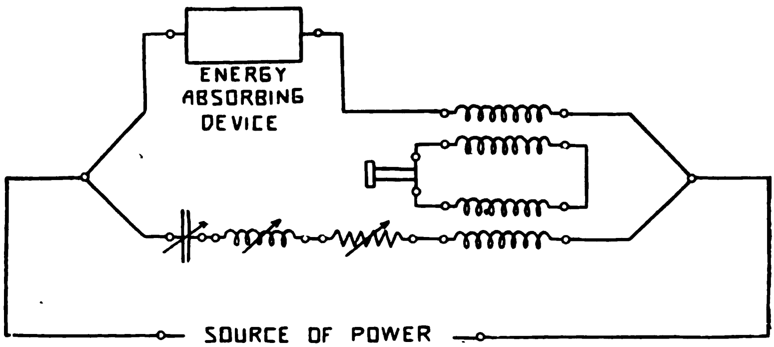
If then the two secondary coils be connected in series with a telephone or other suitable indicator, there will be no indication of energy in the indicator when the currents in the two circuits are equal and in phase. When such is the case, the power consumption in the two circuits is the same. The power in the artificial circuit may easily be measured and equated to the power in the complex circuit.

* Communicated to the Editor, September 28, 1915.

It may be mentioned that it is not necessary to use *both* capacity and inductance in the balancing circuit. In addition to the equivalent resistance, there need be only an equivalent reactance factor:

$$\omega L - \frac{1}{\omega C}.$$

This reactance may be made up of capacity, inductance, or both.



In measuring the power in the balancing circuit, wattmeters of the ordinary types may not be used if the power factors are very low. Thus, in the experiments mentioned power factors as low as 5 per cent arose. As a rule, a high power factor wattmeter may be used in such case provided current potential transformers are available. This device is doubly useful when working at high potentials and low power factors.

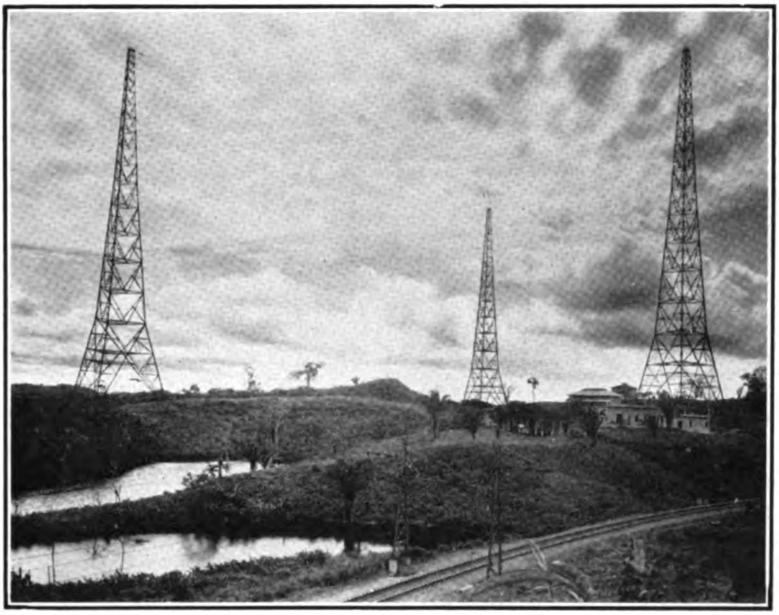
SUMMARY: A null method of measuring power absorption at low power factors in complex circuits at any frequency is described.

THE DARIEN RADIO STATION OF THE U. S. NAVY (PANAMA CANAL ZONE)*

By

Lieut. R. S. Crenshaw

(U. S. N.)



Darien Radio Station of the U. S. Navy

In its system of radio communication for the Canal Zone, the Navy has maintained the high standard set by the Canal in general in having thoroly modern equipment. The layout comprises one coastal station at each end of the Canal for ship to shore work, and one high power station for long distance work.

The Darien radio station is located just twenty-five miles

* Received for publication December 18, 1915. This paper has been censored by the Navy censors.

south of Colon on the Panama Railroad. The railroad forms the east boundary of the reservation which contains eighty-seven and one half acres. The southwest boundary is the Canal itself from which runs a channel twenty feet deep and seventy-five wide into the center of the station plot. Those who may have visited the Canal before the water was allowed to rise may remember this site as being adjacent (to the southeastward) to the old town of San Pablo. On the "relocation" of the railroad, the nearest stop was Caimito Cabin at the North end of the dumps; and the site was known variously as the San Pablo or Caimito radio, or simply as *Radio*, in the early stages. Finally the Department assigned the name Darien when, by request, the Governor named the railroad stop here Darien. Until the name was well known, mail frequently went stray to the southernmost province of the Panama Republic, which is known as the Darien section.

Work was started on construction in December, 1913, by the Quartermaster Department of the Panama Canal. The site was far removed from any of the Canal Zone towns and was mostly jungle. A spur from the railroad was put in, laborers' barracks built; and, as it is unsafe to load cement in the open due to the sudden rains, a cement shed was erected close to the spur. With a hoisting engine on the hill, all material was hauled up a narrow gauge road in De Cauville dump cars. This narrow gauge road continued around the station site for the delivery of material for the buildings and towers. The small locomotives, cars and track were relics of the French construction days. Water for the station was pumped to a tank on a hill by a Worthingston pump, which obtained its steam from a boiler of an old Belgian locomotive side-tracked for that purpose. Drinking water was distilled at this pump station. This equipment supplied the station until the arrival of the electric turbine pump. The Gatun Lake water is now used and merely sterilized for drinking and cooking purposes.

The dwellings on the site are the house for the Radio Officer, cottage for the chief electrician in charge, and barracks for the operators equipped to house seventeen men. Servants' quarters are also provided in the barracks building. Rations are commuted at a dollar a day per man, and a mess is run by the operators.

All the buildings are screened, including the porches. There is such a large breeding area for mosquitoes about the site that the cost would prohibit sufficient sanitation work to keep the

mosquitoes down entirely, and the means adopted are (1) to keep the screening as tight as possible; (2) each morning a sanitary inspector makes the rounds, and catches the mosquitoes inside the living quarters and office; (3) no containers are allowed to collect water; in which there may be breeding on the station site; (4) all drains are kept clear so no water stands in puddles; (5) around the edge of the water the bank is kept skinned to allow the small fish to eat the mosquito larvae (this means is remarkably effective); (6) the force of five laborers allowed the station is kept at work on the grounds to keep the jungle growth cut down as well as possible. When one case of malaria appeared, the whole station was put on a quinine diet for ten days, in order to prevent an epidemic.

The other buildings of the station, with the exception of the boat-house, are of concrete. The boat-house is of old form lumber left over from the concrete work, and corrugated iron roofing robbed from old, abandoned shacks on the site, one of which was a distillery.

The power-house is sixty feet by thirty feet, and contains the motor generators for the main transmitting set.

The main distributing and controlling switchboards are here, with the auxiliary transformers. This building also houses the machine tools, a small lathe, a drill press, milling machine, and emery grinder, and is fitted with a five-ton overhead travelling crane. All wiring is in conduit in wire trenches.

The operating building contains the arc room (where is located the main transmitting set with its auxiliary electric controlling devices), the receiving room and the office, besides a spare room for an auxiliary sending set if needed later. The arc room and the receiving room both have wire mesh embedded in their walls, floor and ceiling, in order to prevent induction from the transmitting set injuring the receivers. The building is fireproof, which is necessary on account of the action of the continuous oscillations used at such high voltage. The charging current into iron in the vicinity of any live lead heats the iron quickly. Some of the reinforcing had to be taken out of one concrete base because the current jumped to it; and one wall 19 inches (47 centimeters) away from the end of the helix heats so that the hand cannot be kept on it after a twenty-minute run. The reinforcement in this wall is merely metal lathing, but it is directly in the field of the main helix.

The contract for the towers was let to the Penn Bridge Co. which in turn sublet the fabrications to the Toledo Bridge and

Iron Works, and the erection to Mr. J. O. Childers. In all three towers there are about 1000 tons of structural steel. They are 600 feet (183 meters) high each; the feet form a triangle 150 feet (46 meters) on a side, and the tower tapers to a 10-foot (3 meters) triangle at the top. An iron ladder runs up the outside of one leg on each tower, having rest platforms about every 50 feet (15.2 meters).

When first erected there was considerable swaying in the bottom long diagonals, but these and others above were stiffened up by cross bracing, and now they are perfectly rigid. When the antenna was hoisted and adjusted to the sag which would give a pull of about 13000 pounds (5500 kilograms), the top of each tower was pulled over only 4 inches (10 centimeters) during hoisting, and settled back to 2 inches (5 centimeters) when hoisting stopped. All the bend was in the upper 200 feet (60 meters).

As mentioned earlier in this article, it was the first intention to locate the towers on the tops of the hills, but on making the actual location it was found that the thrusts (which come on to the footings at the angle of slightly over sixteen degrees from the vertical) would be nearly too parallel to the face of the hills to give solid backing for the footings. They were finally located so that all footings except one butt into the hills. In order to do this, however, the footings were put on about the 120-foot (36 meters) level instead of the 170-foot (52 meters). The surface of Gatun Lake is normally at the 85-foot (26 meters) level. The block for each footing is 16 feet (5 meters) deep and 20 feet (6 meters) square, heavily reinforced with old railroad rail. Each block filled entirely the hole excavated for it without back filling in order to have it bearing in undisturbed earth. The distance between towers is: one and two, 897 feet (273 meters); two and three, 751 feet (254 meters); one and three, 969 feet (295 meters); the antenna covering about six acres.

The Darien Towers, being farther apart, and not so directly beneath the antenna seem not to affect the capacity as at Arlington. Darien evidently has a greater effective height than Arlington.

The antenna was made at the New York Navy Yard and shipped to Darien, each wire on a separate reel and tagged to mark the points where other wire crossed. The cables are all phosphor-bronze; the outside ones being ¾-inch (1.9 centimeter) diameter, the four strain cables thru the mast, ⅜-inch (0.9 centimeter) diameter, and the sixty-six radiating wires of regular

antenna wire. The first 150 feet (48 meters) of the down land of twenty-six wires is a fan and is then grouped by spacing hoops to form the rattail. Each corner is insulated with the Arlington type of Locke insulators with, however, two strings in parallel, as the strain was too near the mechanical breaking limit of the insulators. Lightning has already struck the antenna twice without damage, because of the safety gap feature of these insulators and the towers being grounded. An electric winch on each tower furnishes the power needed for handling the antenna. The feet of each tower are insulated.

The feet rest on 10 porcelain block insulators 11 inches (27.9 centimeters) high, having each three petticoats. Insulators are also placed under the yokes, which secure to the anchor bolts for taking the upward thrusts; and others are placed between the footing and the channel irons projecting from the block to take the side thrusts. However the arc "pulls" better with the towers grounded; so they are operated in that condition by being grounded thru large knife switches to the ground system of the station.

The general ground conditions of this site are excellent, since the Gatun Lake lies on three sides of it, with an arm reaching into the center of the station plot. An artificial ground was laid in addition to cover all the land as follows: 100,000 feet (30,000 meters) of annealed copper wire was laid in the shape of a grid forming rectangles about 50 feet (15 meters) on a side. All intersections were soldered; the ends of all wires, on reaching the water's edge were run 100 feet (30 meters) into the lake, and the main ground plate and the ground plate for each tower are tied into the large grid by busses reaching well out into it. This ground system is buried about 4 inches (10 centimeters) for protection.

The main transmitting set was furnished by the Federal Telegraph Company, and the arc generator is their type of the Poulsen arc.

The signalling is done by short circuiting or opening a compensating helix in series in the antenna circuit. The key for accomplishing this contains 13 pairs of points mounted on a yoke in parallel, so that each pair of points breaks only the voltage due to one turn of the auxiliary helix. This yoke is on the armature of a solenoid, the current controlling which is broken in a strong magnet field; and the key is thus positive and fast in action. The D. C. supply is protected from the radio frequency current by having air core choke coils in both

positive and negative lead to the machines. The arc field spools are in the negative lead, and carbon rod protection in the power-house guards further against high voltages getting into the D. C. generators. This set gives Arlington a signal easily readable thru all but the worst electrical storms.*

The arc can be controlled entirely from the operator's seat; the main generator voltage being controlled there, circuit breakers closed or tripped, the arc struck and starting resistance short-circuited. While running, the arc is regulated to take up the wear of the carbon by foot pedals, so that the operator may not have to interrupt his sending. All circuits are electrically interlocked so that on starting, the correct sequence must be followed.

The regular receiving cabinets with tikkers, as used by the Federal Telegraph Co., were provided with the outfit.

For short wave work, an oscillating audion detector is used on one of the Federal Company's cabinets.

Only government work is handled by this station, and at present there is not enough of this to demand continuous watch so that schedules are run. The complement requires eight operators on watch, two at a time, a chief radio electrician in general charge, a hospital steward of the Navy in charge of sanitation and general health work, a yeoman (who is a clerk for the station and for the Radio Officer), and a machinist. Five laborers are employed on the grounds, which were high jungle when the station was built. The Radio Officer of the Canal Zone lives here, having his office in that of the station. With excellent telegraph and telephone service to all parts of the Isthmus, the station, tho isolated, is in close touch with the Canal Government.

When the Navy Department decided upon the kind of set and the general features of this station most of them were in the experimental stage, and the excellent results obtained at this station have been watched with keen interest and gratification.

SUMMARY: The buildings, sanitary arrangements, towers, ground connection, antenna, and some features of the transmitter and receiver of the Darien radio station of the United States Navy are described.

* (The distance from Arlington to Darien is 1,900 miles (3,000 km.), practically due south.—EDITOR.)

FURTHER DISCUSSION ON "THE TRAINING OF THE RADIO OPERATOR"

By

M. E. Packman

(Instructor in Radio Telegraphy, Dodge Institute of Telegraphy)

(See Proceedings of the Institute of Radio Engineers, Vol. 3, Number 4, Page 311)

M. E. Packman (communicated, November 16, 1915): Mr. Hogan's proposed plan for training students in telegraph receiving thru the use of commercial receiving sets is of course a good one, provided the student is sufficiently far advanced in radio work to be capable of manipulating the instruments. However this is not likely to be the case. It invariably develops that that student who is the most adept in learning to copy telegraphic signals is the slowest in learning to operate radio apparatus. Under these conditions he would make very slow progress. As an extreme case take the new student just out of an office, a store or from a farm and imagine him successfully adjusting a modern receiver in order that he could hear signals. It has been our experience that it is highly desirable to differentiate between the code practice and the theory and operation of radio equipment. Considering this from another point of view, in a school the size of ours where we have from fifty to eighty students in the code work the greater portion of the time, the expense of this number of tuners costing from two hundred and fifty to five hundred dollars each would be entirely out of the question amounting to an outlay of twenty-five or thirty thousand dollars for receiving apparatus alone. Assuming that these tuners would be "modern" for a period of five years it is quite evident that the scheme is not feasible.

As previously mentioned, a plan somewhat similar to Mr. Hogan's arrangement is used for instruction purposes in connection with the study of receiving apparatus. Two methods which we use are shown in Figure 1 and Figure 2. In Figure 1, a receiver is connected to a standard antenna, in the ground lead of which is placed a coil of several turns wound on a rectangular frame. In this last mentioned coil are induced oscillations

from one or more wave meters excited with buzzers and operated from an omnigraph or otherwise. By adjusting these wave meters to different wave lengths it is possible to obtain any degree of interference desired and the student can obtain all the practice in making adjustments of his tuner to prevent the

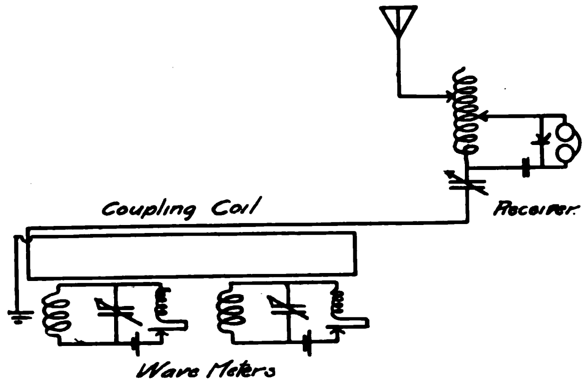


FIGURE 1

interference that is required for any condition of working. This arrangement is probably preferable to the two arrangements shown inasmuch as natural strays or induction are obtained without special arrangements. Actual radio signals can also be received at the same time. In Figure 2 an ordinary

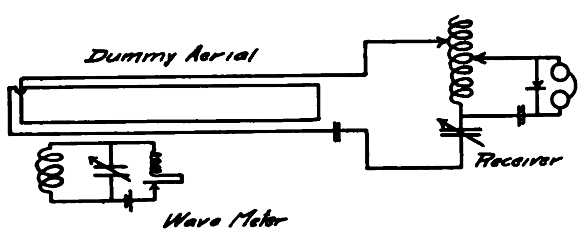


FIGURE 2

dummy aerial system is shown, in which the condenser has the same capacity as an ordinary aerial and the inductance of the loop is of such value that the dummy has any fundamental desired.

One method of producing artificial strays that I have used is shown in Figure 3. *A* and *B* are leads to a power line. *R* is a

current regulating rheostat, P a potentiometer arrangement and I an electrolytic cell or interrupter. With proper adjustment of the interrupter and the rheostat, a very irregular current flows thru the cell. Connections to the receiving code circuits are taken from the potentiometer, condensers being interposed. With this arrangement an almost perfect imitation of strays

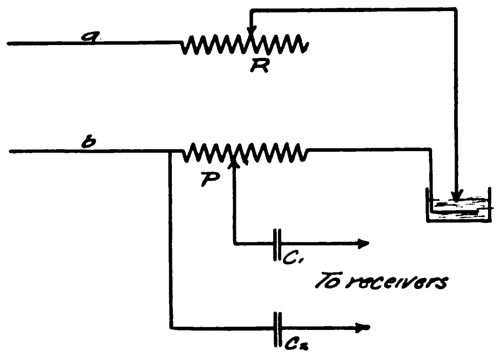


FIGURE 3

of any desired strength can be obtained. This arrangement also permits of the generation of highly damped oscillations which can be induced in any antenna system. This method of producing artificial strays has an advantage over the scheme of using an irregular notched wheel-interrupter in that there is no regular sequence of characters such as would be obtained with a contact moving over the same surface revolution after revolution. Under normal conditions in the region of the Great Lakes there are sufficient strays to render such arrangements somewhat superfluous and hence they are not used in our work to any extent. I might also answer Mr. Bucher's remark regarding the necessity of artificial interference at this time by stating that in this locality conditions are very different from those in the vicinity of New York. Except at night, there is very little interference from commercial stations and I have found it desirable in our case to produce artificial interference as outlined.

I note that both Mr. Sarnoff and Mr. Bucher take exception to my statement that a deplorable condition exists in some branches of commercial radio service. Many improvements have been made during the last few years but there are still some localities in which many changes will eventually be made

before the service can be comparable with that in other quarters. Their remarks are apparently based upon a close observation of conditions in and around New York, where, of necessity, the service has been advanced to its highest efficiency. I am more or less familiar with Mr. Bucher's school and the methods that he uses there, and know that the training he is giving his students in the theoretical and practical work is thoro and adequate to the demands of good service. In many respects the work is similar to that in our institution which is, of course, quite natural. There are, however, some differences especially in the code work since only fairly well advanced students are admitted whereas in our school students having no knowledge of telegraphy or radio apparatus as well as advanced students are enrolled under conditions suitable to all. Here in New York the value of the skilled operator is appreciated and in this particular district as well as possibly some others, efforts are made to obtain such men for the service, but this cannot be said of all districts. It has frequently been remarked to me, "We don't want our men to know too much about the apparatus"; and it has frequently been suggested to me by officials of a commercial company that technical training beyond that necessary for a man to slip by the government examination is not necessary and is, in fact, undesirable. When such a condition as this exists it is not likely that well trained men will be earnestly sought for operators' positions on steamships or in land stations. It is often the case that men who have had experience, no matter how poor telegraphers they may be or how limited their knowledge of radio apparatus, are placed in charge of ship or land stations when men more fit in every way are available. This I term a deplorable condition.

Mr. Sarnoff is quite right in his statements that experience is requisite for the highest efficiency, but the experience that a man acquires in a year or two on a ship fitted with a set of antiquated apparatus where he handles possibly one or two messages a week is not comparable with the experience that he obtains in a good school where he has modern apparatus to work with and every facility for mastering the technicalities of the radio service. Under the present methods of examinations for operators' licenses, it is a very simple matter for any telegrapher to secure a license, and provided he is in the employ of a commercial company during three months of the last six months of the life of his certificate, he will be issued a renewal license without examination thus making it necessary for him to be

actually engaged in radio service only three months in two years. During the life of his license he has no reason to endeavor to make himself more proficient or even to maintain whatever proficiency he may have had at one time. On the other hand, if operators were selected with care, and were examined from time to time by their employers as is done by other commercial institutions, there would be every inducement for them to make themselves as proficient as possible.

The plan followed by the Marconi Company of sending all new men out for a number of trips as second operator is, of course, a good one and one that would be expected. In many cases this is not possible hence it is the function of the radio telegraph school to meet the conditions of commercial service as nearly as possible. To facilitate this and gain the desired end, there must be the closest harmony between the commercial companies and those schools upon whom they depend for their operators. Every facility for properly training these men should be extended to such institutions, both as regards new apparatus and traffic methods.

THE IMPEDANCES, ANGULAR VELOCITIES AND FREQUENCIES OF OSCILLATING-CURRENT CIRCUITS*

By

A. KENNELLY

INTRODUCTION

It is the object of this paper to disclose a simple yet powerful proposition, recently discovered by the writer, which applies to transient currents, charges, discharges, or temporary disturbances, in electric circuits. This proposition, which is believed to be new, may be briefly stated in the following terms: The impedance of any closed circuit or group of circuits, to free oscillations, is zero.† The angular velocity, or velocities, of the oscillations are such as will bring about this condition.

SIMPLE RESISTANCELESS OSCILLATING-CURRENT CIRCUITS

The simplest type of oscillating-current circuit consists of a condenser in series with a reactor of negligible resistance, as indicated in Figure 1. It is known that the angular velocity of the free oscillations in this simple resistanceless circuit is such as

$$p_c = i^2 z_c = \mp i^2 j\sqrt{\frac{l}{c}} \qquad p_l = i^2 z_l = \pm i^2 j\sqrt{\frac{l}{c}} \quad \text{watts} \angle$$

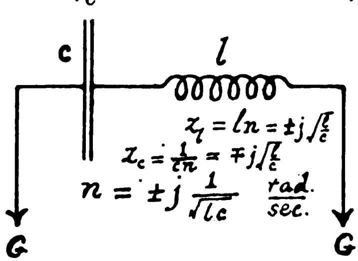


FIGURE 1—Simple Resistanceless Oscillating-Current Circuit of Capacitance and Inductance

* Presented before the Institute of Radio Engineers, New York, November 3, 1915. Manuscript received 30th August, 1915.

† See, however, a paper by George A. Campbell in "Trans. A. I. E. E.," April, 1911; on "Cisoidal Oscillations," Vol. XXX, part II, page 902, to which paper the attention of the present writer has been called since this paper was set in type.

(The notation used in this paper is tabulated at the end of the paper.—EDITOR.)

makes the total reactance zero.* Thus, if l is the inductance of the reactor in henrys, c the capacitance of the condenser in farads, ω the angular velocity of free oscillations thru the circuit, in radians per second, and $j = \sqrt{-1}$; then the reactance of the reactor at this velocity will be $j l \omega$ ohms, that of the condenser $\frac{1}{j c \omega}$ ohms, and the total reaction of the circuit will be $j l \omega + \frac{1}{j c \omega}$ ohms. Equating this total reactance to zero, we obtain:

$$l.j\omega + \frac{1}{c.j\omega} = 0 \qquad \dagger\text{ohms } \angle \quad (1)$$

$$(j\omega)^2 lc + 1 = 0 \qquad \textit{numeric } \angle \quad (2)$$

$$(j\omega)^2 = -\frac{1}{lc} \qquad \left(\frac{\text{radians}}{\text{second}}\right)^2 \angle \quad (3)$$

$$j\omega = \pm j \frac{1}{\sqrt{lc}} \qquad \frac{\text{radians}}{\text{second}} \angle \quad (4)$$

Selecting the positive sign, the reactance of the reactor is:

$$X_l = j\sqrt{\frac{l}{c}} = j l \omega \qquad \text{ohms } \angle \quad (5)$$

And that of the condenser is:

$$X_c = -j\sqrt{\frac{l}{c}} = \frac{1}{j c \omega} \qquad \text{ohms } \angle \quad (6)$$

Thus, a condenser of 0.01 microfarad ($c = 10^{-8}$), is connected in series with a reactor of 0.01 henry or 10 millihenrys ($l = 10^{-2}$). The angular velocity of the free oscillations of this circuit is $\omega = 10^5$ radians per second. The reactance of the reactor will then be $j1000$ ohms, and that of the condenser $-j\,1000$ ohms, making the total reactance zero.

It is furthermore known that if we count time t in seconds from a suitable epoch, either the instantaneous voltage of the condenser, or the instantaneous current in the reactor, may be represented by the instantaneous projection $O p$, of a vector $O P$, Figure 2, on a straight line of reference $X'O X$, the vector revolving with the angular velocity ω radians per second, and therefore describing in time t, a circular angle $X O P = \varepsilon^{j\omega t}$ radians; where ε is the Napierian base 2.71828. . . . Knowing the angular velocity ω, we can thus predict the electrical condition of the system at any assigned subsequent instant.

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*Bibliography (6) page 375.

† The angle sign $\angle$ attached to the unit of an equation indicates a complex quantity or "plane vector." By this means the use of special vector symbols in the equations is dispensed with. They are to be interpreted vectorially, or treated as complex quantities.

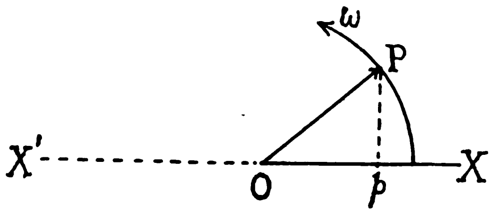

FIGURE 2—Vector Revolving with Angular Velocity ω. Its Projection Op indicates either the instantaneous voltage or the instantaneous current in System of Figure 1, according to the position of the Reference Line $X'OX$

The coefficient $j\omega$ of t, in the angular expression of the characteristic radius vector $\varepsilon^{\pm j\omega t}$, is thus the characteristic angular velocity for the oscillations of the system. The imaginary quantity $j\omega$ indicates oscillation, and is significant of angular velocity in a circle. The double sign indicates that either direction of rotation is possible, and that their sum* is equivalent to a sinusoidal quantity. But suppose that the coefficient of t in the general case is denoted by n; so that n is a generalized angular velocity, which may be either real, imaginary or complex; and so that e^{nt} is the characteristic angular exponential of the system at time t seconds, which determines the voltage or current then existing. Moreover, let us suppose that the impedance offered by an inductance l henrys to an electrical discharge of angular velocity n, is ln ohms, in general a complex quantity; while the impedance offered by a capacitance c farads to the same is $\frac{1}{cn}$ ohms. Then, for the case already considered of a simple resistanceless circuit containing a condenser and reactor in series, if the total impedance is to be zero, we have:

$$ln + \frac{1}{cn} = 0 \qquad \text{ohms} \angle \quad (7)$$

or

$$n^2 . lc = -1 \qquad \text{numeric} \angle \quad (8)$$

and

$$n = \pm\sqrt{\frac{-1}{lc}} = \pm j \frac{1}{\sqrt{lc}} \qquad \frac{\text{radians}}{\text{second}} \angle \quad (9)$$

But this is precisely the coefficient of t which we have found to exist in the simple resistanceless oscillating-current circuit.

According to the above assumptions, therefore, which we shall proceed to justify, an inductance of l henrys offers an impedance of ln ohms, to any generalized alternating current of generalized angular velocity n radians per second, of the type

* The sum of two oppositely directed rotations having the same frequency is well known to be a sinusoidally varying quantity in a straight line.

$a + j\,\omega$, where a is a real quantity, which may be regarded as a hyperbolic angular velocity, or uniform angular velocity in a hyperbola, expressible in hyperbolic radians per second;* while $j\,\omega$ is an "imaginary" angular velocity, which may be regarded as a circular angular velocity, or uniform angular velocity in a circle, expressible in circular radians per second. Similarly, the above assumptions lead to the conclusion that a capacitance of c farads, offers an impedance of $\frac{1}{c\,n}$ ohms to any generalized angular velocity of n radians per second. A pure resistance of r ohms, with negligible inductance or capacity, continues to offer an impedance of r ohms to oscillations of any angular velocity.

It will be noticed that in the case of any simple and sustained alternating current of angular velocity $j\,\omega$ circular radians per second, the assumption above stated reduces to the well known proposition that the impedance of an inductance l henrys is $j\,l\,\omega$ ohms; while that of a capacitance c farads is $\frac{1}{j\,c\,\omega}$ ohms. These impedances, whose sum is, in general, finite, then obey all the laws of direct-current resistances, following the rules of complex quantities. This proposition concerning sustained oscillations was discovered by the writer in 1893, and was first published by him in that year,† forming the basis of our ordinary complex algebra of the alternating-current circuit, in general use at the present day.

The new proposition, here presented, may be looked upon as an extension of the writer's earlier proposition, from sustained alternating currents or oscillations, to unsustained oscillations. From an algebraic standpoint, the value of n is extended from the pure imaginary quantity $j\,\omega$, to the complex quantity $a + j\,\omega$. Altho in engineering practice, sustained oscillations, or simple alternating currents, form the rule, and unsustained oscillations, or transients, form the exception; yet, from a physical point

* (It can be readily shown that, just as

$$\varepsilon^{j\omega t} = \cos \omega t + j \sin \omega t$$

so also

$$\varepsilon^{a t} = \cosh a t + \sinh a t.$$

There is therefore an analogy between the hyperbolic cosine (*cosh*) and the hyperbolic sine (*sinh*) and their corresponding trigonometric functions. Tables and charts permitting the ready use of the hyperbolic functions in engineering calculations have been published by Professor Kennelly, "Tables of Complex Hyperbolic and Circular Functions" and "Chart Atlas of Complex Hyperbolic and Circular Functions," Harvard University Press, Cambridge, Mass., 1914. —EDITOR.)

† Bibliography (2).

of view, the unsustained oscillations of complex angular velocity may be regarded as the general case, and the proposition of 1893, for sustained oscillations, reduces to a mere particular instance of the new proposition.

UNSUSTAINED OSCILLATIONS OF REAL ANGULAR VELOCITY

Condenser in Simple Circuit with Non-Inductive Resistance

Let a condenser of capacitance c farads, be connected in a simple circuit with resistance r ohms and negligible inductance as indicated in Figure 3. Let n be the angular velocity of the

$$p_c = i^2 z_c = -i^2 \times 10^6 \qquad p_r = i^2 z_r = i^2 \times 10^6 \text{ watts.}$$

$10^{-5} f$

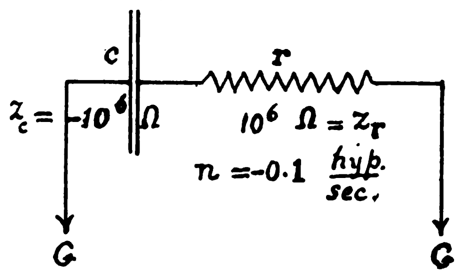

Figure 3—Simple Circuit of Capacitance and Resistance

discharge, as yet undetermined. Then, by assumption, $\frac{1}{cn}$ will be the impedance of the condenser, to this angular velocity, in ohms. The remaining impedance in the discharging circuit will be the resistance r ohms. The total impedance in the circuit will then be $\frac{1}{cn} + r$ ohms. But, by assumption, this total impedance of the circuit during free discharge must be zero: or

$$\frac{1}{cn} + r = 0 \qquad \text{ohms} \quad (10)$$

whence
$$n = -\frac{1}{cr} \qquad \text{hyps. per second} \quad (11)$$

That is, the angular velocity n of discharge is a negative real quantity, which may be regarded, therefore, as a negative angular velocity in a hyperbola, expressible in hyperbolic radians per second or, by abbreviation, in hyps. per sec. The angle described in t seconds of discharge will then be $n\,t$ or $\frac{-t}{cr}$ hyps., which is the

well known exponent of the discharge factor of the system, such that if U is the initial potential difference at condenser terminals, in volts, just before closing the circuit, the potential difference u remaining after t seconds, is

$$u = U\varepsilon^{nt} = U\varepsilon^{-\frac{t}{cr}} \qquad \text{volts} \quad (12)$$

As an example, we may consider a condenser of ten micro-farads ($c = 10^{-5}$), charged to a potential difference of 500 volts ($U = 500$), and allowed to discharge thru a non-inductive resistance of one megohm ($r = 10^6$). Then $n = -0.1$ hyp. per second. The hyperbolic angle described during 5 seconds would be -0.5 hyp. and the voltage across the condenser, remaining at that time, would be $500\varepsilon^{-0.5} = 303.3$ volts, the instantaneous current 0.3033 milliampere, and the instantaneous dissipated power 0.092 watt. The impedance offered by the condenser during discharge is -10^6 ohms, or one megohm negative. We may regard a negative resistance $(-r)$ in a discharge element as involving a dissipative *absorption* of power *into* the circuit of $-i^2r$ watts, under an instantaneous current strength of i amperes. This is just equal to the dissipative *liberation* of power *out of* the circuit of $+i^2r$ watts, in heat or in electromagnetic radiation. In any discharging oscillation system, the instantaneous sum of the negative or absorbed powers, and the positive or liberated powers is zero; or $\sum i_n^2 z_n = 0$ watts; where i_n is the instantaneous current in the oscillation impedance z_n of branch n. In general, the instantaneous power $i_n^2 z_n$ is a complex quantity. The real component is dissipative power. The imaginary component is reactive or non-dissipative power; i. e., the power of storing energy. In the case considered, the instantaneous power absorbed into the circuit from the dielectric of the condenser is -0.092 watt, and the instantaneous thermally liberated power in the resistance r is $+0.092$ watt.

The same conditions apply to the sustained oscillations of alternating-current circuits; but with $\alpha = 0$, negative real components of impedance and power do not present themselves.

Reactor in Simple Circuit with Non-Condensive Resistance

If a reactor of inductance l henrys, (Figure 4) is connected in a simple discharge circuit of total resistance r ohms, with negligible side-capacitance, then, according to assumption, the impedance of the reactor to any generalized angular velocity n, will be ln ohms, and the impedance of the remainder of the circuit will be r ohms; so that the total discharge impedance

$$p_l = i^2 z_l = -i^2 \times 10^2 \qquad p_r = i^2 z_r = i^2 \times 10^2 \text{ watts.}$$

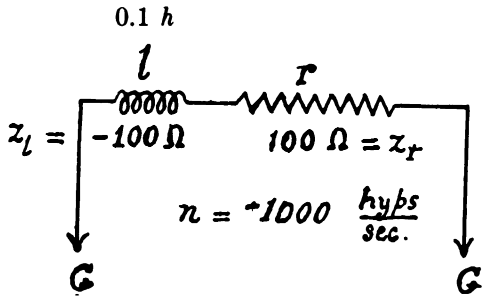

FIGURE 4—Simple Circuit of Inductance and Resistance

will be $l\,n + r$ ohms. If now the value of n so adjusts itself in free discharge that this sum total reduces to zero,

$$l\,n + r = 0 \qquad \text{ohms} \angle \quad (12a)$$

whence

$$n = -\frac{r}{l} \qquad \frac{\text{hyps.}}{\text{sec.}} \quad (13)$$

The angle described in t seconds of discharge will be $n\,t$ or $-\frac{r\,t}{l}$ hyps. which is the well known exponent of the discharge factor $\varepsilon^{-\frac{rt}{l}}$ of the system, such that if I is the initial current in the reactor just before discharge, the current remaining after t seconds is

$$i = I\,\varepsilon^{n\,t} = I\,\varepsilon^{-\frac{rt}{l}} \qquad \text{amperes} \quad (14)$$

Thus, if a reactor of inductance 100 millihenrys ($l = 0.1$) discharges thru a total resistance of 100 ohms ($r = 100$), with an initial current of 2 amperes, the angular velocity of discharge will be $n = -1000$ hyps. per second. The hyperbolic angle described in 0.5 millisecond will be -0.5 hyp. and the current flowing at that instant will be $2\,\varepsilon^{-0.5} = 1.213$ amperes. The discharge impedances of the reactor, assumed resistanceless, will be $l\,n = -100$ ohms. The instantaneous rate of dissipative energy absorption from the reactor's magnetic field into the circuit is $-100\,i^2 = -147.2$ watts, and the corresponding rate of dissipative energy liberation from the circuit in the resistor r is $+100 i^2 = +147.2$ watts. This power is affected with a damping factor $\varepsilon^{-\frac{2rt}{l}}$. There is no reactive power component and therefore no oscillatory storage of energy.

We have seen that our proposition leads to the deduction that the discharges of either condensers or reactors, thru simple resistances, involve real negative values of n; or what may be represented as uniform angular velocities in a hyperbola. We may consider briefly the geometry of this representation.

Let the rectangular coördinate axes OX, OY, Figure 5, be

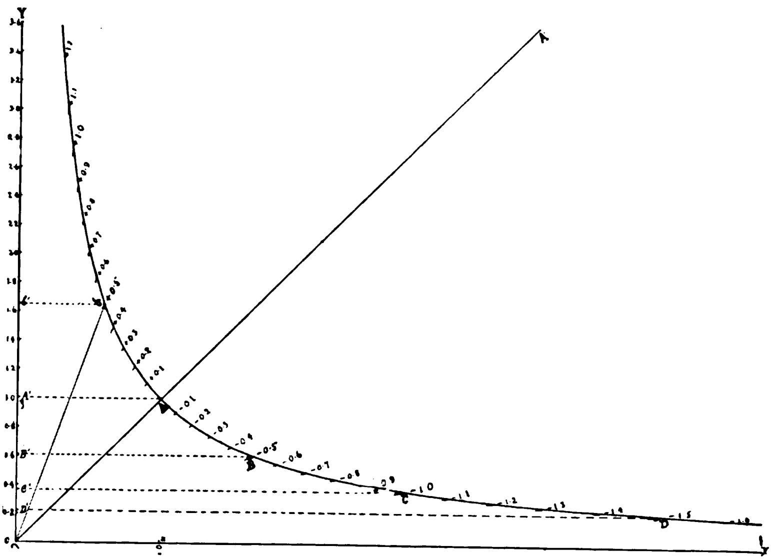

FIGURE 5—Rectangular Hyperbola with Successive Equal Increments of Hyperbolic Angle and Their Exponential Projections

the asymptotes of the rectangular hyperbola $c\ b\ A\ B\ C\ D$. Let this hyperbola have the axis OAA' and pass through the point A, whose coordinates are $x = 1, y = 1$. Then let a radius vector Ob, starting from the position OA, move with center O over the curve in the positive direction, so as to include an assigned hyperbolic angle θ, defined by the area of the sector OAb, these angles being marked off in Figure 5 along the curve. The ordinate Ob' of the extremity of the vector Ob, is known to be ε^{θ} units in length. In the case presented, $\theta = 0.5$ hyp., and $O\,b' = \varepsilon^{0.5} = 1.649$. In this sense, the value of ε^{θ} may be said to define the hyperbolic angle θ. Similarly, if starting from the initial position OA, the radius vector moves over the curve in a negative or clockwise direction, so as to occupy successively the positions OB, OC, $O\,D$, which include respectively -0.5, -1.0, and -1.5

hyps., the corresponding ordinates OB', OC', OD', measure $\varepsilon^{-0.5}$, $\varepsilon^{-1.0}$, and $\varepsilon^{-1.5}$ units. An exponential $\varepsilon^{-\theta}$, in this sense defines projectively a negative hyperbolic angle $-\theta$. As, therefore, a radius vector OB moves over the hyperbola with uniform hyperbolic velocity $+n$ hyps. per second, describing equal areas in equal times, the ordinate of the moving extremity of the radius vector follows the exponential ε^{+nt} units of length. Conversely, the expression ε^{-nt} may be interpreted geometrically as defining a radius vector which moves over a hyperbola with a uniform negative hyperbolic angular velocity $-n$ hyps. per second. The quantity $-n$ is often called a "damping constant" and the expression ε^{-nt} a damping factor, or damping coefficient; but the conception of n as an angular velocity seems better adapted for our present purposes.

Circular Angles and their Exponential Symbols

It is well known that the exponential quantity $\varepsilon^{j\beta}$ defines the position B (Figure 6) of a point situated in a plane, and on a

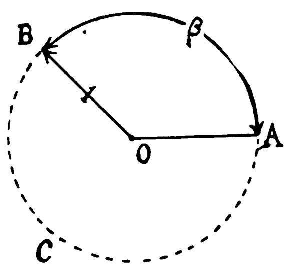

Figure 6—Representation of the Exponential $\varepsilon^{j\beta}$

circle $A\ B\ C$ of unit radius, which is at a distance of β units of length from A, the initial point on the circle. It therefore likewise defines the number of circular radians β, which the radius vector OB has described in its rotation or rotations, about the center O, from the initial position $O\ A$. The exponential $\varepsilon^{j\beta}$ thus defines the total circular radians described in passing from OA to OB. Consequently the exponential ε^{jnt} may be interpreted geometrically as defining a radius vector which moves over a circle of unit radius with a uniform circular angular velocity n circular radians per second.

Complex Angular Velocities and Their Exponential Symbols

Since $\varepsilon^{(\pm a \pm j\omega)t} = \varepsilon^{\pm at} \times \varepsilon^{\pm j\omega t}$, it follows that the exponential $\varepsilon^{(\pm a \pm j\omega)t}$ may be interpreted as representing the product of two

angles, each increasing uniformly with time, one a plus or minus hyperbolic angle, and the other a plus or minus circular angle. Or, we may interpret it as a radius vector rotating in a plane with uniform circular angular velocity $\pm \omega$ circular radians per second, the radius vector at the same time changing in length, in projective accordance with uniform hyperbolic angular velocity $\pm \alpha$ hyperbolic radians per second. The path of such a moving point is known to be an equiangular spiral. Such angular velocity may be described as complex, being defined by the complex quantity $(\pm \alpha \pm j \omega)$ radians per second.

Proof of Oscillation Impedance Theorems

The following considerations will probably suffice to establish the three propositions: (1) that the oscillation impedance of an inductance l henrys is $l\,n$ ohms; (2) that the oscillation impedance of a capacitance c farads is $1/(c\,n)$ ohms; (3) that the total oscillation impedance of a circuit or path to free oscillation is zero.

(1) Let pure inductance of l henrys, assumed devoid of either resistance or capacitance, carry an instantaneous oscillating current of i amperes, which obeys the law

$$i = I\,\varepsilon^{nt} \qquad \text{amperes} \angle \quad (15)$$

where I is an initial current at time $t = 0$ and $n = -(\alpha \pm j\,\omega)$, a generalized angular velocity. Then the instantaneous back emf. of self induction, opposed to the current, is:

$$-e_l = -l\frac{d\,i}{d\,t} = -l\,I\,n\,\varepsilon^{nt} = -l\,n\,i \qquad \text{volts} \angle \quad (16)$$

The instantaneous driving emf. which is therefore necessary to overcome this back emf. is:

$$e_l = l\,n\,i \qquad \text{volts} \angle \quad (17)$$

The instantaneous apparent resistance offered by the inductance at its terminals is then:

$$z_l = \frac{e_l}{i} = l\,n \qquad \text{ohms} \angle \quad (18)$$

(2) Let a pure capacitance of c farads carry the same instantaneous oscillating current i, above considered under (1). Then the instantaneous back emf. of condensance, opposing the current, is:

$$-e_c = -\frac{1}{c}\int i\,d\,t = -\frac{1}{c\,n}I\,\varepsilon^{nt} = -\frac{i}{c\,n} \qquad \text{volts} \angle \quad (19)$$

The instantaneous driving emf. to overcome this is:

$$e_c = \frac{i}{c\,n} \qquad \text{volts } \angle \quad (20)$$

The instantaneous apparent resistance of the capacitance is then:

$$z_c = \frac{e_c}{i} = \frac{1}{c\,n} \qquad \text{ohms } \angle \quad (21)$$

(3) At any instant the total driving emf. of disturbed energy must be equal to the total back emf. in the circuit, including $r\,i$ drop in the circuit. Otherwise the unsatisfied driving emf. would create a greater current than actually flows at the instant considered. That is, at each and every instant,

$$i\,.\,\Sigma z = 0 \qquad \text{volts } \angle \quad (22)$$

where $i\,.\,\Sigma z$ signifies the vector sum of all the oscillation impedances drops in the circuit, including simple ohmic drops of the type $i\,r$ volts.

Dividing (22) by i, we obtain:

$$\Sigma z = 0 \qquad \text{volts } \angle \quad (23)$$

Or the sum of all the oscillation impedances in the path of the current i is zero at every instant.

In any closed loop of an oscillating-current system, the total instantaneous emf., including $i\,r$ drops, must be zero; or, if z_n is the oscillation impedance of conductor n carrying instantaneous current i_n and forming part of a closed loop, $\Sigma\, i_n\, z_r = 0$ volts. In the case of sustained oscillations, or impressed alternating currents ($a = 0$), this reduces to the extended complex or two-dimensional form of Kirchoff's loop-voltage law* first published by Steinmetz in 1893.

Case of a Simple Circuit of Capacitance, Inductance and Resistance

We may now proceed to consider more complicated cases of unsustained oscillations. In Figure 7, the reactor of inductance l henrys is in series with a total resistance of r ohms (including that within the reactor) and the condenser of capacitance c farads. If n is the generalized angular velocity of unsustained oscillation, the condenser will have an impedance of $1/(c\,n)$ ohms, and the reactor an impedance of $l\,n$ ohms. Equating the total impedance to zero, we have:

$$\frac{1}{c\,n} + r + l\,n = 0 \qquad \text{ohms } \angle \quad (24)$$

* Bibliography (3).

or $$n^2 . cl + ncr + 1 = 0 \qquad \text{numeric } \angle \tag{25}$$

whence $$n = -\frac{r}{2l} \pm \sqrt{\left(\frac{r}{2l}\right)^2 - \frac{1}{cl}} \qquad \frac{\text{hyp. radians}}{\text{second}} \tag{26}$$

or $$n = -\frac{r}{2l} \pm j\sqrt{\frac{1}{cl} - \left(\frac{r}{2l}\right)^2} \qquad \frac{\text{radians}}{\text{second}} \angle \tag{27}$$

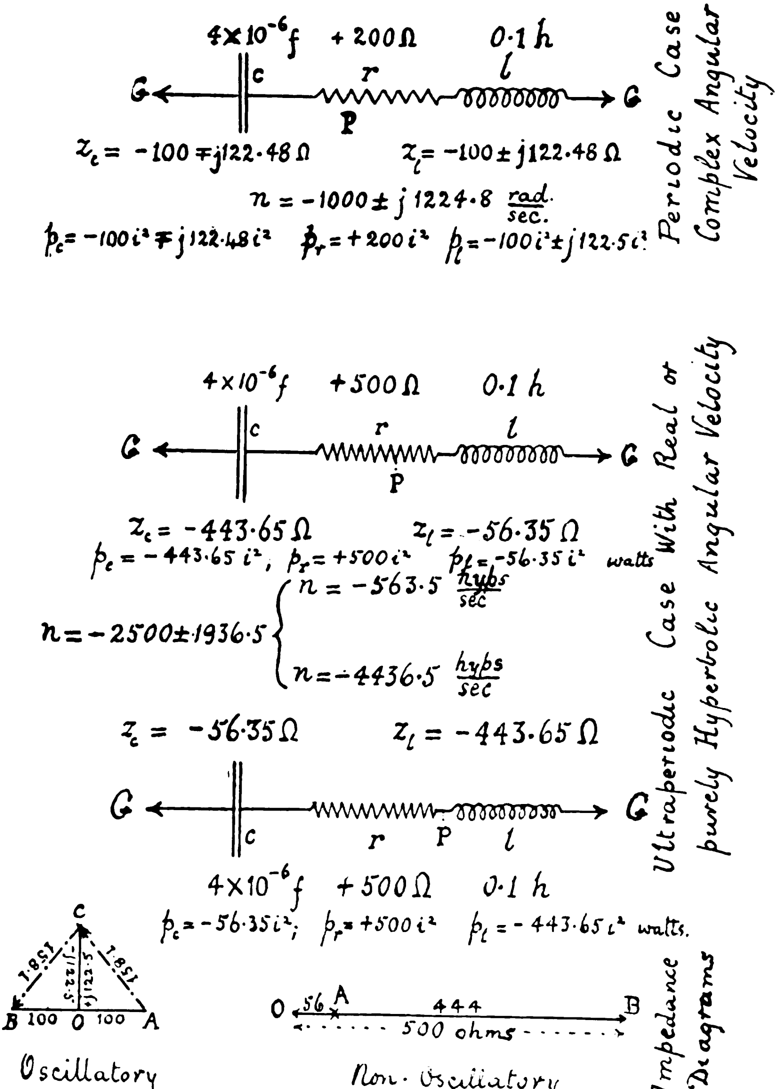

FIGURE 7—Condenser and Resistive Reactor. Periodic and Ultraperiodic Cases

according as $\left(\frac{r}{2l}\right)^2$ is greater or less than $\frac{1}{cl}$. In the former case, the discharge is ultraperiodic, and the angular velocity wholly hyperbolic.† In the latter case, the discharge is periodic, and the angular velocity is partly hyperbolic and partly circular. In the intermediate condition, with $\frac{r}{2} = \sqrt{\frac{l}{c}}$, the discharge is aperiodic. If we denote $r/(2l)$ by a, and $\sqrt{\frac{1}{cl} - \left(\frac{r}{2l}\right)^2}$ by ω, then, in the periodic case,

$$n = -a \pm j\,\omega \qquad \frac{\text{radians}}{\text{second}} \angle \quad (28)$$

As before, the dual sign of the circular angular velocity $j\,\omega$ indicates that either direction of rotation may be adopted, and the sum of two opposite rotations gives rise to a sinusoidal quantity.

The quantity of electricity, q coulombs, in the condenser at any instant t seconds from the beginning of the discharge is known to be:

$$q = A\,\varepsilon^{(-a+j\omega)t} + B\,\varepsilon^{(-a-j\omega)t} \qquad \text{coulombs} \quad (29)$$

where A and B are arbitrary constants depending upon the initial conditions. This general and well known result was first published in 1853* by Lord Kelvin, from an analysis based on energy relations. It is evident that the impedance equation (24) leads directly to the angular velocity of discharge. The oscillation frequency is $f = \frac{\omega}{2\pi}$ cycles per second, and the damping constant $-a$.

As an example, let $c = 4 \times 10^{-6}$ farad, $l = 0.1$ henry, and $r = 200$ ohms. Then $n = -1000 \pm j\,1224.75$ radians per second, and the frequency of oscillation is $f = \frac{1224.75}{2\pi} = 194.92$ cycles per second, accompanied by a damping constant of 1000; or a damping factor of ε^{-1000t}. The discharge impedance, or oscillation impedance of the reactor, assumed resistanceless, is $-100 \pm j\,122.48$ ohms, and that of the condenser $1/(c\,n) = -100 \mp j\,122.48$ ohms. If the resistance $r = 0$; or were entirely removed from the circuit, the angular velocity would, by (9), become sustained

† Bibliography (5).
* Bibliography (1).

at $n = \pm j\,1581.1$ radians per second, the impedance of the reactor $\pm j\,158.11$ ohms, and that of the condenser $\mp j\,158.11$ ohms. The frequency would then be sustained at $1581.1/(2\pi)$ or 251.65 cycles per second. If this frequency were sustained by an independent alternator or impressing source, only the upper signs would be applicable under international notation; i. e., the reactor's impedance would be $+j\,158.11$ and the condenser's impedance $-j\,158.11$ ohms. The dual signs presenting themselves in the solutions of free oscillations may be attributed to the absence of an independent source of impressed current. Either the condenser or the reactor may become the source of discharges, and either direction of current the direction of reference. With this understanding, the dual signs of imaginary (circular) angular velocities need give rise to no ambiguity or uncertainty.

If the resistance r of the circuit were increased to say 500 ohms, then (26) would apply, and $n = -2500 \pm 1936.5$ radians per second $= -563.5$ or -4436.5 hyps. per second. There are thus two hyperbolic angular velocities present, and two damping factors, $\varepsilon^{-563.5t}$ and $\varepsilon^{-4436.5t}$. The impedance of the reactor to the lower angular velocity is shown in Figure 7, to be -56.35 ohms, and that of the condenser -443.65. At the higher velocity, these values interchange, the reactor taking -443.65, and the condenser -56.35 ohms. In the complete analysis of this ultraperiodic case, it is optional either to assign a certain share of the discharge to each independent hyperbolic angular velocity: or to combine them into the single hyperbolic angular velocity 1936.5 hyps. per second, associated with the damping factor ε^{-2500t}. The results in either case are the same.*

Combination of Condensers and Reactors in Series Circuit

If a circuit contains a plurality of condensers in simple series with a plurality of reactors and resistances, the angular velocity of disturbance in the circuit is readily found.

Let $C_1, C_2, C_3 \ldots$ be the respective capacitances in the circuit (farads).

Let $l_1, l_2, l_3 \ldots$ be the respective inductances in the circuit (henrys).

Let $r_1, r_2, r_3 \ldots$ be the respective resistances in the circuit (ohms).

Let the capacitance-reciprocals, or elastances, of the condensers be found, $s_1 = 1/c_1$, $s_2 = 1/c_2$, $s_3 = 1/c_3 \cdots$ These may be ex-

* See Bibliography (6), Page 411, for a more detailed analysis of this case.

pressed in darafs. Then the total elastance of the circuit is $S = s_1 + s_2 + s_3 + \cdots$ darafs. The total inductance is $L = l_1 + l_2 + l_3 + \cdots$ henrys, and the total resistance $R = r_1 + r_2 + r_3 + \cdots$ ohms. Then the oscillation impedance of the total elastance is $\frac{S}{n}$ ohms, that of the total inductance Ln ohms, and of the total resistance R ohms.

Consequently $$\frac{S}{n} + Ln + R = 0 \qquad \text{ohms} \angle \quad (30)$$

whence, as in (27), $$n = -\frac{R}{2L} \pm j\sqrt{\frac{S}{L} - \left(\frac{R}{2L}\right)^2} \qquad \frac{\text{radians}}{\text{second}} \angle \quad (31)$$

assuming that the resistance R is less than $2\sqrt{LS}$ ohms; i. e., that the disturbance is oscillatory; otherwise the roots of (30) are real, as in (26).

As an example let $S = 25$ darafs; $L = 0.1$ henry; $R = 200$ ohms; then $n = -1000 \pm j\,1224.75$ radians per second.

Oscillation Angular Velocity of Resistanceless Discharging Elements in Parallel

The simplest case of discharging elements in parallel, producing oscillations, is perhaps that indicated in Figure 8. A discharging element may be defined as an element capable of containing electromagnetic energy, and therefore capable of having the amount of its energy content disturbed. A discharging element may therefore be a reactor of inductance l henrys, which may contain magnetic energy of $li^2/2$ joules, when traversed by a current of i amperes. It may also be a condenser, of capacitance c farads, which may contain electric energy of $ce^2/2$ joules, when charged to a potential difference of e volts. The oscillations here considered may be those accompanying either an increase, or a decrease of energy in any element; i. e. accompanying either charge or discharge; but discharge is the easier phenomenon to analyze; because in charge, a final steady state has ordinarily to be superposed upon that transient state of disturbance which is the immediate subject of discussion. We may, therefore, confine our discussion to cases of discharge, with the understanding that the results apply also, with reversal of currents and powers, to cases of charge, if the subsequent steady state is independently superposed.

In Figure 8, let a number of condensers of capacitances $c_1, c_2, c_3 \ldots$ farads, respectively, be connected in parallel to common bus-bars BB', bb'. Let any number of reactors be also

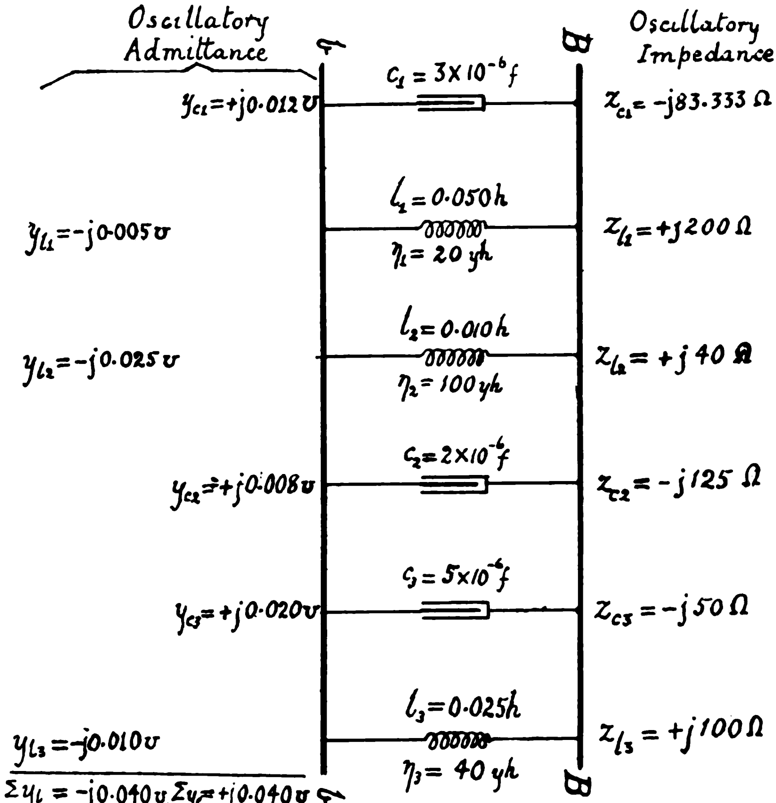

Figure 8—Parallel Connection of a Number of Discharge Elements, with Negligible Resistance

connected in parallel to the same bars, and let the resistance of each and all the elements of the circuit be negligibly small. When reactors of inductances l_1, l_2, l_3 . . . henrys are connected in parallel, it is convenient to use the reciprocals of these values $\eta_1 = 1/l_1$, $\eta_2 = 1/l_2$, $\eta_3 = 1/l_3$, . . . for arithmetical purposes. These reciprocal inductances may be called *ductances*, for want of a better term. A ductance may be expressed in (henrys)$^{-1}$; or, as Karapetoff suggested, say, in yrnehs. An inductance of 0.1 henry is therefore a ductance of 10 yrnehs. The total ductance of a number of ductances in parallel is then their numerical sum, just as the total capacitance of a number of capacitances in parallel is their numerical sum.

It is evident that the system of Figure 8, assumed resistanceless, is equivalent to a single condenser of capacitance $C = c_1 +$

$c_2 + c_3 +$ farads, in simple series with a single ductance $E = \gamma_1 + \gamma_2 + \gamma_3 +$ yrnehs. The case of Figure 8 thus reduces to that of Figure 1, with C in place of c farads, and $1/E$ in place of l henrys. The discharge impedance of the combined condenser is then $1/(C\,n)$ ohms, and that of the combined ductance n/E ohms. Consequently

$$\frac{1}{C\,n} + \frac{n}{E} = 0 \qquad \text{ohms } \angle \quad (32)$$

and

$$n^2 C + E = 0 \qquad \text{yrnehs} \quad (33)$$

or

$$n = \pm j\sqrt{\frac{E}{C}} \qquad \text{radians/sec. } \angle \quad (34)$$

which result is in agreement with (9), and is almost self-evident in view of (9). Our proposition states, however, that the discharge impedance of the system to any element must be zero. Consider the element c_1 as having its energy suddenly disturbed, and as discharging thru the rest of the system. The discharge impedance of c_1 is $1/(c_1 n)$ ohms. That of the remaining condensers is $1/(C - c_1)\,n$ ohms, and that of the ductances n/E ohms, as already considered. The remaining condensers are in parallel with the ductances, and their joint impedance must be taken in relation to c_1; so that

$$\frac{1}{c_1 n} + \frac{\frac{n}{E} \times \frac{1}{n(C - c_1)}}{\frac{n}{E} + \frac{1}{n(C - c_1)}} = 0 \qquad \text{ohms } \angle \quad (35)$$

from which

$$n^2 = -\frac{E}{C} \qquad \left(\frac{\text{radians}}{\text{sec.}}\right)^2 \angle \quad (36)$$

or

$$n = \pm j\sqrt{\frac{E}{C}} \qquad \frac{\text{radians}}{\text{sec.}} \angle \quad (37)$$

This is the same result as was reached in (34). It means that the angular velocity of discharge oscillations is the same in each individual condenser as in the system as a whole; so that there is one and only one oscillation frequency $f = n/(j\,2\pi)$ cycles per second. Moreover, this frequency is such that the impedance of the system is zero, taking each condenser in turn as the main path of discharge.

Similarly, taking any one ductance, say γ_1, as the main path of discharge, this element only having its energy suddenly disturbed; then its impedance is n/γ_1 ohms, and that of the remaining ductances, in parallel, $n/(E - \gamma_1)$ ohms.

Consequently

$$\frac{n}{\gamma_1} + \frac{\dfrac{n}{E-\gamma_1} \times \dfrac{1}{nC}}{\dfrac{n}{E-\gamma_1} + \dfrac{1}{nC}} = 0 \qquad \text{ohms} \angle \quad (38)$$

from which
$$n^2 = -\frac{E}{C} \qquad \left(\frac{\text{radians}}{\text{second}}\right)^2 \angle \quad (39)$$

or
$$n = \pm j\sqrt{\frac{E}{C}} \qquad \frac{\text{radians}}{\text{second}} \angle \quad (40)$$

again the same result as in (34) and (37). There is thus one and the same oscillation frequency in all branches of the system. If resistances are injected into the various branches, this simple relation is destroyed, and altho the same principles and method of procedure apply, the result is usually an equation of the nth degree, for n discharging elements, giving on solution, n roots, every one root corresponding to the angular velocity of each discharge element, considered in turn as the main path. The number of distinct oscillation frequencies is, however, usually distinctly less than n; because each pair of conjugate complex roots gives rise to but a single oscillatory frequency.

If we take as an example the following values:— $c_1 = 3 \times 10^{-6}$, $c_2 = 2 \times 10^{-6}$, $c_3 = 5 \times 10^{-6}$, $C = 10^{-5}$, $\gamma_1 = 20$, $\gamma_2 = 100$, $\gamma_3 = 40$, $E = 160$; then $n = \pm j\,4000$ circular radians per second, and the oscillation frequency $f = 4000/(2\pi) = 636.6$ cycles per second. The impedances and admittances of the various elements at this frequency are indicated in Figure 8, just as if the frequency were independently sustained in an alternating-current circuit. It may be observed that the total admittance of the branches of the system is zero, and this we shall find to be a general law, whether resistances are present in the various branches, or not.

Condenser, Reactor, and Resistance, in Star Connection

We may next consider the case represented in Figure 9, of a condenser, reactor and non-inductive resistance in star connection; or, what is of course the same, connected in parallel between bus-bars. Here we have two discharge elements and an inert or energyless resistance leak, all in parallel. It is optional to consider either discharge element as the main path and the two others, in joint connection, closed on it. Let c be the capacitance in farads, l the inductance in henrys containing a resistance of r ohms, and g the conductance of the leak in

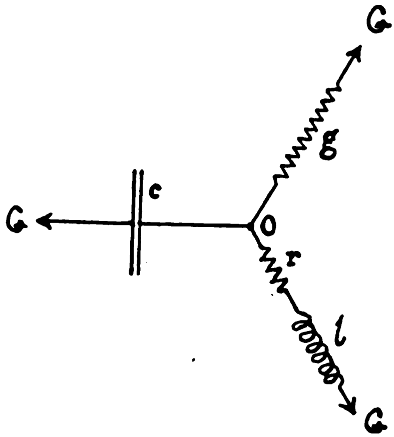

FIGURE 9—Fleming's Case of a Condenser c Shunted by a Leak g and Discharging thru a Reactor l

mhos. Then taking the condenser as the main path, we obtain:—

$$\frac{1}{cn} + \cfrac{1}{g + \cfrac{1}{r + ln}} = 0 \qquad \text{ohms} \angle \quad (41)$$

whence $\quad n^2 c\, l + n\,(c\,r + g\,l) + (1 + g\,r) = 0 \qquad$ numeric $\angle$ (42)

$$\text{and } n = -\left(\frac{r}{2l} + \frac{g}{2c}\right) \pm j\sqrt{\frac{1+gr}{cl} - \left(\frac{r}{2l} + \frac{g}{2c}\right)^2} \quad \frac{\text{radians}}{\text{second}} \angle \quad (43)$$

$$= -\left(\frac{r}{2l} + \frac{g}{2c}\right) \pm j\sqrt{\frac{1}{cl} - \left(\frac{r}{2l} - \frac{g}{2c}\right)^2} \qquad \text{“} \quad \angle \quad (44)$$

Formula (44) was derived by Fleming, from a different method, in 1913.* If we prefer to take the reactor as the main path of discharge: then

$$l\,n + r + \frac{1}{c\,n + g} = 0 \qquad \text{ohms} \angle \quad (45)$$

whence $n^2 c\, l + n\,(c\,r + g\,l) + (1 + g\,r) = 0 \qquad$ numeric $\angle$ (46)

which is identical with (42) and therefore leads to the same result.

In view of the practical instance cited by Fleming, no example of this case needs to be discussed arithmetically.

Two Resistive Reactors and a Condenser, in Star or Parallel Connection

In the case presented in Figure 10, we have three discharge elements in parallel, two of them reactors of l_1 and l_2 henrys,

* Bibliography (7).

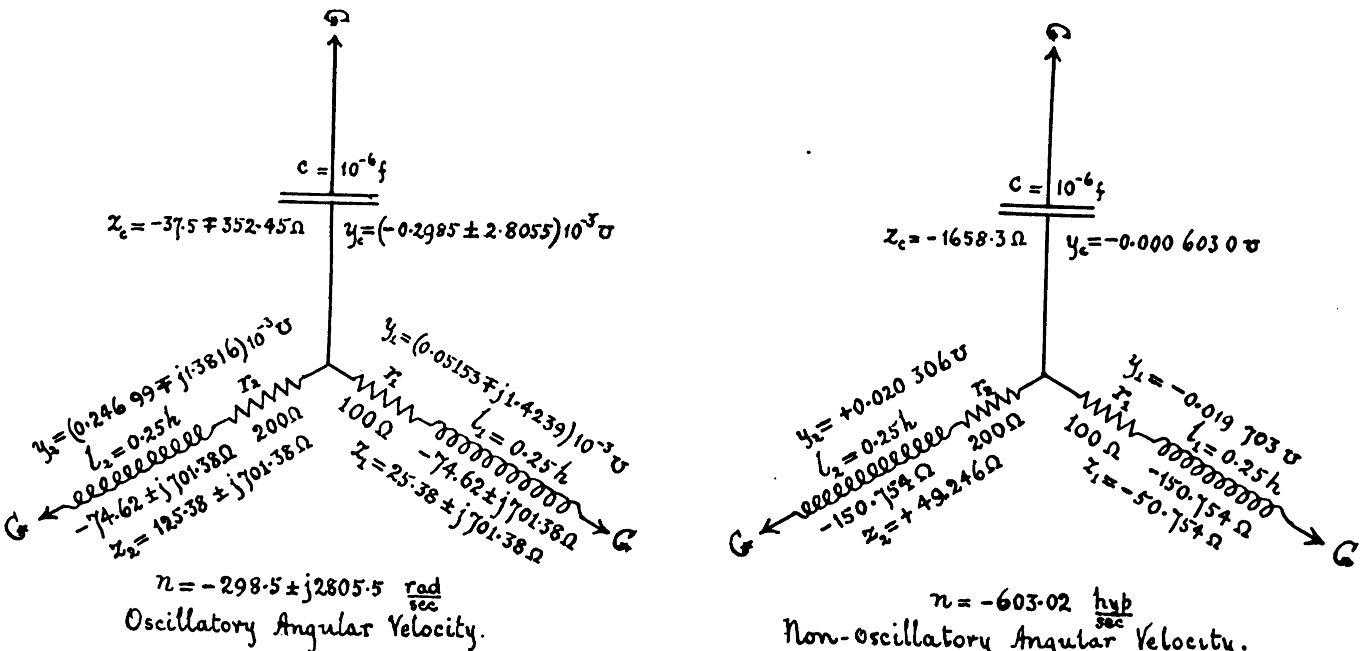

FIGURE 10—Case of Condenser Connected to Pair of Unlike Reactors in Parallel

r_1 and r_2 ohms respectively, and the third a condenser of capacitance c farads. It is a matter of indifference which element we take as the main path of discharge; but taking the condenser element, we have:

$$\frac{1}{cn} + \frac{(r_1 + l_1 n) \times (r_2 + l_2 n)}{(r_1 + l_1 n) + (r_2 + l_2 n)} = 0 \qquad \text{ohms} \angle \quad (47)$$

whence

$$n^3 + n^2\left(\frac{r_1}{l_1} + \frac{r_2}{l_2}\right) + n\left(\frac{1}{c\,l_1} + \frac{1}{c\,l_2} + \frac{r_1 r_2}{l_1 l_2}\right) + \frac{r_1 + r_2}{c\,l_1 l_2} = 0 \qquad \left(\frac{\text{radians}}{\text{second}}\right)^3 \angle \quad (48)$$

a cubic equation having one real and two complex roots. There are thus three values of the angular velocity n, which, substituted in (47), will enable that equation to hold. The real value may be regarded as pertaining to the discharge from one reactor thru the other, and thru the resistance $r_1 + r_2$, in their circuit. The two conjugate complex values may be regarded as pertaining to the discharge from the condenser into a certain single resistance and inductance, equivalent to the pair of parallel reactors.

As an example, we may take $c = 10^{-6}$ farad, $r_1 = 100$ ohms, $l_1 = 0.25$ henry, $r_2 = 200$ ohms, $l_2 = 0.25$ henry. Then (48) becomes

$$n^3 + 1200\,n^2 + 8.32 \times 10^6\,n + 4.8 \times 10^9 = 0 \qquad \left(\frac{\text{radians}}{\text{second}}\right)^3 \quad (49)$$

This equation may be solved by the use of an auxiliary hyperbolic angle in the well known manner; but it is easy to find the roots by first plotting the value of (49), as ordinates, against arbitrarily selected values of n, as abscissas, in the regular way, as indicated in Figure 11, which shows that the graph passes through the zero line of ordinates near $n = -600$. A few more arithmetical trials, close to this value of n, will give a more nearly correct value of -603.02. This is the numerical value of the real root. Dividing (49) by $(n + 603.02)$, we obtain as the quotient:

$$n^2 + 596.98\,n + 7.96 \times 10^6 = 0 \qquad \left(\frac{\text{radians}}{\text{second}}\right)^2 \angle \quad (50)$$

an ordinary quadratic equation, of which the solution is:—

$$n = -298.49 \pm j\,2805.5 \qquad \frac{\text{radians}}{\text{second}} \angle \quad (51)$$

This equation gives the two remaining complex roots of (49).

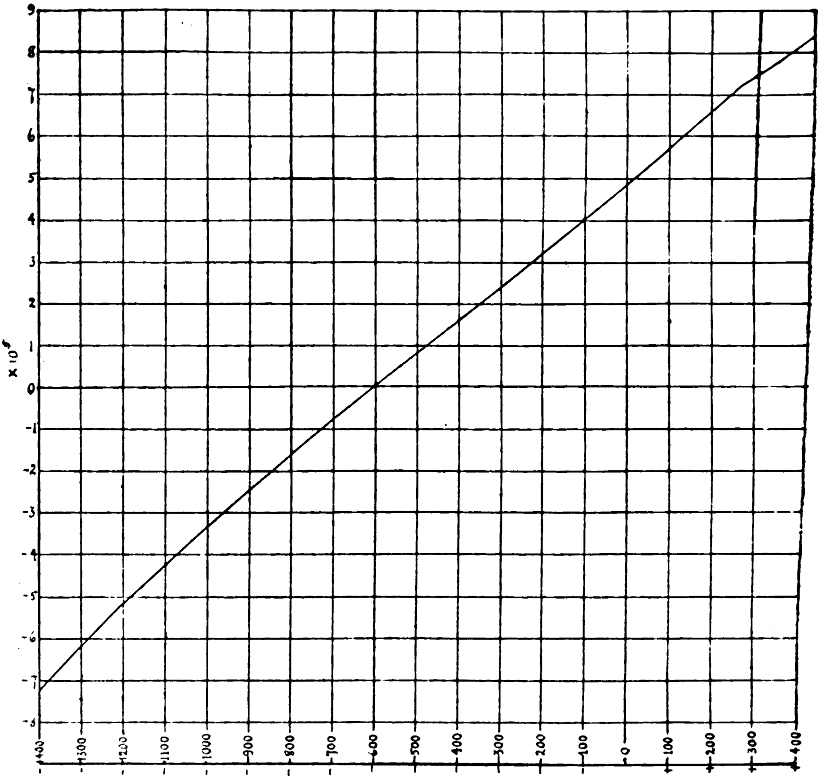

FIGURE 11—Graph of Expression $n^3 + 1200\,n^2 + 8.32 \times 10^6\,n + 4.8 \times 10^9$ Between the Values $n = -1400$ and $n = +400$

If the condenser were disconnected from the system, leaving the two reactors connected thru $r_1 + r_2 = 300$ ohms, we find, by (13), that their free angular velocity would be $n = -600$ hyps. per second; so that the presence of the condenser merely modifies this to -603.02. Again, if the two resistances were removed ($r_1 = r_2 = 0$), leaving the condenser in series with the reactor, we find, by (9), that the free angular velocity would be $n = \pm j$ 2828.4 radians per second. The presence of the resistances reduces this to $-298.49 \pm j\,2805.5$.

The system of Figure 10 in the example considered, thus dissipates disturbance energy in two different modes. One is a non-oscillatory discharge of -603.02 hyps. per second, or accompanied by a damping factor of $\varepsilon^{-603.02t}$. This is the discharge between the two reactors, slightly modified by the presence of the condenser. The other is an oscillatory discharge

of angular velocity $-298.49 \pm j\,2805.5$ radians per second, having a frequency of $2805.5/(2\pi) = 446.5$ cycles per second, accompanied by a damping constant of 298.49, or a damping factor of $\varepsilon^{-298.49t}$. This damped oscillatory discharge is between the condenser and the joint ductance, as modified by the presence of the resistances.

It is shown in Figure 10 that at $n = -603.02$, the condenser has an impedance of -1658.3 ohms, one reactor -50.754 ohms, and the other $+49.246$. Taking the admittances, or reciprocals of these quantities, the condenser has -0.60302 millimhos, one reactor -19.703 millimhos, and the other $+20.306$. The sum of these admittances is zero.

Similarly, at $n = -298.49 \pm j\,2805.5$ radians per second, Figure 10 shows that the sum of the three branch admittances is zero. We may proceed to establish this proposition generally.

The Sum of the Oscillation Admittances About any Branch Point is Zero

In Figure 12, a number of branches, to ground or common connection, meet at the point O Each branch may contain a

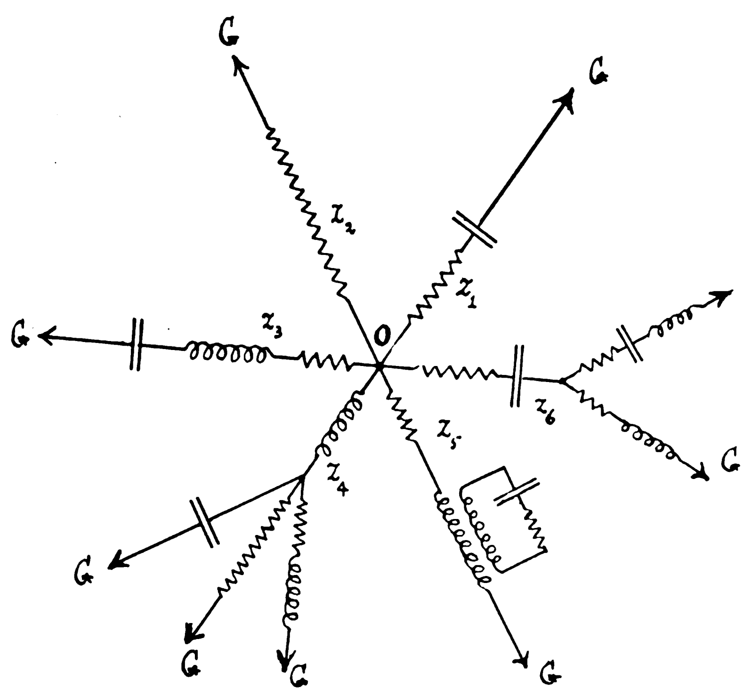

FIGURE 12—Group of Oscillatory Impedances meeting at a Knot Point O

plurality of discharge elements, or resistances, or sub-branches. Then let the discharge impedance of these branches be z_1, z_2, z_3 . . . etc., each being formed on the understanding that an inductance has ln ohms, and a capacitance $1/(c\,n)$ ohms, n being the subsequently determined generalized angular velocity. Then, in order that the total discharge impedance in the path of any one branch, say z_1, shall be zero, we must have:

$$z_1 + \cfrac{1}{\cfrac{1}{z_2} + \cfrac{1}{z_3} + \cfrac{1}{z_4} + \,.\,.} = 0 \qquad \text{ohms} \angle \quad (52)$$

or if $y_1 = 1/z_1$, $y_2 = 1/z_2$, $y_3 = 1/z_3$, $y_4 = 1/z_4$. . are the respective discharge admittances,

$$z_1 + \frac{1}{y_2 + y_3 + y_4 + \,.\,.} = 0 \qquad \text{ohms} \angle \quad (53)$$

whence $$y_2 + y_3 + y_4 + \quad = - y_1 \qquad \text{mhos} \angle \quad (54)$$

or $$y_1 + y_2 + y_3 + y_4 + \quad = 0 \qquad \text{"} \quad (55)$$

or $$\Sigma y = 0 \qquad \text{"} \quad (56)$$

This relation must hold for each and all values of n which may satisfy (52). It applies not merely to a subdivided circuit: but also to a single undivided circuit; such as that in Figure 7, if any available point P be selected as a branch point of two branches. Knot-point cases* may often be solved advantageously by using this rule.

A number of less simple oscillating-current networks have been worked out by the methods here presented, and checked by independent means. No discrepancies have yet been found.

Inductively Coupled Circuits

If two circuits are inductively coupled by a mutual inductance μ henrys, as in Figure 13, the primary having constants c_1, l_1, r_1, and the secondary c_2, l_2, r_2, it was shown by the writer in 1893† that the impedance z'_{12} ohms of the primary circuit to sustained oscillations in the presence of the closed secondary circuit, is:

$$z'_{12} = z'_1 - \frac{(\mu j \omega)^2}{z'_2} \qquad \text{ohms} \angle \quad (57)$$

*Thus, the case presented in Figure 10, with equation (47), may be stated as follows:—

$$c\,n + \frac{1}{r_1 + l_1 n} + \frac{1}{r_2 + l_2 n} = 0 \qquad \text{mhos} \angle \quad (56a)$$

which reduces to (48).

†Bibliography (4).

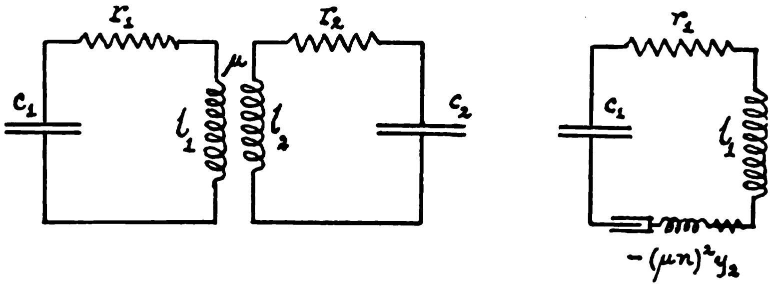

Figure 13—Pair of Inductively Connected Oscillatory Circuits and the Equivalent Single Primary Circuit

where z'_1 is the impedance of the primary circuit with the secondary circuit open, z'_2 the impedance of the secondary circuit with the primary circuit open (ohms $\angle$), and ω is the angular velocity of the impressed alternating current, in radians per second. Knowing the impedance of the primary circuit from (57), the current in that circuit to any impressed alternating emf. is immediately obtained. The emf. induced in the secondary circuit is then found by multiplying the primary current with $-\mu j \omega$ ohms.

The corresponding rule for free oscillations of generalized angular velocity n radians per second is:

$$z_{12} = z_1 - \frac{(\mu n)^2}{z_2} = z_1 - \mu^2 n^2 y_2 \qquad \text{ohms } \angle \quad (58)$$

so that, considering the primary circuit as the discharging circuit, with zero oscillation impedance, z_{12} must be equated to zero; or:

$$z_1 - \frac{(\mu n)^2}{z_2} = 0 \qquad \text{ohms } \angle \quad (59)$$

is the condition for determining n, it being understood that z_1 is the impedance of the primary circuit to angular velocity n, when the secondary circuit is open, and z_2 the impedance of the secondary circuit to angular velocity n, when the primary circuit is open.

The proposition may be proved as follows:

Let the instantaneous oscillating current i_1 in the primary circuit follow the law

$$i_1 = I_1 \varepsilon^{nt} \qquad \text{amperes } \angle \quad (60)$$

where n is a generalized or complex angular velocity, and I_1 the initial value of the primary current when $t = 0$. Then the

instantaneous induced emf. in the secondary circuit will be

$$e_2 = -\mu \frac{d\,i_1}{d\,t} = -\mu\, n\, I_1\, \varepsilon^{nt} = -\mu\, n\, i_1 \qquad \text{volts } \angle \quad (61)$$

and if the oscillation impedance of the secondary circuit is z_2 ohms, the instantaneous secondary current strength will be

$$i_2 = \frac{e_2}{z_2} = -\frac{\mu\, n\, i_1}{z_2} = -\mu\, n\, i_1\, y_2 \qquad \text{amperes } \angle \quad (62)$$

The instantaneous emf. induced in the primary circuit by the rate of change in secondary current will be

$$-e_1 = -\mu \frac{d\,i_2}{d\,t} = \mu^2 n^2 i_1 y_2 \qquad \text{volts } \angle \quad (63)$$

The instantaneous driving emf. in the primary circuit, needed to overcome $-e_1$ will be

$$e_1 = -\mu^2 n^2 i_1 y_2 \qquad \text{volts } \angle \quad (64)$$

The instantaneous impedance in the primary circuit due to the reaction of the secondary will be

$$z_{12} = \frac{e_1}{i_1} = -\mu^2 n^2 y_2 = -\frac{\mu^2 n^2}{z_2} \qquad \text{ohms } \angle \quad (65)$$

a result which includes (57), when $\alpha = 0$ and $n = j\,\omega$.

General Considerations

An equation in n of the second degree can be satisfied either by two real roots ($-\alpha_1$, $-\alpha_2$, non-oscillating angular velocities) or by a pair of conjugate complex roots, of the type ($-\alpha \pm j\,\omega$), entailing one oscillation frequency. An equation in n of the third degree, or of any odd degree, indicates at least one real root $-\alpha_1$, which can ordinarily be evaluated in the manner exemplified by Figure 11. The remaining two roots, if conjugate, represent one oscillation frequency. Similarly, an equation in n, with lowest terms, and of the fourth degree, may indicate the presence of two independent oscillation frequencies, and an equation of the sixth degree, three oscillation frequencies. An inspection of the oscillation system connection-diagram may help in forming a judgment as to the number of independent oscillation frequencies present.

In view of the close analogy which exists between the arithmetics of electric oscillations in oscillatory-current circuits, and of small mechanical oscillations in mechanically vibrating systems,* it is evident that the rules above discussed apply, in general, also to mechanical oscillation-systems, provided the

* Bibliography (8).

elastic forces are proportional to the corresponding displacements and the frictional forces to the first powers of the velocities.*

In the case represented in Figure 13, the full expression of (59) yields an equation of the fourth degree in n, with two pairs of conjugate complex roots, corresponding to two oscillation frequencies and damping constants. The complete solution of this fourth-degree equation is, in general, very tedious; but full results for engineering purposes may be obtained by abbreviated methods. The detailed discussion of oscillation frequencies in mutually coupled circuits calls however, for a separate paper, and need not be continued here.

CONCLUSIONS

(1) The oscillation impedance of a circuit traversed by free electric oscillations is zero.

(2) The oscillation impedance of a pure resistance is equal to its ohmic resistance.

(3) The oscillation impedance of a capacitance c farads, to angular velocity n, is $1/(c\,n)$ ohms $\angle$. In other words, its oscillation admittance is $c\,n$ mhos $\angle$.

(4) The oscillation impedance of an inductance of l henrys, to angular velocity n, is $l\,n$ ohms $\angle$.

(5) The oscillation impedances of the elements of a circuit or system of circuits follow the laws of resistances in such circuits when traversed by continuous currents, subject to the rules of complex quantities, or of plane-vector arithmetic.

(6) The impedance of a circuit, or system of circuits, to sustained oscillations, or impressed alternating currents, is a particular case under the general laws above stated, ($a = 0$, $\Sigma z \neq 0$).

(7) Any free oscillation in a circuit, or system of circuits, selects such an angular velocity, n radians per second, as will reduce its total impedance to zero.

(8) A generalized angular velocity n is a complex quantity containing a real and an imaginary component. The real component is the damping constant, and may be regarded as the projection of a hyperbolic angular velocity. The imaginary component is a circular angular velocity, of 2π times the oscillation frequency. Its projection, on an axis of reference, gives a sinusoidal quantity.

* Since the printing of this paper, the author's attention has been directed to a statement by Mr. H. W. Nichols, which indicates that certain mathematical propositions concerning mechanical oscillations, bearing closely on this matter, are already known to physicists.

(9) The sum of the oscillation admittances of the branches of a multiple oscillation circuit, at a knot point, is zero.

(10) The oscillation angular velocities of mutually coupled circuits can be expressed in terms of their mutual impedances.

(11) The sum of the instantaneous oscillation-impedance drops ($\Sigma i_n z_n$) around any closed loop in an oscillation system is zero. In the case of sustained oscillations, i. e., alternating currents, with $a = 0$, this reduces to Steinmetz's extension of Kirchoff's law into two dimensions.

(12) The instantaneous power of discharge in an oscillation impedance z is $i^2 z$ watts $\angle$, the phase angle of the instantaneous current i being taken as zero. Negative power values signify powers absorbed into the circuit. Positive values signify powers liberated out of the circuit. Real components signify dissipative powers. Imaginary components signify non-dissipative and transformed or reactive powers. The same conditions apply to the sustained oscillations in alternating-current circuits, except that with $a = 0$, negative real components do not present themselves.

(13) The total instantaneous discharge power in an oscillation system ($\Sigma i_n^2 z_n$) is zero.

(14) The share of oscillating current which a discharging element delivers to any one of a group of oscillation admittances in parallel is proportional to the oscillation admittance of that path, computed according to the rules of complex quantities or plane vectors.

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LIST OF SYMBOLS EMPLOYED

A, B	Integration constants of initial electric quantity (coulombs).
α	Projected hyperbolic angular velocity; (hyps. per sec.) or damping constant.
β	Circular angle described by a rotating unit radius vector (radians).
$C = c_1 + c_2 + c_3 +$	Sum of capacitances in parallel (farads).
c	Capacitance of a condenser (farads).
c_1, c_2, c_3	Capacitances of individual condensers (farads).
d	Sign of differentiation.
$E = \gamma_1 + \gamma_2 + \gamma_3 +$	Sum of pure ductances in parallel (yrnehs).
$\gamma = 1/l$	Ductance, or reciprocal of a pure inductance (yrnehs).
$\gamma_1, \gamma_2, \gamma_3$	Ductances of individual pure reactors (yrnehs).
e	Instantaneous emf. of a discharging element (volts $\angle$).
$-e_1, e_2$	Instantaneous primary and secondary induced emfs. (volts $\angle$).
$-e_c$	Instantaneous back emf. of a capacitance (volts $\angle$).
$-e_l$	Instantaneous back emf. of an inductance (volts $\angle$).
$\varepsilon = 2.71828\ldots$	The Napierian base (numeric).
$f = \omega/2\pi$	Oscillation frequency (cycles per second). In drawings, the symbol for a farad.
G	In drawings, the symbol for a ground connection, assumed perfect.

g Conductance of a leak (mhos).

h In drawings, the symbol for a henry.

θ Hyperbolic angle (hyperbolic radian or hyp.).

I Initial current strength (amperes $\angle$).

I_1 Initial primary current strength (amperes $\angle$).

i Instantaneous current strength (amperes $\angle$).

i_1, i_2 Instantaneous primary and secondary currents (amperes $\angle$).

$j = \sqrt{-1}$ Sign of "imaginary" quantity.

$L = l_1 + l_2 + l_3 +$ Sum of individual inductances in series (henrys).

l An inductance (henrys).

l_1, l_2, l_3 Individual inductances (henrys).

μ Mutual inductance between two circuits (henrys).

$n = -(a \pm j\omega)$ A generalized complex angular velocity (radians per second $\angle$).

p_c, p_l, p_r Instantaneous powers in a capacitance, inductance, or resistance (watts $\angle$).

$\pi = 3.14159\ldots$ (Numeric).

q Quantity of electricity in a condenser (coulombs).

$R = r_1 + r_2 + r_3 +$ Sum of a number of pure resistances in series (ohms).

r A pure resistance (ohms).

$S = s_1 + s_2 + s_3 +$ The sum of individual elastances in series (darafs).

$s = 1/c$ Elastance of a condenser of capacitance c (darafs).

s_1, s_2, s_3 Elastances of individual condenser (darafs).

Σ Sign of summation.

t Time elapsed from an epoch, or original condition (seconds).

U Initial difference of potential across a discharging condenser (volts).

u Instantaneous difference of potential across a condenser (volts).

$X_l = l\omega$ Reactance of an inductance to sustained angular velocity ω (ohms).

$X_c = 1/(c\omega)$ Reactance of a capacitance to sustained angular velocity ω (ohms).

x, y Rectangular Cartesian coordinate of a point in a plane (cm).

$y = 1/z$ Admittance of an impedance z (mhos $\angle$).

$y h$ In drawings, a symbol for yrnehs.

y_2 Admittance of a secondary oscillation circuit (mhos $\angle$).

y_1, y_2, y_3 Individual admittances in parallel (mhos $\angle$).

z An oscillation impedance (ohms $\angle$).

z_1, z_2, z_3 Individual oscillation impedances (ohms $\angle$).

z_1, z'_1 Oscillation impedance of a primary circuit with the secondary open or removed (ohms $\angle$).

z_2, z'_2 Oscillation impedance of a secondary circuit with the primary open or removed (ohms $\angle$).

z_{12}, z'_{12} Oscillation impedance of a primary circuit with the secondary closed or present (ohms $\angle$).

z_c Oscillation impedance of a capacitance c (ohms $\angle$).

z_l Oscillation impedance of an inductance l (ohms $\angle$).

z_r Oscillation impedance of a resistance r (ohms).

Ω In drawings a symbol for ohms.

℧ In drawings a symbol for mhos.

ω Circular angular velocity (radians per second).

$\angle$ Angle sign appended to a unit, indicating the existence of a complex quantity or plane vector.

SUMMARY: Corresponding to the usual angular velocity (2π times the frequency) of an alternating current is the generalized angular velocity of an oscillating current. The generalized velocity is a complex quantity; the real portion determining the damping constant, the imaginary portion the frequency of the current. The author shows that the oscillation impedances of resistances, inductances and capacities are formed in the same way from generalized angular velocities as from the usual angular velocity. The oscillation impedance of any circuit or system of circuits is found by the usual law of resistances for continuous currents, due regard being paid to the rules of complex quantities. It is then shown that free oscillations of any system of circuits select such angular velocities as to reduce the total oscillation impedance to zero. A number of cases of parallel and series oscillating circuits are treated by this method with much simplicity. The total oscillation admittance at a knot point is shown to be zero, as also is the sum of the instan-

taneous oscillation-impedance drops around a closed loop. The instantaneous discharge power in any oscillation impedance is readily derived and shown to be zero in a pure oscillation system. The problem of coupled circuits is given a preliminary treatment by these methods.

J. A. Fleming (communicated): In reference to the paper of Dr. A. E. Kennelly, I may mention that for about ten years past I have been accustomed to give to my students in lectures a proposition which is very nearly identical with the one which forms the basis of his paper.

I have usually put the matter as follows:

If a circuit has capacity (C) and inductance (L) in series and is submitted to a simple periodic E. M. F. having a frequency n and *pulsation* or angular velocity $p = 2\pi n$, then the circuit is non-inductive for a frequency equal to the natural frequency of the circuit.

I have also been accustomed to employ the idea of a complex angular velocity or complex pulsation P in connection with damped oscillations.

In teaching the elements of alternating current theory, the students are, of course, taught that the quantity $L p$, called the reactance, is of the dimensions of a resistance and can be measured in ohms and that the product of this quantity and the current (I) is called the reactance voltage LpI. They also learn that the quantity $\frac{1}{C p}$ which I have always called the "captance" is a quantity of the dimensions of a resistance and that the product of captance and current $\frac{I}{C p}$ is of the dimensions of an E. M. F. and is measured in volts.

Hence for a circuit of ohmic resistance R and reactance $L p$ and captance $\frac{1}{C p}$ the resultant vector impedance is

$$R+j\left(L p-\frac{1}{C p}\right)$$

and the size of this vector is

$$\sqrt{R^2+\left(L p-\frac{1}{C p}\right)^2}$$

Accordingly, if the frequency is such that $L p-\frac{1}{C p}=0$, the circuit is non-inductive. But the natural frequency is given by the condition $p=\frac{1}{\sqrt{LC}}$, which is identical with the above condition for non-inductivity. I think that this equality is, however, only exact if we can neglect the resistance of the circuit in comparison with its reactance.

A generalised proof of the proposition may be obtained as follows: Maxwell showed in his "Treatise on Electricity and Magnetism," Vol. II, that the equations which Lagrange established for the dynamics of mechanical systems could be applied to electrokinetic systems with certain modifications in the meaning of the symbols.

The Maxwell-Lagrange equations are as follows: If T is the conserved energy of the system and H is the rate of dissipation of energy in the system, and if x is any current in any mesh or circuit in which the impressed electromotive force is E, then we have

$$\frac{d}{dt}\cdot\left(\frac{dT}{dx}\right)+\frac{1}{2}\frac{dH}{dx}=E \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

If the system has an electromotive impulse given to it, and if it is then left to itself, there is then no impressed E. M. F., and, therefore, the time of free oscillation must be given by solving the above equation for p or n when the left hand-side is equated to zero. Hence the free period is obtained from the equation:

$$\frac{d}{dt}\left(\frac{dT}{dx}\right)+\frac{1}{2}\frac{dH}{dx}=0 \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

Now the dissipation function, H is a quadratic function of the currents. If R is the resistance of the circuit then $H=Rx^2$ and $\frac{1}{2}\frac{dH}{dx}=Rx\cdot$ Hence for a single circuit of resistance R, the equation (1) takes the form

$$\frac{d}{dt}\left(\frac{dT}{dx}\right)+Rx=E \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

Hence the condition for the circuit being non-inductive is

$$\frac{d}{dt}\left(\frac{dT}{dx}\right)=0 \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

If R is small, the condition (4) is nearly the same as the equation (2) which determines the free frequency. In other words, if the resistance is small or negligible compared with the reactance, then the circuit is non-inductive for a frequency equal to its natural frequency of oscillation.

Let us take as an example the simple case of a condenser of capacity C in series with a coil of resistance R and inductance L. Let n be the frequency of the impressed E. M. F., and $p=2\pi n$. Then the energy function T is

$$T=\frac{1}{2}Lx^2+\frac{1}{2}\frac{q^2}{C} \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

where x is the current at any instant and q is the charge in the condenser at the same instant. But $x=\frac{d\,q}{d\,t}$ because the coil is in series with the condenser. Moreover x is independent of q. Hence we have

$$\frac{d}{dt}\left(\frac{dT}{d\,x}\right)=L\frac{d\,x}{d\,t}+\frac{q}{C} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (6)$$

and

$$H=R\,x^2$$

therefore

$$\frac{1}{2}\frac{d\,H}{d\,x}=R\,x\cdot \quad . \quad . \quad . \quad . \quad . \quad . \qquad (7)$$

Equation (1) becomes then

$$L\frac{d\,x}{d\,t}+\frac{q}{C}+R\,x=E \quad . \quad . \quad . \quad . \quad . \quad . \qquad (8)$$

The circuit is therefore non-inductive if

$$L\frac{d\,x}{d\,t}+\frac{q}{C}=0$$

or if

$$L\frac{d^2\,x}{d\,t^2}+\frac{x}{C}=0 \quad . \quad . \quad . \quad . \quad . \quad . \qquad (9)$$

Also the frequency is derived from the equation

$$L\frac{d^2\,x}{d\,t^2}+R\frac{d\,x}{d\,t}+\frac{x}{C}=0 \quad . \quad . \quad . \quad . \quad . \quad . \qquad (10)$$

for a solution of the above is $x=\varepsilon^{m}$ where m is found from

$$m^2+\frac{R}{L}m+\frac{1}{LC}=0 \quad . \quad . \quad . \quad . \quad . \quad . \qquad (11)$$

or

$$m=-\frac{R}{2L}\pm j\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}$$
$$=a+j\,\beta.$$

This gives the solution of (10) in the form

$$x=\varepsilon^{-at}\left\{A\ \cos\beta\,x+B\sin\beta\,x\right\}$$

from which it follows that the frequency is

$$n=\frac{1}{2\pi}\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (12)$$

or if R is negligible

$$n=\frac{1}{2\pi\sqrt{L\,C}} \quad \text{or} \quad p^2=\frac{1}{LC} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (13)$$

If, however, x is a simple periodic current, then $\frac{d^2\,x}{d\,t^2}=-p^2\,x$, and equation (9) becomes $p^2\,L=\frac{1}{C}$ or $p^2=\frac{1}{L\,C}\cdot$

Hence if the applied frequency is the same as the *free* frequency the circuit is non-inductive.

The mechanical equivalent of this proposition is as follows: A mechanical system having inertia and elastic constraints acts as an inertia-less system towards applied impulses having a frequency equal to its free oscillations.

This can be proved as above shown from Lagrange's equations. This is merely another form of the principle of resonance, viz., that very large displacements can be produced by infinitely small impulses, if these latter are applied at intervals exactly equal to the free frequency of oscillation of the system.

I may say also that I have long been accustomed to make use of the conception of a complex pulsation or angular velocity, and to explain to students that the expression ε^{jPt} represents:

(1) An undamped simple periodic oscillation,

(2) A dead beat motion, or

(3) A damped oscillatory motion,

according as P is a real, imaginary, or complex quantity; and have used this idea to calculate the frequency of the oscillations in a condenser circuit having in it inductance and a leaky condenser.

(See my "Wireless Telegraphists' Pocket-Book of Notes, Formulae and Calculations."—The Wireless Press, Ltd., London, page 165.)

Problems connected with damped oscillations in circuits having resistance are easily treated by assuming that the currents and voltages are proportional to the real part of the expression ε^{jPt}, where $P=(p+j\alpha)$ and $p=2\pi n$ and $\alpha=n\delta$, where δ is the decrement per complete period.

It is worth noticing that the expression ε^{jPt} is an operator which when applied to a vector $\alpha+j\beta$ causes that vector to rotate thru an angle pt and at the same time to shrink in size in the ratio of $1:\varepsilon^{-\alpha t}$.

The free extremity of such a vector pivoted at one end therefore describes a logarithmic spiral and the projection of the free end on a line moving uniformly parallel to itself gives us the decrescent wavy curve which graphically denotes a damped oscillation.

Arthur G. Webster (communicated): Professor Kennelly's paper is, like all of his contributions, extremely clear and

helpful to the student. He is one of a group of engineering teachers who have the gift of making difficult things seem easy, and of presenting clear directions for the solution of problems. I am sorry to say, however, that these gentlemen do not always make it plain as to what is new, and what is merely put in a new way. Since in the first paragraph of his paper Professor Kennelly states that he believes his proposition to be new, and later that a particular case was discovered by him in 1893, I take the liberty of making a few historical observations. The proposition with which this paper is concerned appears explicitly in a paper by Heaviside (to whom the notion of impedance is due) published in 1884, and is found in Vol. I of his "Electrical Papers" (page 415, equation 137). The whole matter is treated in his paper on "Resistance and Conductance Operators," published in 1887, and found in Vol. II, page 355, in an extremely general manner.

I am free to say that it has always seemed to me that it is not enough to know that a thing is so, but that one should know why it is so, and that rather than put off the student with these apparently simple methods it would be better to advise him to learn the small amount regarding differential equations that would enable him to understand how these problems are really to be handled. The reason for the simplicity of all these matters is that the differential equations involved are all *linear* with constant coefficients, and as has been known since the time of Cauchy all such equations may be solved by exponential functions, since these functions preserve their form on differentiation. That is the reason for the appearance of the function ε^{nt}. Now, since it seems to be thought by many persons in this country that the application of the complex variable or rotating vector to the study of alternating currents was invented by Mr. C. P. Steinmetz, I will say a few words on that matter. The formula $\varepsilon^{j\theta} = \sin\theta + j\cos\theta$ was given by Euler about a century and a half ago. When the complex diagram was introduced by Argand in about 1800, or by Wessels a little earlier, it became evident that the real part of $\varepsilon^{j\omega t}$, that is $\cos\omega t$, could be used to represent any harmonically varying quantity. I find such use by Cauchy in 1821 in optics, where we have oscillations, and all thru his works he makes great use of this imaginary exponential. But lest it be said that this is not electricity, I may say that Helmholtz, in a paper on the telephone, published in 1878, uses the imaginary exponential to represent sustained vibrations. But the method of solving all cases of

oscillations in nets containing any sort of apparatus is given by Maxwell in his great paper on "A Dynamical Theory of the Electromagnetic Field," published in 1864, in which he applies the method of Lagrange to electric circuits. Now Lagrange solved all these problems in his "Mécanique Analytique," published in 1788, and the equations can be seen on page 375 of Vol. I. So this is really the age of the theorem.

Without going into Lagrange's method, let us apply the simple method of action and reaction to the problem of two coupled circuits. We know that the electromotive force necessary to fill a condenser c with a charge q is $q\,c$. But if the charge comes from a current I, we have $q=\int I\,dt$. Also the back electromotive-force from a coil of inductance L is $L\frac{dI}{dt}$, and if there is an influencing current I_2 the effect of that is $M\frac{dI_2}{dt}$. If there is resistance, we need to overcome the electromotive force $R\,I$. Putting all together, we need

$$E_1=\frac{1}{C_1}\int I_1dt+R_1I_1+L_1\frac{dI_1}{dt}+M\frac{dI_2}{dt}.$$

Now treating the other circuit in the same way, we need

$$E_2=\frac{1}{C_2}\int I_2\,dt+R_2\,I_2+M\frac{dI_1}{dt}+L_1\frac{dI_2}{dt}.$$

Suppose that there is no electromotive force impressed in the second circuit, $E_2=0$, and that there is a simple harmonic E. M. F. in the first. This can be represented by the real part of $E\,\varepsilon^{j\omega t}$ and, for very simple reasons, we may take as a solution the real parts of $I_1=A\,\varepsilon^{j\omega t}, I_2=B\,\varepsilon^{j\omega t}$. Now since differentiation of the exponential multiplies it by $j\,\omega$ and integration divides it by the same, we find, dividing out the $\varepsilon^{j\omega t}$,

$$E=\left(\frac{1}{j\,\omega\,C_1}+R_1+j\,\omega\,L_1\right)A+j\,\omega\,MB$$

$$0=j\,\omega\,M\,A+\left(\frac{1}{j\,\omega\,C_2}+R_2+j\,\omega\,L_2\right)B\cdot$$

So that our problem of calculus has disappeared, and we have merely algebra. We have now arrived at the state treated by Professor Kennelly. If we eliminate B we get

$$\frac{E}{A}=\frac{1}{j\,\omega\,C_1}+R_1+j\,\omega\,L_1-\frac{j^2\,\omega^2\,M^2}{\frac{1}{j\,\omega\,C_2}+R_2+j\omega L_2}=Z$$

which is the impedance of the first circuit, influenced by the second. Now if there is no impressed E. M. F. in the first circuit either, we have $E=0$, and since A is not zero, $Z=0$, which is Heaviside's equation. The method is perfectly general, and is described in my book on "Electricity and Magnetism," §241, as well known at the time of publication, 1897. The method of elimination of the constants A, B is perfectly general, and leads to Lagrange's equation for the periods, as described by Kennelly.

From such considerations as arise from the definition of impedance by the equation $E=ZI$, it follows at once that impedances in series are additive, and in parallel their reciprocals are additive, and this gives the application of Kirchhoff's two laws, which is far older than Steinmetz.

With regard to Professor Pupin's experimental method, it is exactly the application of this theorem, that is, if we neglect the resistances. If not, the values of n given by the period equation can never be purely imaginary, and thus the impedance for the actual current can not be exactly zero, but as Professor Pupin says the difference is very small in practise. I have made a remark on this in my book, page 499.

In conclusion, I cannot let pass the opportunity to criticise Professor Kennelly's practise of printing the units in the margin, as I consider it a cardinal principle in writing formulae that the formula should be true whatever the units. Fancy writing

$$s=v\,t \qquad\qquad \text{feet}$$

when the formula is equally true for miles, centimeters, or what not, all the units being properly taken.

Joseph G. Coffin (communicated): I have read Professor Kennelly's article with great interest, and also with surprise. Professor Kennelly has no doubt discovered a simple yet powerful proposition, but I am surprised that he has not discovered that it is not new and that it is known to many of us. I shall merely refer to Perry's "Calculus for Engineers," (1897), pages 231–261, a work not mentioned in his article and with which he seems to be unfamiliar. I refer especially to pages 236 and 237 where may be found the words:

"In any network of conductors we can say exactly what is the actual resistance (for steady currents) between any point A and another point B if we know all the resistances r_1, r_2, etc., of all the branches. Now if each of these branches has self-induction l_1, etc., and capacity k_1, etc., what we have to do is

to substitute $r_1+l_1\theta+\frac{1}{k_1\theta}$ instead of r_1 in the mathematical expressions, and we have the resistance right for currents that are not steady."

The following is a direct demonstration of the theorem, based upon Perry's statement and well-known mathematical results; which may be of interest to others as the standpoint is somewhat different.

In any network of linear conductors in which steady currents are circulating, two laws, well known as Kirchhoff's laws hold.

1. At any point where two or more conductors meet (branch point), the sum of the currents all taken as flowing into (or out of) that point is zero. This means merely that there is no accumulation of electricity at any such point.

2. Around any (closed) circuit of the network taken at random the sum of the Ri drops is equal to the sum of the impressed E. M. F.'s in that circuit.

These two rules of great importance are expressed by the equations

$$\Sigma i=0 \qquad (1)$$

$$\Sigma Ri=\Sigma e \qquad (2)$$

For variable currents these equations still hold at any instant, and hence at all instants, provided there is added to the ohmic drop, Ri, the back E. M. F.'s due to self-induction, $L\frac{di}{dt}$, and the back E. M. F.'s due to charges on condensers, $\frac{\int i\,dt}{C}$, and the back E. M. F.'s due to mutual induction, $M\frac{di}{dt}$.

For example, the integro-differential equation for the current in a single circuit containing R, L and C in series is

$$L\frac{di}{dt}+Ri+\frac{1}{C}\int i\,dt=e \qquad (3)$$

where e, any given function of the tune, is the impressed E. M. F. on the circuit. In all cases of any importance, e is the exponential function $E\varepsilon^{mt}$; as it is well known that $e=0$, $e=E\varepsilon^{mt}$, $e=E\sin\omega t$ and $e=E\varepsilon^{-at}\sin\omega t$ are obtained by making $E=0$, $m=n$, $m=\pm j\omega$ and $m=-a\pm j\omega$ respectively.

The important thing then is the solution of equation (3) when $e=E\varepsilon^{mt}$ where m can be real, purely imaginary or complex. Taking the case of free action, let $e=0$. Equation (3) becomes

$$L\frac{di}{dt}+Ri+\frac{1}{C}\int i\,dt=0$$

or $$\left\{L\frac{d}{dt}+R+\frac{1}{C}\int(\ )dt\right\}i=0 \tag{4}$$

It is well known that, assuming

$$i=e^{nt}, \tag{5}$$

where n is as yet an undetermined constant, real, purely imaginary or complex, we obtain

$$Ln+R+\frac{1}{Cn}=0 \tag{6}$$

as the condition that n must satisfy to make (5) a solution.

If one wishes to call the expression on the left of (6), the impedance of the circuit, there is certainly no objection.

To understand Perry's symbolic resistances, consider the equation for the current thru an ohmic resistance R; it is

$$Ri=e. \tag{7}$$

One may say that R operating on i gives the E. M. F.; or that $\frac{1}{R}$ operating on e gives the current.

The equation for the current in a circuit containing an inductance L is

$$L\frac{di}{dt}=e. \tag{8}$$

Note: Let us abbreviate, as does Perry, by letting

$$\frac{d}{dt}(\ )\equiv\theta(\ ). \tag{9}$$

Equation (8) is then

$$L\theta i=e. \tag{10}$$

This means that differentiating i with respect to the time and multiplying by L gives a result always equal to the impressed E. M. F. e.

By comparison with equation (7), one may say that $L\theta$ operating on i is a result equal to the impressed E. M. F. $L\theta$ is therefore analogous to a resistance and may be called the symbolic resistance due to L.

The equation for the current in a circuit containing a capacity C is given by

$$i=C\frac{de}{dt}=C\theta e \tag{11}$$

or by $$q=\int i\,dt=Ce. \tag{12}$$

So that we see not only that symbolically

$$\frac{1}{C\,\theta}\,i=e, \tag{13}$$

but that $\frac{1}{\theta}$ means integration with respect to the time. One may say that $\frac{1}{C\,\theta}$ is the symbolic resistance due to C. $\frac{1}{\theta}$ is called the inverse operator to θ.

From this point of view, (4) may be written

$$L\,\theta\,i+R\,i+\frac{i}{C\,\theta}=e$$

or

$$\left(L\,\theta+R+\frac{1}{C\,\theta}\right)i=e. \tag{14}$$

One may say finally that

$$L\,\theta+R+\frac{1}{C\,\theta}$$

is the symbolic resistance due to L, R and C in series.*

Considering equations (1) and (2), it is now seen that they become for varying currents

$$\Sigma\,i=0 \tag{15}$$

and

$$\Sigma\left(L\,\theta+R+\frac{1}{C\,\theta}\right)i=\Sigma\,e, \tag{16}$$

putting aside the consideration of mutual induction for the moment.

Equations (15) and (16), the generalized Kirchhoff equations, which are well known, give the solution in the following manner. Deduce the resistance around any chosen circuit or between any two points of the network, in terms of the resistances R_1, R_2, etc., of the separate branches as if there were merely ohmic resistances; in the resulting expression, replace R_1, R_2, etc., by the symbolic resistances

$$L_1\frac{d}{d\,t}+R_1+\frac{1}{C_1}\int(\ \ )\,dt$$

or by

$$L_1\,\theta+R_1+\frac{1}{C_1\,\theta},\ \text{etc.}$$

It is easily seen that the symbol θ always obeys the following laws in combination with i (to the first power only), and with a and b (constants).

$$(\theta+d)\,i=\theta\,i+d\,i$$

$$\theta\,(i_1+i_2)=\theta\,i_1+\theta\,i_2 \qquad \textit{Distributive Law.}$$

$$\theta\,a\,i=a\,\theta\,i \qquad \textit{Commutative Law.}$$

* See Perry, bottom of page 236.

and if $\theta\,\theta\, i$ is written $\theta^2 i=\frac{d^2 i}{dt^2}$, etc., it is easily seen that

$$\theta^n\, \theta^m\, i=\theta^{n+m}\, i \qquad\qquad \textit{Index Law.}$$

When any quantity obeys these laws, that quantity enters into expressions in combination with i's and constants exactly as do ordinary algebraic quantities.

For instance, it is easy to see that

$$(\theta+a)\,(\theta+b)\, i=[\theta^2+(a+b)\,\theta+a\,b]\, i$$

or that $(\theta+a)\,(\theta+b)$ is the same thing as

$$(\theta+b)\,(\theta+a), \text{ etc.}$$

It follows from this that the expression obtained when the R's are replaced by the $\left(L\,\theta+R+\frac{1}{C\,\theta}\right)$'s may be simplified, and the result will be

$$f_1\,(L\,R\,C\,\theta)\, i=f_2\,(L\,R\,C\,\theta)\, e, \tag{17}$$

where f_1 and f_2 are polynomials in θ with constant coefficients.

In the case of free action $e=0$ and (17) becomes

$$(d_o\,\theta^m+d_1\,\theta^{m-1}+\quad \cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\quad +d_{m-1}\,\theta+d_m)\, i=0 \tag{18}$$

It is well known since the time of Euler that the solution of (17) is made up of two parts: (**a**) the solution when $e=0$, i. e. of (18), plus (**b**) a particular solution of (17). The first part is the natural or free action of the system left to itself and the second is the forced part.

At present, consider the free action of the system, i. e. of (18).

The well-known method is to assume a solution of the form

$$i=\varepsilon^{n\,t} \tag{19}$$

where n is as yet an undetermined constant which may be real, purely imaginary or complex.

There results on substitution of (19) in (18) since

$$\theta\, i=\theta\,\varepsilon^{n\,t}=n\,\varepsilon^{n\,t}=n\, i$$
$$\theta^m\, i=n^m\, i,$$

that is $\theta^m = n^m$ when applied to an exponential function.

$$(a_o\, n^m+a_1\, n^{m-1}+\quad \cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\quad +a_{m-1}\, n+a_m)\, i=0 \tag{20}$$

which shows that if n has for its value any of the m solutions of the *algebraic* equation

$$a_o\, n^m+\quad \cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\quad +a_m=0 \tag{21}$$

then
$$i=A_1\,\varepsilon^{n_1 t}+A_2\,\varepsilon^{n_2 t}+\qquad +A_m\,\varepsilon^{n_m t} \tag{22}$$

satisfies (18) because each term on the right satisfies it separately.

This is the general solution as it contains m arbitrary constants. In general, there will be real roots either positive or negative and imaginary roots either pure or complex, the latter always occurring in conjugate pairs.

The A's in (22) are arbitrary constants. In any particular case, they are determined by the initial or final state, of the system, which are supposed to be known. As a matter of fact, it can be shown that the real roots in any electrical case are always negative, and that the real parts of the imaginary roots are always negative.

For every real root $(-n_1)$ (excepting equal roots), there is a solution of the form

$$i_1 = I_1 \varepsilon^{-n_1 t} \tag{23}$$

For each pair of conjugate complex roots (always in pairs),

$$\begin{aligned} n_2 &= -a + j\,\omega \\ n_3 &= -a - j\,\omega \end{aligned} \qquad (j \equiv \sqrt{-1})$$

there is a solution

$$i_2 = A_2\, \varepsilon^{(-a+j\omega)t} + A_3\, \varepsilon^{(-a-j\omega)t},$$

which by means of Euler's equation,

$$\varepsilon^{\pm jx} = \cos x \pm j \sin x$$

reduces to

$$i = I\,\varepsilon^{-at} \sin(\omega t + \phi) \tag{24}$$

where I and ϕ are constants determined by the initial conditions.

Solution (23) is a decaying current and (24) a damped harmonic oscillation of period $\dfrac{2\pi}{\omega}$; (24) is a sustained harmonic oscillation if $a=0$; i. e. if n_2 and n_3 are purely imaginary.

Now equation (21) is the result of placing the impedances,

$$L n + R + \frac{1}{C n}, \qquad \text{etc.,}$$

treated as resistances in the circuit of the network equal to zero; hence the theorem. The whole thing is a case of Kirchhoff's Laws, and properties of the exponential function.

Consider the case exemplified in Figure 12 of Professor Kennelly's paper. The points G are in reality a single point, so that the problem is one in which there are a number of circuits connected in parallel between O and G.

By Kirchhoff's law, the sum of the currents entering O is zero

$$\Sigma i = 0$$

Also the $R i$ drop in each branch is the same hence

$$R_1 i_1 = R_2 i_2 = R_3 i_3, \qquad \text{etc.}$$

If we call $\frac{I}{R}$, the admittance, $=A$, we may write

$$\frac{i_1}{A_1} = \frac{i_2}{A_2} = \quad . \quad . \quad . \quad . \quad . \quad . = \frac{\Sigma i}{\Sigma A}.$$

But as $\Sigma i = 0$, then must

$$\Sigma A = 0;$$

which is Professor Kennelly's statement.

In the case of two mutually inductive circuits, Figure 13, the equations of Kirchhoff give

$$R_1 i_1 + M \theta i_2 = e,$$

$$M \theta i_1 + R_2 i_2 = 0,$$

where the R's stand for $L\theta + R + \frac{1}{C\theta} = Z$

From the second equation

$$i_2 = -\frac{M\theta}{R_2}.$$

Substituting in the first

$$\left(R_1 - \frac{M^2 \theta^2}{R_2}\right) i = e.$$

If $e = 0$, we obtain

$$R_1 - \frac{M^2 \theta^2}{R_2} = 0$$

or

$$Z_1 - \frac{M^2 n^2}{Z_2} = 0 \qquad \text{(Preceding article, Equation 59)}$$

as the condition to be fulfilled by θ, i. e. n, for the free action of the primary of such a system.

These methods of solution have been familiar to me for at least sixteen years, and are the result of applying the matter in Perry's "Calculus" to these problems.

Professor Perry was interested more particularly in the result when $e = E \sin \omega t$, as free action has become of more importance only since radio work has become important.

But he has distinctly stated the case for free action in the cited pages of his book. It is an extremely powerful method and the most satisfactory one for the solution not only of the free action in any network but for the case where the network has alternating currents impressed upon it.

H. W. Nichols (communicated October 22, 1915): We owe most of our practical methods of treating electrical circuit problems to Heaviside, who originated in 1887 ("Phil. Mag.," December, 1887, and "Electrical Papers," Volume II, pages 355 to 374 and elsewhere) as a special case the so-called "complex method" of treating alternating current problems, later popularized by Steinmetz and others. Unfortunately his papers are hard to read, a remark on which his own characteristic comment was that "they were even harder to write."

It is, however, a fact which has been discovered by many later investigators that most apparently new methods in circuit problems are really all in the book, and this method just described is no exception, being very clearly stated in the paper above cited (page 371 ff. of the "Papers").

It is also in common use by some telephone engineers, and we find it again stated explicity in a paper by G. A. Campbell (A. I. E. E., April, 1911, page 902) in the words:

> "The characteristic feature of free oscillations is that, thruout the part of the network over which the oscillation extends, the driving point impedance is equal to zero. This follows from the fact that as the driving point impedance is equal to the impressed electromotive force divided by the current, it vanishes when the electromotive force vanishes, provided the current does not vanish. The criterion for free oscillations is therefore $A=0$.
>
> "The solution of this equation contains all the possible values of the time coefficient p. Each possible oscillation is aperiodic or not, according as p is purely imaginary or not; p cannot be real for any actual system, since energy must be dissipated in any oscillation which may occur in such a system."

One has only to read the Heaviside paper of 1887 and the ones referred to in it to see displayed the whole theory; but since engineers do not usually read Heaviside, it is doubtless of service to have him interpreted, and for that reason we are indebted to Dr. Kennelly for his interesting paper.

A. E. Kennelly (communicated): In dealing with the impedance of the simple alternating-current circuit with sustained oscillations of a frequency imposed by the generator, the impedance can only occupy one half of the plane; namely, that on the positive real side of the axis of imaginaries. That is, the impedance can only be $+R \pm jX$ ohms, where R is an essentially positive resistance, and jX a reactance either plus

or minus. It is known, however, that by the aid either of electromagnetic induction (transformer action), or of the virtual impedance of a synchronous E. M. F., such as that of a synchronous motor, it is possible to invade the other half of the impedance plane, and to secure, in the steady state of operation, a representation of* negative resistance $(-R)$. As a consequence, however, of the reasoning set forth in the paper here presented, the negative half of the impedance plane is assigned to the impedance of inductances and capacitances during oscillations; so that the whole of the impedance plane comes into service in dealing with the simple alternating-current circuit, one half in the sustained oscillations, and the other half in unsustained or transient oscillations.

Since the paper was communicated, a new book by Professor Fleming ("The Wireless Telegraphists' Pocket-book of Notes, Formulae and Calculations") has reached this country. The date of the book's going to press is, however, earlier (May, 1915). The book contains passages relating to oscillatory frequencies bearing closely on the matters presented† in the paper; so that in tracing the history of the development of the propositions presented in the paper, the publications of Professor Fleming, including that cited in the Bibliography must certainly be taken into account.

Without belittling any scientific authority, or disparaging any kind of scientific work, it should be pointed out that the knowledge which requires determinants, differential equations, and a maze of symbols for its expression is not the kind of knowledge which can be readily apprehended and applied by the engineer. Propositions may be stated in such broad general terms as to possess no appreciable meaning in particular. Thus, the law of the conservation of energy may in a certain sense cover and include all future discoveries in physics. If the propositions set forth in the paper have been made known in prior publications, it is amazing how little use has been made of them up to this time.

The statements cited by Mr. Joseph G. Coffin from Professor Perry's admirable "Calculus for Engineers" are directed to forced or sustained oscillations; altho we can now see that they may also be used in connection with free or transient oscillations.

* D. C. and J. P. Jackson's "Alternating Currents and Alternating-Current Machinery," Macmillan Co., 1913 (page 238).

† See the section on "Time Period of Electric Oscillations in Circuits Having Inductance Resistance and Capacity" (page 120), and the chapter (VII) on "High-Frequency Cymometer Measurements" (page 166).

On page 238, the remark is made "In all this we are thinking only of the forced vibrations of the system," and on page 239—"We are now studying this latter part, the forced part only. In most practical engineering problems the exponential terms rapidly disappear."

In regard to Professor Webster's criticism on the use of marginal units, it is undeniable that in pure mathematics, dynamics and physics, such units constitute needless limitations to the equations, perhaps hampering rather than helping the reader. But in mathematics applied to engineering, the insertion of the units greatly assists the reader. In this paper, most of the equations are complex or plane-vector equations; but a few are simple scalar equations. The danger of confusing vectors and scalars is avoided by the use of the marginal unit. What engineer has not wasted ill-spared hours, over technical papers, in striving to discover the formula units connoted but concealed by the writers?

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A MAGNETIC AMPLIFIER FOR RADIO TELEPHONY*

By

E. F. W. Alexanderson

(Consulting Engineer, General Electric Company)

Assisted by

S. P. Nixdorff

GENERAL CONSIDERATIONS

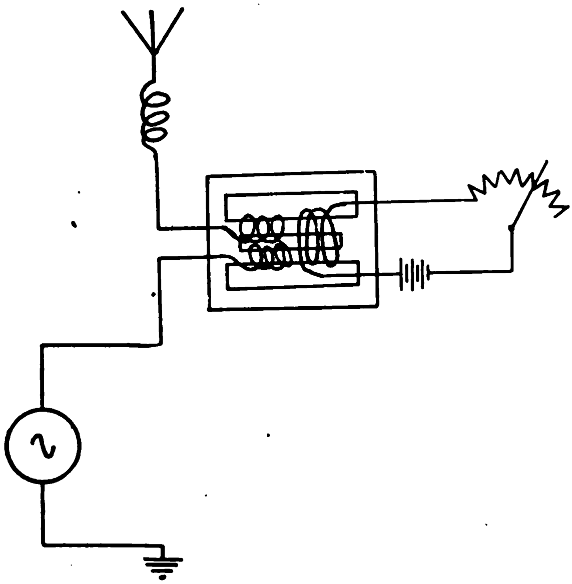
The name of "magnetic amplifier" has been given to a device for controlling the flow of radio frequency currents because this name seems to describe its function when it is used for radio telephony better than would any other. As the same device can be used for a variety of other purposes the above name may in some cases not seem so appropriate. However, the essential part of the theory that will be given refers to the amount of amplification which is possible of attainment and the methods of securing a higher ratio of amplification than would be given by the device in its simplest form.

The fundamental principle of varying an inductance by changing the permeability of its iron core is suggested in the early work of Fessenden as a means for changing the tuning of a radio antenna. The magnetic amplifier constructed as shown in Figures 1 and 2 was, on the other hand, developed as an accessory to an alternator having a solid rotor in order to take advantage of the better mechanical construction of a solid steel rotor and yet produce the results that could be obtained by field control in a machine having a completely laminated magnetic circuit. The aggregate of the constant field alternator and the stationary controlling device has, as will be shown, the effect of a machine with variable field excitation. This analogy refers not only to the proportionality between excitation and electromotive force but also to such phenomena as self-excitation and instability.

If two windings (e. g., *A* and *B* in Figure 2) are related to each other and a common magnetic structure as shown in Figures 1 and 2, it is apparent that there is no direct transformation of energy possible from one winding to the other. Each turn in the controlling or exciting winding *B* includes

* Presented before The Institute of Radio Engineers, New York, February 2nd, 1916.

both the positive and negative branch of the flux produced by the A. C. winding, A, and hence there is no voltage induced in B. The current in either winding A or B, on the other hand, influences the permeability of the common magnetic material; and, therefore, changes the inductance of the other winding. If a current flows in either winding sufficient to saturate the iron, it is thereby rendered practically non-magnetic and the



AMPLIFIER IN SERIES WITH ALTERNATOR

FIGURE 1—Combination of Alternator and Amplifier in a Simple Form

inductance of the other winding is reduced to the value it would have if the coil included only air. If, on the other hand, a current flows in the other winding which gives a magnetomotive force equal and opposite to the first, the iron is rendered magnetic again. Inasmuch as the two branches of winding A are wound relatively opposite to winding B, the one branch will oppose the ampere-turns of winding B on one half cycle and the other branch during the next half cycle. In order to have any large flux variation in winding A, the opposing ampere-turns must be at least equal to the ampere-turns in winding B. The relation of currents in these windings is substantially the same as that between the primary and secondary current in a transformer,

altho in this case one is an alternating and the other a direct current, or a current of a different frequency. It is thus obvious how the current flow in winding A can be regulated in proportion to the controlling current in winding B. When the magnetic amplifier is used in shunt to the alternator (Figure 2), it has the immediate object of controlling the voltage rather than the current. The combined characteristics can be derived from the

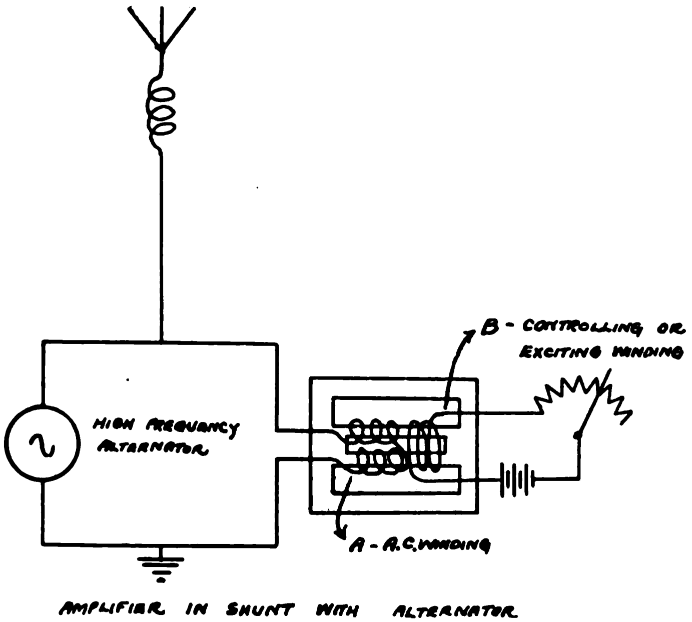


FIGURE 2—Combination of Alternator and Amplifier in a Simple Form

characteristics of the alternator when operating on an antenna, and at the same time controlled by a variable shunt across its terminals, as shown in Figure 5.

As indicated in Figures 1 and 2, it is possible to connect the amplifier either in series with the alternator or in shunt to the alternator. Of these two arrangements, the shunt connection is preferable because the effect of the amplifier on the alternator is the same as if the electromotive force of the alternator in the antenna circuit had been reduced; whereas the amplifier in the series connection does not influence the electromotive force in the antenna circuit but changes the tuning of the antenna. The result of this change in antenna tuning has an undesirable

effect on the speed characteristics of the alternator, because it is found upon further analysis that the control does not become effective unless the alternator is operated on the upper or unstable side of the tuning curve of the antenna. If, on the other hand, the alternator is operated on the stable side of the tuning curve, the change in tuning partly neutralizes the intended controlling effect.

RATIO OF AMPLIFICATION

The method of arriving at a theory for the ratio of amplification can perhaps be best explained by the following mechanical analogue.

A throttle valve in a steam pipe may be designed so that it is perfectly balanced and it might move on ball bearings so that an infinitely small effort would be sufficient to throttle an infinitely large flow of power. If, on the other hand, the valve were to be opened and closed 1,000 times in a second, the accelerating of the moving parts against their inertia would intermittently absorb considerable energy. Altho this would be "wattless" energy (inasmuch as the energy consumed in accelerating would be given back in retarding), the device which performs this movement must control considerable power. In addition, if there be frictional resistance to motion, still more energy will be required *thruout* the cycle, and this is not "wattless" energy. In analogy to this, we must ask ourselves what are the corresponding "wattless" and "watt" energy in our magnetic valve which must be overcome in opening and closing it at the frequency of a telephone current. The answer is: the "wattless" energy is that required to create the magnetic field neglecting hysteresis (and eddy current losses). The "watt" energy is that lost during any number of cycles because of hysteresis. This energy is the integrated area of the saturation curve between the limiting points between which the changes take place. The energy of the controlling field is not necessarily equal to the energy of the radio frequency field but of somewhat the same order of magnitude. The wattless flow of energy is proportional to the energy per cycle and the number of times per second the energy must be delivered and returned. It can, therefore, be said that the ratio of amplification is proportional to the ratio between the frequency of the radio current and that of the controlling current.

However, the assumption that the energy per cycle is the same in the radio frequency and in the controlling circuit is

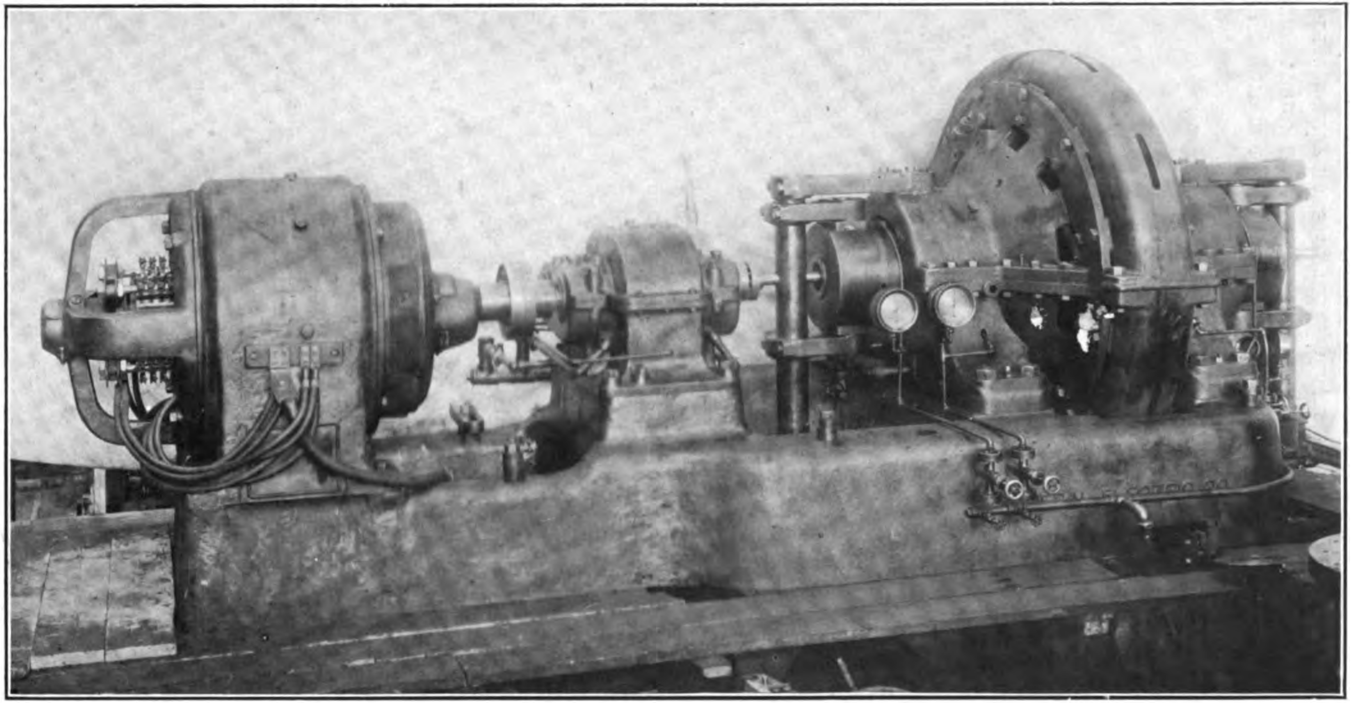


FIGURE 3—75-Kilowatt Radio Frequency Alternator Used for Tests

only a first approximation. The object of design and improved arrangements is evidently to make this energy ratio as favorable as possible; in other words, to produce a maximum flux variation in the radio frequency circuit for a minimum variation in the controlling circuit.

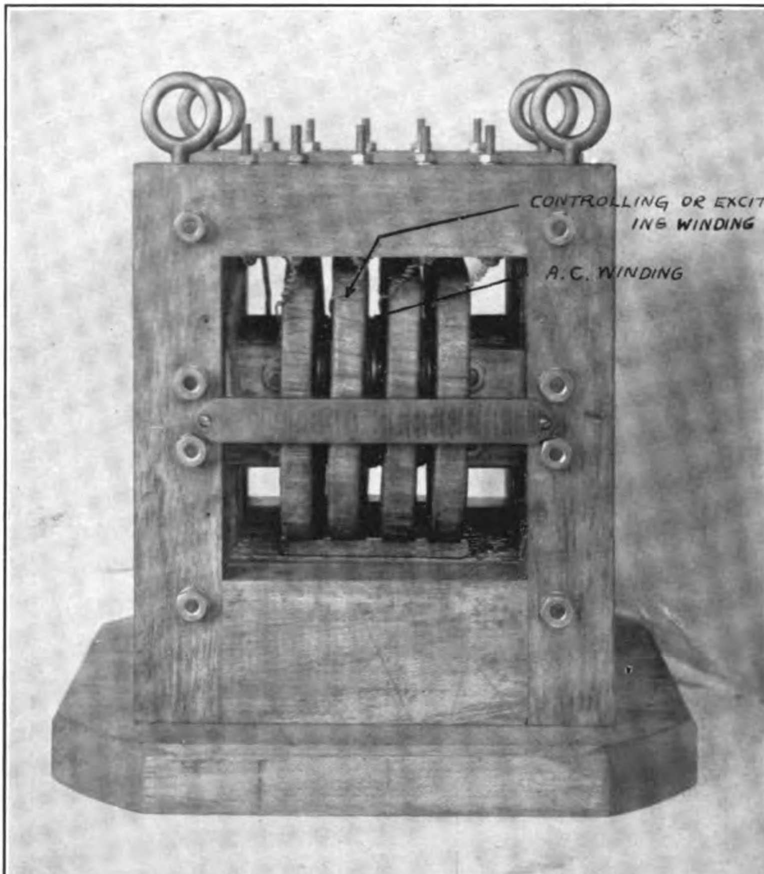


FIGURE 4—Amplifier Coil Used for Tests

In order to understand these relations it is necessary to make a further study of the laws governing changes of permeability. The object of the magnetic amplifier, when used for radio telephony is not only to control the radio energy but also to reproduce a telephone current in its true shape. An important part of the analysis is, therefore, a study of the conditions that

lead to linear proportionality between the controlling and the controlled current.

MAGNETIC THEORY AND CHARACTERISTICS

The magnetic amplifier can be operated in two ways, as indicated by the diagram on Figure 6. In one case, when the two A. C. windings are in series, the current in both windings is definite; and the flux in the corresponding branches of the

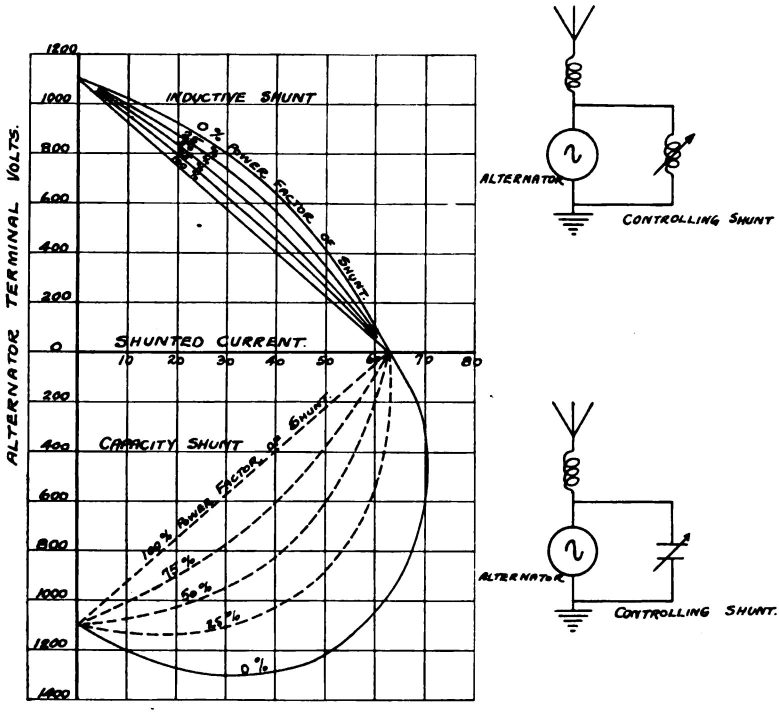


FIGURE 5—Characteristics of Alternator When Controlled by Variable Shunt Impedance

core adjusts itself accordingly. In the second case when the two A. C. windings are in multiple the currents in each winding are not immediately obtainable; because a cross current of a strength not yet defined may flow between the two windings. We know on the other hand that the flux variations in the two branches of the core must be identical, inasmuch as they produce the same terminal voltage in the multiple connected windings.

The characteristics of the amplifier winding in series and multiple connection, as obtained from tests, are shown in Figure 6. The upper curve represents series and the lower, multiple connection. The curves are plotted in ampere-turns and volts per turn, so as to make the conclusions independent of the number of turns. Both curves represent the same D. C. excitation.

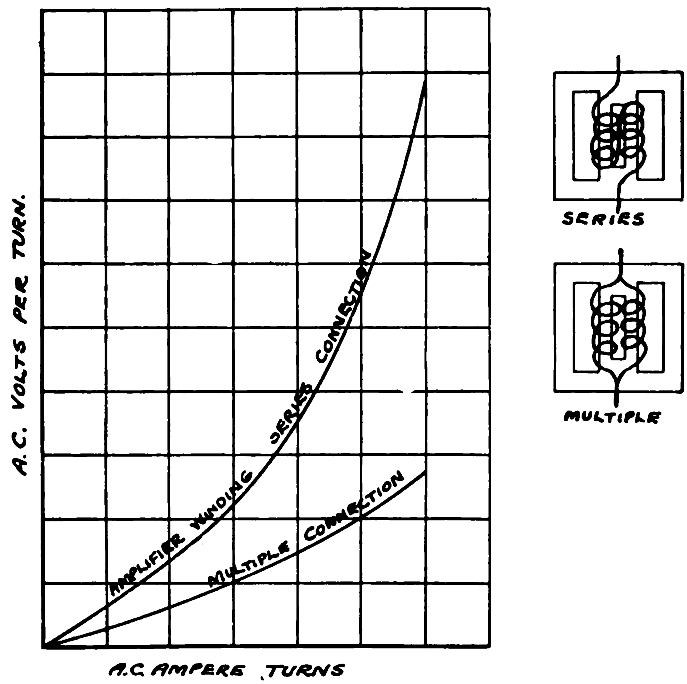


FIGURE 6—Comparative Volt Ampere Curves With the Same D. C. Excitation

tion. These curves, as well as theoretical considerations, show that the multiple connection gives a lower curvature and a lower impedance. For zero D. C. excitation it is evident that the volt-ampere curves for these two connections must be identical. It thus follows that the change of impedance corresponding to a given D. C. excitation is greater with the multiple connection. The multiple connection is, therefore, altogether more advantageous because a lower impedance with a certain control excitation means greater sensitiveness; and a lower curvature means that larger currents can be carried without causing instability, as will be shown later. While thus the second mode

of operation with multiple A. C. winding appears to have better characteristics, there are some other considerations which must be taken into account before it can be concluded that this connection could be used.

OBJECT OF SHORT CIRCUIT CONDENSER

In the multiple connection, the flux variations are forced, as already explained, by the short circuit that is formed between

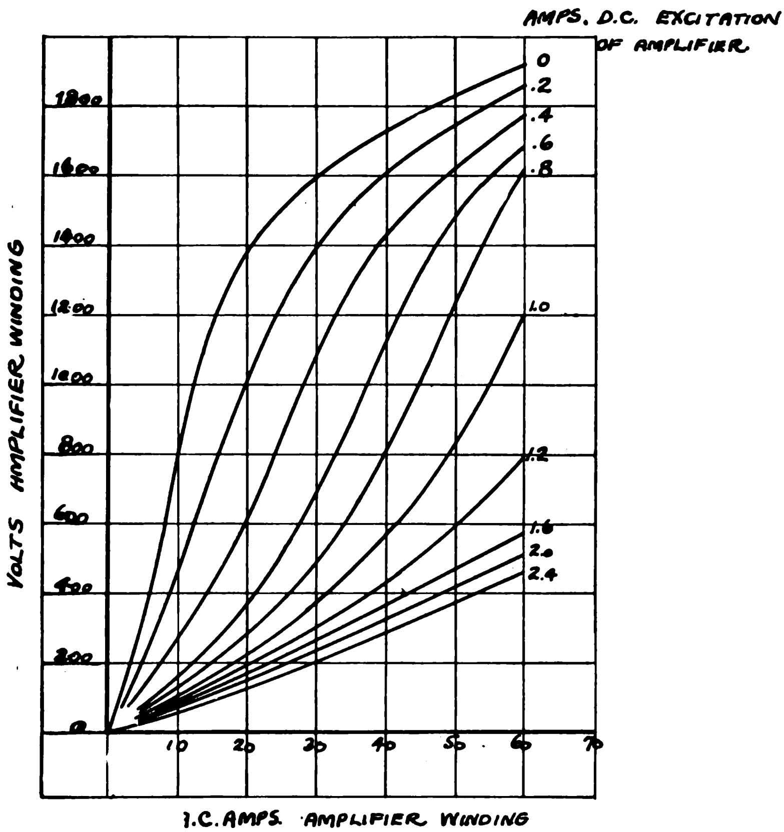


FIGURE 7—Characteristics of Amplifier Coil Windings in Multiple

the two multiple coils. The induced current in this short circuit tends to oppose any changes in the average flux, and thus a telephone current in the controlling winding would simply cause a corresponding short circuit current between the two A. C.

coils without producing the desired flux variations. This difficulty can, however, be overcome by taking advantage of the fact that the A. C. winding needs to operate only at radio frequencies, which are very much higher than the frequency of the telephone current. It is, therefore, possible to find a condenser of such value that it acts as a short circuit for the radio

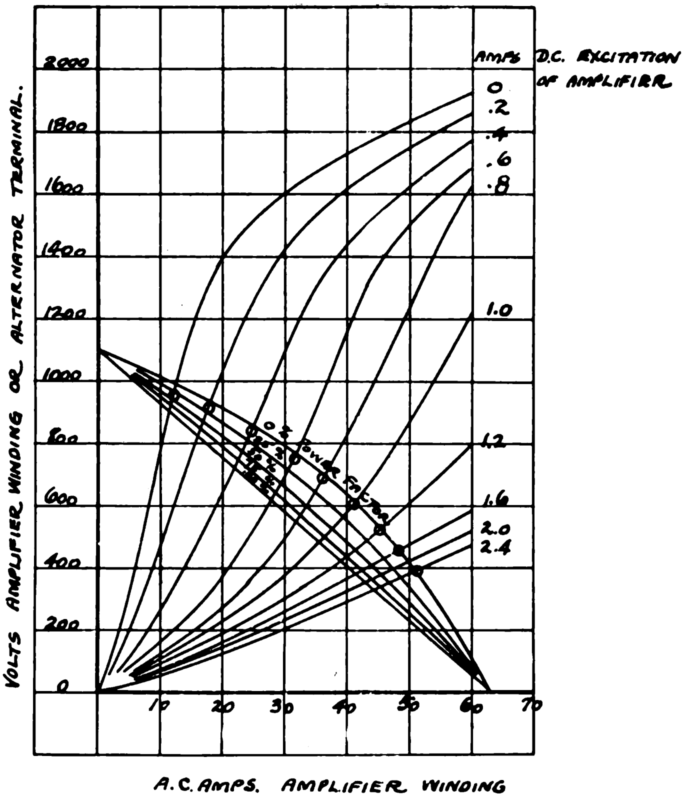


FIGURE 8—Alternator and Amplifier Characteristics Superimposed

currents and an open circuit for the telephone current. Accordingly a condenser is introduced in series with each of the A. C. coils as shown in Figure 9.

COMBINATION OF ALTERNATOR AND AMPLIFIER

In order to demonstrate how the magnetic amplifier can be used for controlling the voltage of an alternator, reference may again be made to the alternator characteristics as shown in

Figure 5. The alternator voltage is plotted against the current in the shunt circuit. The magnetic amplifier is used as this shunt circuit and the volt-ampere characteristics of the amplifier can, therefore, be directly combined with the volt-ampere characteristics of the alternator. The volt-ampere characteristics of the amplifier are shown in Figure 7. Figure 8 shows the alternator and amplifier characteristics superimposed. The intersections between the sets of curves give the alternator voltages at the corresponding amplifier excitations, and thus another curve can be plotted between alternator volts and amplifier excitation. This curve as obtained from test is the upper curve in Figure 9, which approaches the X axis asymp-

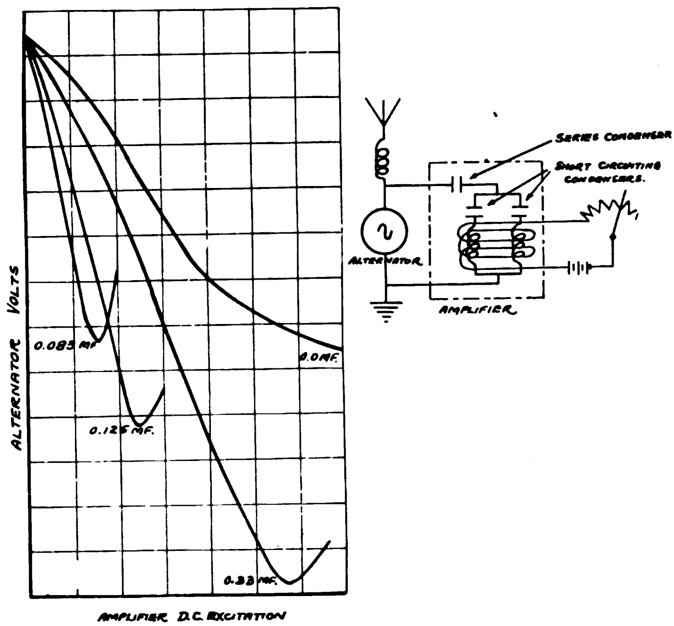


FIGURE 9—Curves Obtained from Test Showing Sensitiveness of Alternator Voltage Control with Different Series Condensers

totically with increasing excitation of the amplifier. It is possible in this way to reduce the voltage practically to zero without using an excitation which is excessive from the point of view of heat capacity of the exciting winding. A magnetic amplifier may be used in this way as a controlling device for radio telegraphy. However, in this form, it is not well adapted for tele-

phony; because, as shown by the curves, the relation of volts of alternator to amperes of excitation departs too far from the desired linear proportionality. Such proportionality can be obtained by the introduction of a series condenser as shown in Figure 9, while at the same time the sensitiveness of the amplifier is greatly increased so that a much smaller control current is

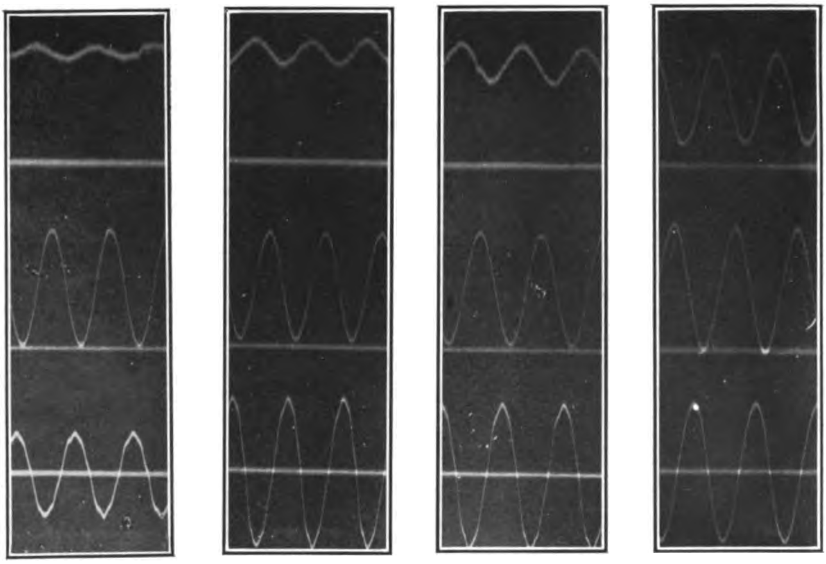


FIGURE 10—Oscillograms Showing Ratio of Amplification with Different Series Condensers.

needed. If the condenser is chosen so that it exactly neutralizes the inductance of the amplifier winding at some definite value of excitation, the resulting impedance at this excitation becomes a minimum; and the impedance at any lower excitation is determined by the difference between the inductive reactance of the amplifier coil and the capacity reactance of the series condenser. The smaller this difference, the lower will be the amplifier excitation that gives minimum impedance and the corresponding minimum of the alternator voltage. This means that the sensitiveness of the amplifier is increased because a smaller excitation is needed to reduce the alternator voltage. The increase of

sensitiveness that can be obtained in this way is, however, not unlimited. If the minimum impedance is obtained as a result of a large inductive and a large capacitive reactance, the core loss due to hysteresis and eddy currents becomes appreciable and appears as an equivalent resistance which cannot be neutralized. Figure 9 shows, from results of tests, the variations of alternator voltage that can be obtained by different values of series condenser and the corresponding increase of the sensitiveness of the amplifier. The sensitiveness is represented by the steepness of the curves. It can be seen from the shape of these curves that the increased sensitiveness is gained at the expense of range of control or difference between maximum and minimum voltage. However, all the curves show a practically linear proportionality between excitation and voltage over almost the whole range available. The difference in sensitiveness with various series condensers is also illustrated by oscillograms, Figure 10. The upper curve represents alternator voltage. The two lower curves represent amperes and volts, respectively, impressed upon the amplifier controlling winding, the frequency of the controlling current being 500 cycles. The effect of departure from linear proportionality and the consequent distortion of wave shape is shown in Figure 16.

The amplification ratio is defined as the difference between the maximum and minimum kilowatts output divided by the effective alternating volt-amperes supplied to the controlling winding. The ratio of amplification can be derived directly from the oscillograms with reference to the calibration of the oscillograph curves. The ratio of amplification for operation suitable for telephone control ranges from 100:1 to 350:1.

INSTABILITY

The voltage which results from the combination of alternator and amplifier can be determined as has been explained by the intersection of the alternator and amplifier characteristics. When the curves have a definite and sharp intersection point a definite alternator voltage results from each excitation of the amplifier. If, on the other hand, the curves should have such a shape that the alternator and amplifier characteristic curves become parallel in some place, the intersection becomes indefinite and the result is instability and generation of self-excited oscillations. This is a condition that must be avoided for telephone control; whereas it may have useful applications for other

purposes. The conditions that lead to instability can be graphically analyzed as shown in Figures 11 and 12. Figure 11 cor-

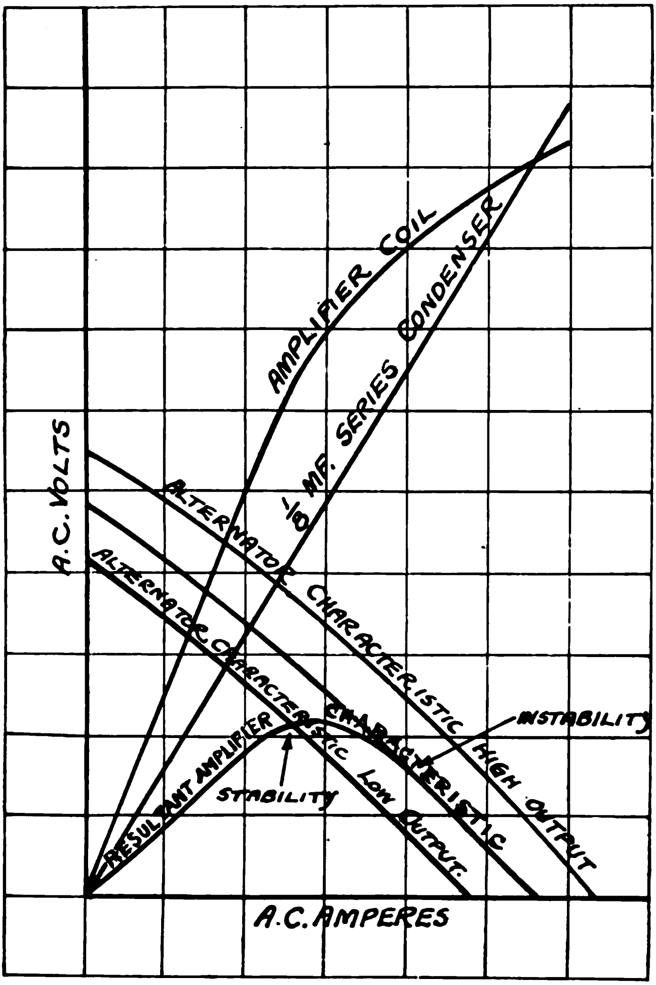


FIGURE 11—Graphic Analysis of Instability

responds to a series condenser of 1/8 μf. (microfarad) which leads to instability at higher generator outputs; whereas, Figure 12 corresponds to a series condenser of 1/3 μf. and represents a condition which is stable for all voltages at which the generator can be operated. The upper curve in each diagram is the volt-ampere curve of the amplifier coil and the straight line thru the

origin represents the series condenser. The lower curve is the difference between the volts amplifier coil and volts series condenser, which is for the present purpose a sufficiently close

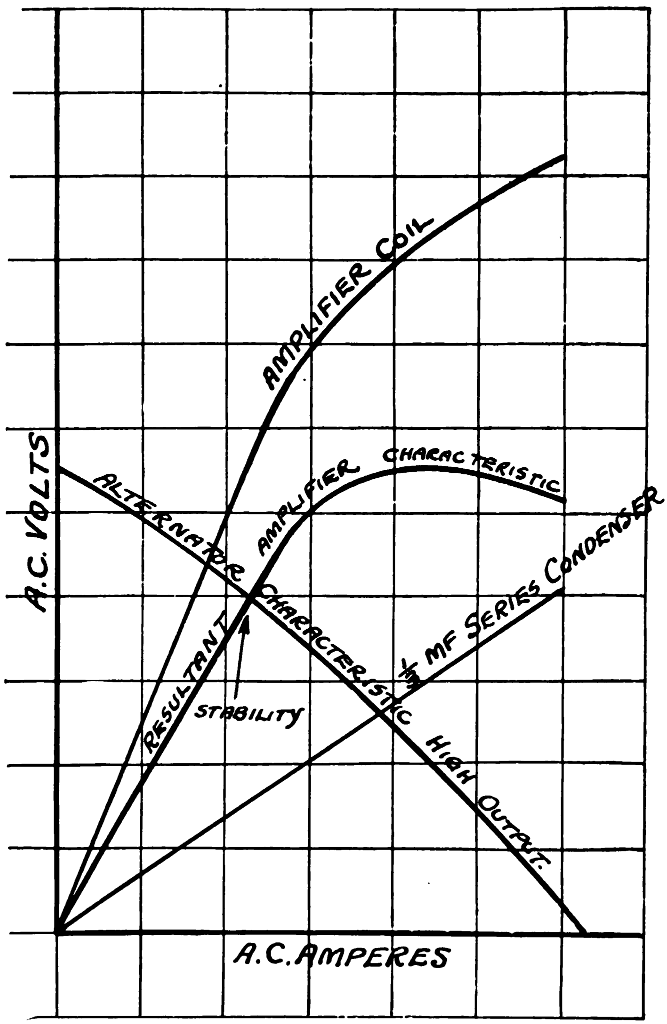


FIGURE 12—Graphic Analysis of Instability

approximation of the volt-ampere curve of the combination. This resultant volt-ampere curve in Figure 11 rises to a maximum, then falls again and crosses the zero line. The crossing of the zero line means change from inductive to capacitive impedance.

With reference to any circuit which has a volt-ampere characteristic with a bend in it, it can be said that as long as the volt-ampere curve is rising, the circuit is stable where it is connected to a source of constant potential, and wherever the volt-ampere curve is drooping, the circuit is unstable on a constant potential. The rising part of the curve corresponds to a

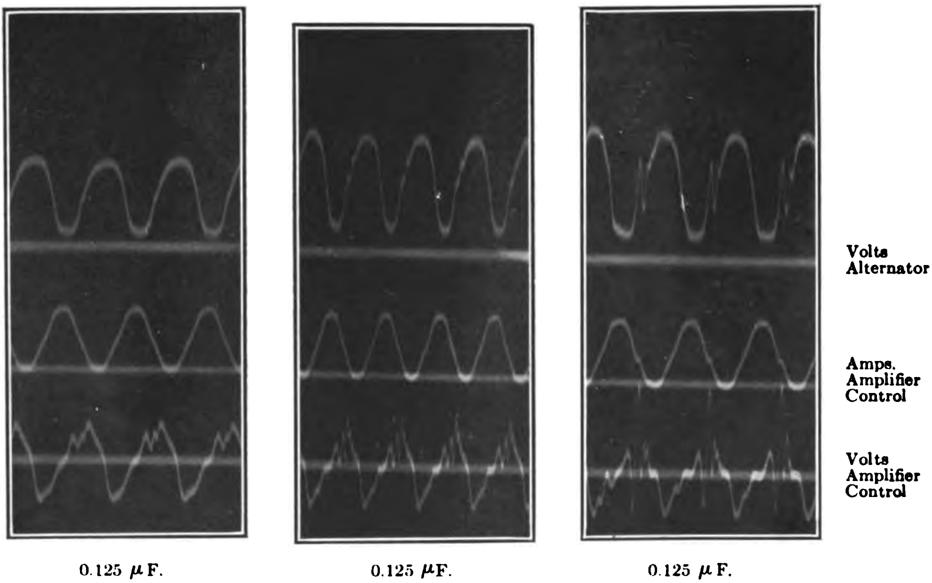


FIGURE 13—Oscillograms Showing Instability at Higher Alternator Voltage

positive resistance and the drooping side to a negative resistance. Well-known types of negative resistance are electric arcs or series commutator generators. A circuit of this character is stable only when operated on a source of potential which has characteristics equally or more drooping than the drooping volt-ampere curve. These same curves (Figures 11 and 12) show the volt-ampere characteristics of the alternator. In Figure 12, the resultant characteristic is only slightly drooping at the end, whereas, in Figure 11 the condition for instability is indicated by the place where the volt-ampere curve of the amplifier is more drooping than the volt-ampere curve of the alternator. Figure 11 also shows that the alternator curve corresponding to low output intersects the amplifier curve at the stable portion and the characteristics for increased output reach the unstable portion

of the resultant amplifier curve. This change from stability to instability is shown by the series of oscillograms on Figure 13. The instability as shown by the self-excited oscillations re-occurs at the same place of the wave which is the point at which the characteristic curves are tangent, as shown on Figure 11.

SHUNT CONDENSER

A further improvement in sensitiveness can be obtained by using a combination of shunt and series condenser. The shunt condenser is so proportioned as to make the amplifier take lead-

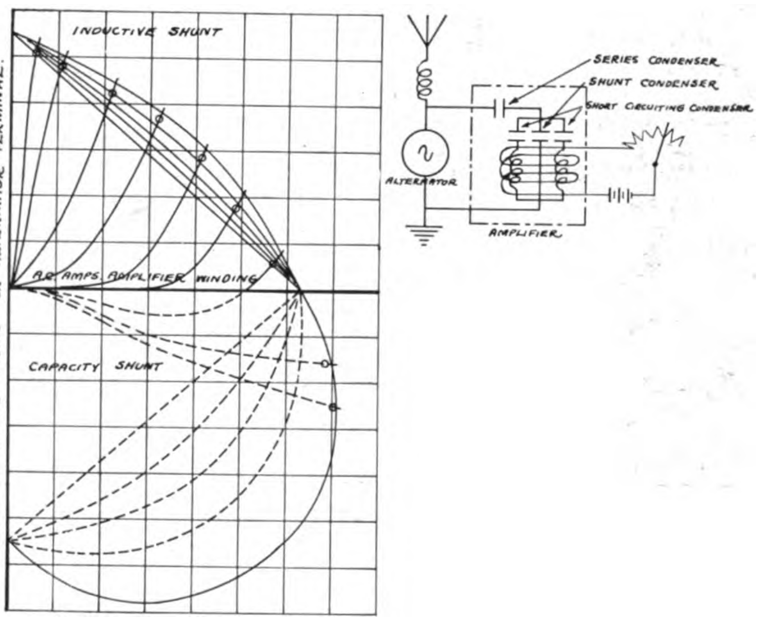


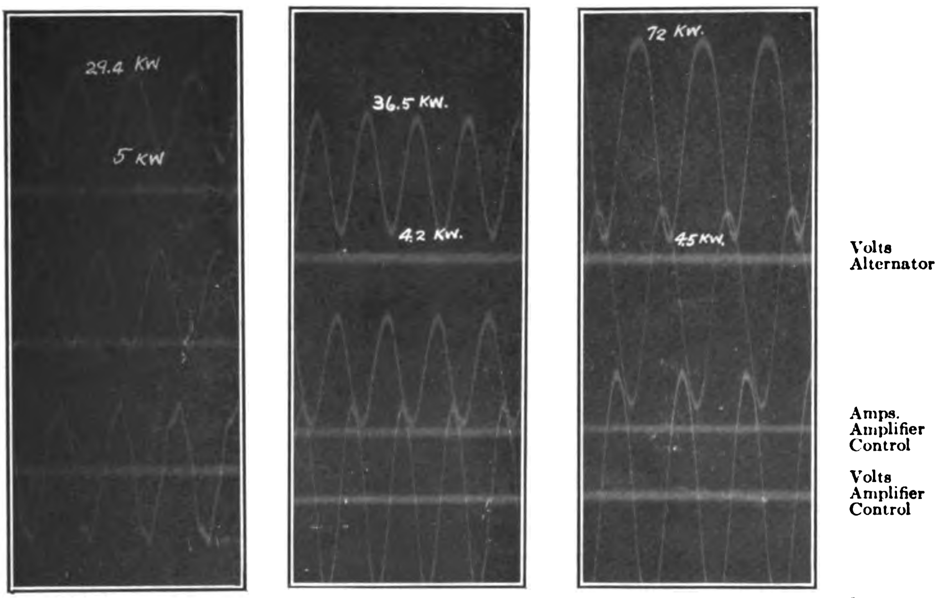
FIGURE 14—Volt-ampere Characteristics of Amplifier with Shunt and Series Condenser Showing Intersection with Alternator Characteristic

ing instead of lagging current at low excitation. Complete characteristic curves of the amplifier with shunt and series condenser and the superimposed alternator characteristics are shown in Figure 14. The series of oscillograms, Figure 15, show the effect of different amounts of shunt condenser. While the series condenser is used within the limits of stability to increase the sensitiveness, the shunt condenser has the object of allowing

the alternator to assume its full maximum voltage. The last oscillogram, Figure 15, shows an alternator output of 72 kilowatts.

APPLICATIONS OF THE AMPLIFIER

The oscillograms of Figure 17 show telephone control of the alternator output. The two curves on the oscillogram, which curves are relatively upside down, show that the variation of



None	0.056 μF.	0.33 μF.	Shunt Condenser
0.33 μF	0.33 μF.	0.33 μF	Series Condenser
224	360	236	Ratio of Amplification

FIGURE 15—Oscillograms Showing Effect of Different Shunt Condensers

the alternator voltage is in all details an almost exact reproduction of the controlling telephone current.

While a specific method of adapting the magnetic amplifier to an alternator as described above has been worked out both theoretically and experimentally in greater detail, there are obviously a variety of possibilities of adapting the same devices and theories to other conditions. Outside of telephony, the magnetic amplifier will probably be found of value as a non-

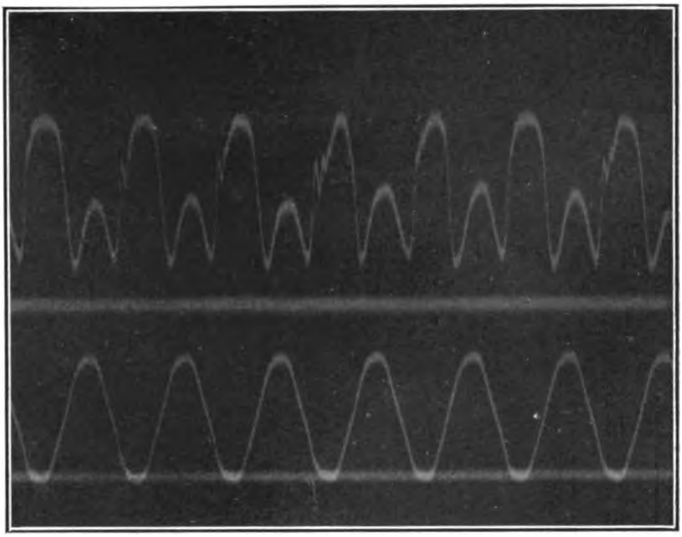


FIGURE 16—Oscillogram Showing Distortion when Range of Linear Proportionality of Control is Exceeded

arcing key for telegraphy, and particularly will make possible high speed telegraphy at the same rate and with the same means as high speed automatic telegraphy on land lines. Oscillograph

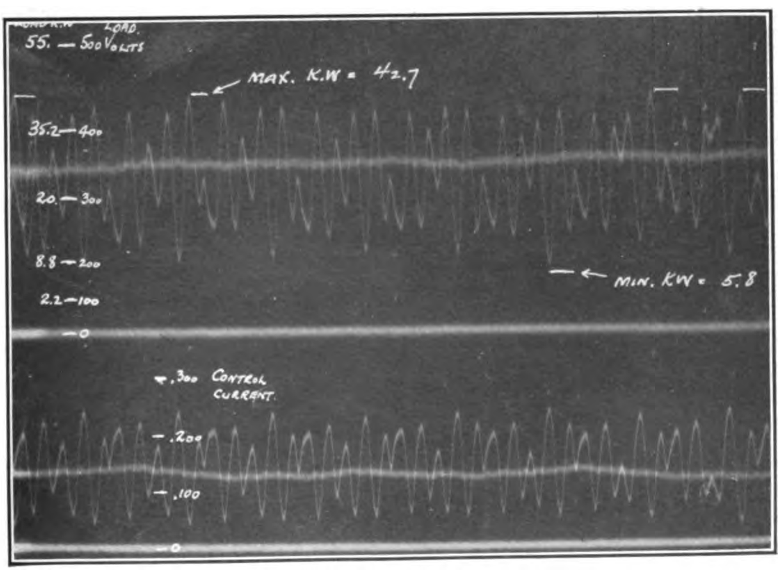


FIGURE 17—Oscillograms Showing Telephone Control

records have been taken of telegraphic control transmitting from 500 to 1,500 words per minute.

The structure and the mode of operation of the magnetic amplifier which has been described is such that there appears to be no limit to the power that might be controlled in this way if the apparatus is designed with suitable dimensions. The 72 kilowatt control which has been demonstrated may be sufficient for most purposes, but there would be nothing surprising if several times this amount of energy were to be used in trans-Atlantic radio telephony or high speed telegraphy in order to make the service thoroly reliable.

SUMMARY: A magnetic amplifying device is described for permitting the control of radio frequency currents by varying the saturation of the iron core of an inductance included in the radio frequency circuit. The arrangement of windings of the amplifier is such that the controlling winding has no radio frequency E. M. F. induced in it, nor does it induce currents in the radio frequency coils.

Various arrangements of this amplifier in connection with a solid steel rotor, radio frequency alternator are shown, notably those in series with the alternator and those in parallel. "Short-circuiting" condensers are connected to each of the radio frequency coils. A shunt condenser across both coils and their "short-circuiting" condensers increases the sensitiveness for reasons which are given. Another condenser inserted in series with the entire amplifier is employed to obtain linear proportionality of amplification and increased sensitiveness. The ratio of amplification is found to be proportional to the ratio of the frequency of the radio current to that of the controlling current.

The control of the output of a 75 K. W. alternator of radio frequency for telephonic purposes is then shown by oscillograms to be accurate, and the numerical characteristics of alternator and amplifier separately and in combination are given.

DISCUSSION

M. I. Pupin: It is not clear to me how the control circuit current is varied. How is this control current obtained?

E. F. W. Alexanderson: Figures 1 and 2 show the controlling current regulated by rheostats in order to produce variations in the controlling current. The rheostat may represent a microphone or any other source of amplified telephone current. In the experiments referred to, the amplification was accomplished by a vacuum tube relay.

M. I. Pupin: Then the variation in the output from 4 to 45 kilowatts is produced by a variation in the saturation of the iron of the shunt impedance?

E. F. W. Alexanderson: The control is accomplished entirely by the variation of saturation in the iron.

J. Zenneck: Mr. Hogan has called attention to the fact that the problem discussed by Mr. Alexanderson is closely analogous to the problem of the frequency doublers. The device used by Mr. Alexanderson consists substantially of two iron cores, which are magnetised by a direct current in the same direction, whilst two radio frequency coils are wound on them in opposite directions. If we put on each of these iron cores a secondary coil and connect these secondary coils in series, we have nothing but the ordinary frequency doubler; in the secondary coils an E. M. F. of double frequency is induced.

This arrangement of frequency doubler has been used some three years ago for radio telephony by the Gesellschaft für drahtlose Telegraphie (Telefunken). Just as has been done by Mr. Alexanderson, they controlled the direct current directly or indirectly by means of a microphone. Just as obtained by Mr. Alexanderson, they got a relay action or amplifying effect; since, in their arrangement, any change of the direct current influenced the E. M. F. induced in the secondary circuit and also the impedance of this circuit and therefore the tuning. In a paper entitled "A Contribution to the Theory of Magnetic Frequency Doublers" which I read before THE INSTITUTE OF RADIO ENGINEERS in September, 1915, this amplifying action was indicated by the fact that, in the equation for the secondary current I_2, the numerator (representing the secondary E. M. F.), as well as the denominator (representing a complicated form of impedance), contained the direct current I_o.

The main difference between Mr. Alexanderson's device and that of the Gesellschaft für drahtlose Telegraphie is that in the former the primary current is affected by the audio frequency control current whereas in the latter the secondary current of the frequency doubler is affected. In other words, Mr. Alexanderson uses the unloaded frequency doubler, the Gesellschaft für drahtlose Telegraphie the loaded doubler.

A detailed description of the devices used by the Telefunken Company has been given in the "Elektrotechnische Zeitschrift," 1914, number 29, and in the "Jahrbuch der drahtlosen Telegraphie," volume 9, 1915, page 502 by L. Kühn. It is well known that this arrangement has proven to be very satisfactory in operation. In 1912 or 1913, a good radio telephonic connection between Berlin and Vienna was thus obtained.

Of course the fact that a magnetic amplifying arrangement has been previously used for radio telephony does not in any way detract from the credit due Mr. Alexanderson; his device showing so many interesting features differing widely from those hitherto used.

Louis Cohen (communicated): Hitherto it has been the practice of radio engineers to avoid the use of iron in any form in oscillatory circuits, because of the eddy current and hysteresis losses that would be thus introduced. Mr. Alexanderson has shown, however, by his splendid researches that this is not generally true. If proper attention is given to lamination and design, iron may be used in radio frequency circuits without the accompanying losses hitherto thought inevitable. The investigations of Mr. Alexanderson on the effect of frequency on circuits containing iron paved the way for the development of the magnetic amplifier discussed in this paper. The authors are to be congratulated on their achievement in the development of this device. It unquestionably represents fine engineering skill in the conception of the method and the working out of the design.

It appears to me that the fundamental principle of the method for amplification discussed in the paper, namely: the variation of inductance of a tuned circuit by a change in current in an auxiliary circuit, will find its application to other problems in connection with radio frequency circuits. One that suggests itself immediately is the amplification of incoming signals in radio telegraphy. Suppose the controlling circuit B is connected in the antenna, and the coil A is part of a separate tuned circuit

excited by an alternator, or coupled to an arc circuit (any suit. able arrangement will do so long as the oscillations in the circuit comprising coil A are forced), then any signal acting on the antenna will by the current generated thereby in coil B produce a change in the permeability of the iron core and thus change the inductance of coil A. If the circuit containing coil A was originally tuned to the frequency of the impressed E. M. F., then a change in inductance will cause a considerable change in the current in that circuit, which may be a great many times larger than the current in the antenna. It may be that the authors have already considered the use of their method for this particular purpose, and perhaps found it unsuitable; but if so I am sure that other radio engineers as well as myself would be glad to hear from them on this point.

In regard to the use of the shunt condenser shown in Figure 14, it appears to me that the improvement produced by it is due to the fact that a loop circuit is thus introduced. The condition may be represented diagrammatically as shown in Figure 1. If

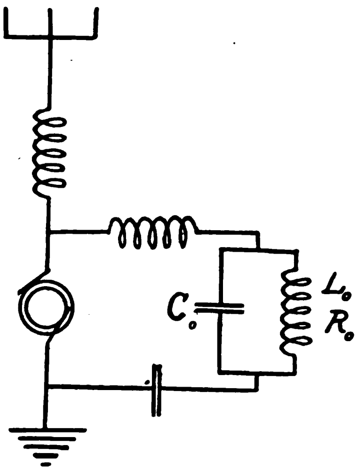


FIGURE 1

the circuit is tuned as a whole to the frequency of the impressed E. M. F., then a change in the inductance L_o of the loop circuit not only detunes the circuit, but also introduces a change in resistance of the circuit which may under certain conditions be very large. In fact the impedance of the loop circuit ($R_o\, L_o\, C_o$) is

$$Z = \frac{R_o}{(1 - L_o C_o \omega^2)^2 + R_o^2 C_o^2 \omega^2} + j \frac{L_o \omega (1 - L_o C_o \omega^2) - R_o^2 C_o \omega}{(1 - L_o C_o \omega^2)^2 + R_o^2 C_o^2 \omega^2}.$$

If the value of $L_o C_o \omega^2$ differs from unity; that is, the loop circuit is not tuned separately to the frequency of the alternator, then we may neglect the terms $R_o^2 C_o^2 \omega^2$ and $R_o^2 C_o \omega$ as being very small in comparison with the term $(1 - L_o C_o \omega^2)$, and we have

$$R' \text{ (effective resistance)} = \frac{R_o}{(1 - L_o C_o \omega^2)^2}.$$

It is obvious, therefore, that a change in the inductance of the loop circuit not only changes the tuning constant of the circuit, but causes also a change in the effective resistance of the circuit. If the authors could give us the data on the constants of the circuits they used, it would be interesting to calculate the change in resistance of the circuit thus produced.

Now, while we are on the subject of the use of iron in radio frequency circuits, I may be permitted to mention a somewhat different method for amplification by the use of iron in the circuit, that I have thought of some time ago. I never had the opportunity to make any experiments, and I am offering this in the form of a suggestion.

The principle of the method is indicated in Figure 2. The loading coil of the antenna contains an iron core (a), preferably

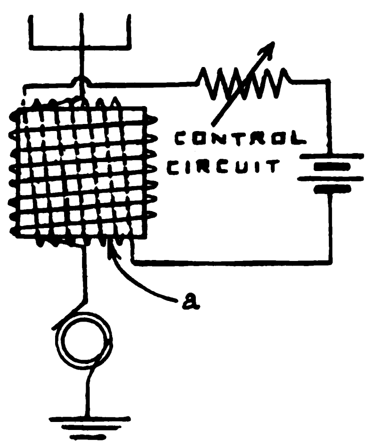


FIGURE 2

in the form of a bundle of fine iron wires, and which forms part of the control circuit. It is evident that the magnetic fields in the core produced by the currents from the antenna and control circuit are at right angles. A greater current flow in the control

circuit causes a greater twist of the iron molecules at right angles to the alignment produced by the current in the antenna circuit, and this would represent a change in the magnetic flux of the loading coil; which means, of course, a change in the value of the inductance, thus causing a detuning of the antenna circuit. I have made some preliminary calculations which lead me to believe that considerable amplification could be obtained by this method. To obtain the best results, however, considerable care will have to be exercised in the design, the dimensions of coil, the lamination of the core, etc.; so that a small current in the controlling circuit may produce sufficient cross magnetisation to change appreciably the inductance of the loading coil in the antenna circuit.

Alfred N. Goldsmith: Professor Zenneck has pointed out a similarity between the direct-current-controlled frequency doubler and the Alexanderson magnetic amplifier. The enormous advantage of the Alexanderson device is that the direct-

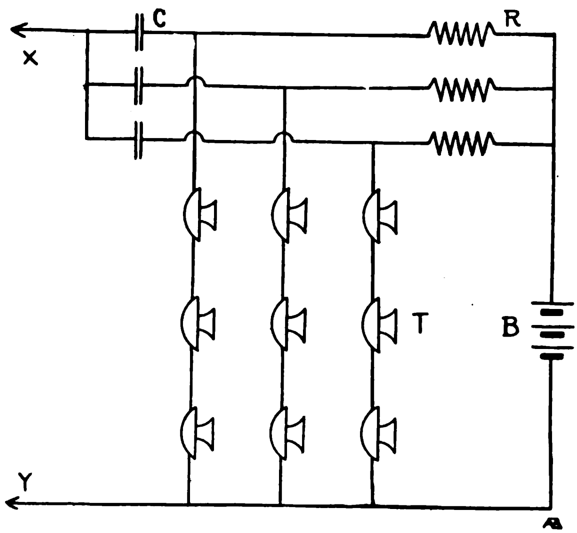


FIGURE 3

current-control circuit has no large radio frequency electromotive forces induced in it. On referring to Dr. Kühn's paper, cited by Professor Zenneck, it will be found that very special means had to be adopted to prevent the radio frequency currents

induced in the control circuit from becoming extremely dangerous.

In connection with the obtaining of a sufficiently powerful control current, Dr. Kühn used the ingenious arrangement shown in the figure. Here the transmitters T, arranged in sets of say three in series, are worked in parallel by the use of steadying resistances R which prevent current overload on any one set of transmitters. By the use of the fixed condensers C, all the transmitter sets are, in effect, placed in parallel so far as the alternating current obtained from them is concerned. The output, which is the control current, is tapped from the points X, Y.

J. Zenneck: It seems to me that Professor Goldsmith's statement that no radio frequency electromotive forces are induced in the control circuit of the Alexanderson amplifier is incorrect. It would be correct if the iron cores were not unsymmetrically saturated. But, under the actual working conditions, an E. M. F. of double frequency would be obtained in the control circuit of the Alexanderson amplifier just as in the Telefunken device.

On the other hand, a quantitative difference arises from the fact that in the Alexanderson amplifier not all the current supplied by the alternator flows thru the radio frequency winding on the iron cores. It is, however, difficult to say whether the double frequency E. M. F. in the direct current circuit will be less than in the case of the controlled frequency doubler.

E. F. W. Alexanderson: Professor Zenneck points out that an electromotive force of double frequency is induced in the controlling circuit of the magnetic amplifier on account of the iron being unsymmetrically magnetised. This is true when the two alternating current coils are used in the series connection as shown in the upper diagram of Figure 6. On the other hand, the preferred multiple connection, as shown in the lower diagram, has the characteristic of suppressing the second harmonic by forming a local short circuit. It is the short circuit for the second harmonic which affects the resulting permeability of the iron in the way illustrated in the characteristic curves of Figure 6, and results in increase of sensitiveness. Measurements have been made of the induced double frequency voltage in the control windings with the alternating current coils in series as well as in multiple. If conditions are made favorable for the generation of the second harmonic, very high voltages appear in the control windings which were, in one case, actually damaged by this voltage.

But, inasmuch as the object of the magnetic amplifier is not to produce a second harmonic, it is easy in various ways to suppress the same and the method shown in Figure 6 has in addition the advantage of improved characteristics of the amplifier.

Lee De Forest: Mr. Alexanderson's paper is indeed exceedingly interesting but I regret that it was too short. Just where it became the most interesting for me, it stopped. I would have liked to have heard some more of the exact part played by the amplifying audion, that interesting device with so many new names.

I would like to direct comparison between this high power radio telephone method and the method recently developed by the Western Electric Company and tried out at Arlington, where the audion amplifier principle was worked to the limit. In that case they started, as Mr. Alexanderson did, with a microphone and amplified the voice currents thru an audion amplifier, and kept on amplifying them. In the first oscillating audion circuit the radio frequency currents were modulated by the microphone, then these modulated radio frequency currents were amplified in a bank of some twenty bulbs, and finally the current from this bank was amplified in a bank of 500 bulbs.

This ensemble of amplifying audion tubes put a total energy of 11 kilowatts in the antenna; beautifully controlled by the voice, it is true, but having the modest upkeep renewal cost of something like ten thousand dollars per month. So as a practical engineering proposition, there is absolutely no comparison between that method and the method worked out by the General Electric Company. I believe that I am entitled to express that sort of opinion of the audion if anyone is! This is the situation as it stands to-day. No one can say, however, that the situation will not be altered very materially in one, two or three years, after we learn how to build oscillions for large power outputs, say for 5 or 10 kilowatts each. That will create a very different situation. It is difficult to say, therefore, which of the two methods discussed possess the greatest practical promise.

It is obvious that there are going to be two classes or two lines of development of high-power radio telegraphy and telephony. Since the present seems to be an era of word coining, I propose the term "*sans-ferric*" as applying to methods such as the oscillion and Poulsen arc, in which no iron whatever is involved in the radio frequency generating apparatus, as distinguished from the several alternator methods which are now being developed.

The method of generating sustained oscillations by the oscillion, on a large scale, has been thus far worked out by telephone engineers who were familiar, first of all, with the audion amplifier in long distance telephone lines. They were deeply impressed by the possibilities inherent to this device, and they worked out their problem as telephone engineers could be expected to work it out, from the telephone engineers' point of view, rather than that of a radio engineer.

On the other hand, the General Electric engineers have tackled the problem from a power engineering standpoint. The paper of Mr. Alexanderson therefore is a little less clear to radio engineers than it might have been. Too little has been said therein about the tuning problems involved, particularly when the amplifier circuit including series and shunt condensers is connected, across the radio frequency alternator. It would be very interesting to us to know just what are the oscillation constants of this amplifier circuit in combination with the antenna, and alternator, for different frequencies and just what amount of detuning is involved to obtain the extraordinary variations of radiated energy amplitude which Mr. Alexanderson's curves illustrate. For, after all, it is largely a question of detuning, of signaling by means of change of wave length.

E. F. W. Alexanderson: In connection with the subject of amplifying telephone currents, it should be pointed out that the magnetic amplifier can itself be used for amplifying in several stages. The radio frequency alternator is used for furnishing the energy for the first stage amplifications and the radio frequency energy which is modulated by a magnetic amplifier is then rectified. The amplified telephone current thus obtained is used for controlling a magnetic amplifier in a second stage. I do not wish to express any opinion as to the relative merits of this method of amplification compared with a vacuum tube relay; however, there are a number of acceptable methods of amplifying the telephone current to the magnitude necessary for controlling the high power device described in the paper.

The suggestions made by Mr. Cohen are much to the point. The possibility of using the magnetic amplifier in a receiving circuit has been thought of and the portion of the paper dealing with audibility was partly included to give suggestions in that direction. In regard to the change of effective resistance accompanying the change of inductance, these relations can be treated mathematically as Mr. Cohen

points out. As an approximate basis, it can be assumed that the power factor of the iron core inductance is 35 per cent, when not saturated, and that the effective resistance decreases proportionately to the square of the inductance when saturated.

VARIATIONS IN NOCTURNAL TRANSMISSION*

By

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INTRODUCTION

The following is an account of certain experiments in nocturnal transmission, carried out by radio stations "9XN" (University of North Dakota, Grand Forks) and "9XV" (Washington University, St. Louis, Missouri), in an attempt to test the interference theory of "fading" and "swinging" effects, and to correlate, if possible, transcontinental weather conditions with radio transmission.

The radiation current at Grand Forks being limited to 13 aerial amperes, and that at St. Louis to 7, the observations were necessarily limited to the periods of twilight and total darkness; and since the distance between the stations is 1,250 kilometers (780 miles), no observations were possible during the mid-summer period of violent strays and bad transmission.

Incomplete as the work is, it is hoped that certain features of it are of sufficient interest to be presented at this time.†

PRELIMINARY DISCUSSION

An immense number of observations on transmission variations must have been made by operators, amateurs, and others, but comparatively little of this data is available for study and criticism.

A number of important papers have however been published.

*Presented before The Institute of Radio Engineers, New York, January 5, 1916, by Professor Taylor for the joint experimenters and authors.

†It should be stated that work in this direction was undertaken earlier between the University of North Dakota, then "9YN," and "8XA," the University of Michigan; but was tacitly abandoned on account of various experimental difficulties due principally to lack of sufficient radiation at that time at "9YN," and to the use of a wave whose length was poorly adapted to the Michigan aerial. A breakdown of the Michigan primary condenser putting a stop to such tests as were then being attempted, the work was taken up under more favorable conditions at Grand Forks, with St. Louis.

Austin's investigations have established an empirical formula for transmission (over salt water) which is identical in form with the Sommerfeld equation for intermediate distances, except that the exact value of the absorption term and its manner of dependance on wave length seem to be still a matter of dispute.

A wide study of seasonal variations in transmission has been made by Marriott[1] using what might be called a statistical rather than a quantitative method. The paper is full of valuable material, and shows clearly the well-known seasonal variation. It refers mainly to over-sea transmission.

Austin has curves showing quantitatively the seasonal variation in daylight overland transmission between Washington and Philadelphia, 185 kilometers, and Washington and Norfolk, 235 kilometers.[2] His results are in good agreement with those of Marriott, altho wide variations in individual observations are noticeable.

Overland transmission is more irregular, and experiences a wider seasonal variation than that over the sea. It shows a very high daylight absorption and variations depending on the nature of the country traversed.[3] Nocturnal overland transmission is especially erratic, and seems to depend to a certain extent on the weather conditions of the preceeding day.[4] One of the writers has shown that cloudy weather during the daytime makes for good nocturnal transmission, especially if the cloudiness has been quite general, and markedly so if the cloudiness has prevailed in the neighborhood of sender only.[5]

Cloudiness, in the immediate vicinity of receiver only, has little or no effect.

It has been suggested[6] that the effect was due to a modification of the soil conditions by rain, but this has been disproved by observations taken at Grand Forks on "VBA," Port Arthur, Ontario, during a six weeks period in the fall of 1913, when, altho often cloudy, not a drop of rain was recorded in the territory between Port Arthur and Grand Forks. The beneficial effect of cloudiness was nevertheless almost uniformly noted in this period.

The received energy in nocturnal transmission often attains

[1]"Radio Range Variation," "Proc. I. R. E.," March, 1914.

[2]"Seasonal Variations in the Strength of Radiotelegraphic Signals," "Proc. I. R. E.," June, 1915.

[3]A. H. Taylor, "Diurnal and Annual Variations in Radio Transmission," "Phys. Rev.," Nov., 1914.

[4]A. H. Taylor, "Wireless and Weather," "Electrical World," Aug. 30, 1913.

[5]A. H. Taylor, "Radio Transmission and Weather," "Phys. Rev.," May, 1914.

[6]"Electrical World," Aug. 30, 1913. Editorial note.

values so high that divergence according to the inverse square law seems impossible. That this may be due to a combination of reflection and refraction has been suggested by Eccles,[1] Kennelly,[2] Barkhausen,[3] and Kiebitz.[4] The first three papers mentioned deal with refraction and reflection in very high ionized layers. In connection with this it is interesting to note Fleming's recent suggestion that electrons driven off from the sun by radiation pressure may reach the earth's atmosphere.[5] According to Humphreys,[6] cosmic dust may also play a role in ionizing the outer layers of the earth's atmosphere. Thus ultra-violet light may not be the only agent tending to produce a difference in wave velocity at different levels.

The theory of Kiebitz deals with the increase in refractive index brought about by the presence of water vapor at relatively low altitudes. The effect would be to refract the waves unfavorably for good transmission.

None of these theories would seem to have anything to do with the favorable effect of clouds at low levels, except that of Kiebitz, which predicts an effect opposite to that observed.

One of the writers[7] has therefore suggested that a portion of the energy is reflected from a surface of electrical discontinuity at the cloud level, thus passing from sender to receiver as between two roughly parallel surfaces of earth and clouds, and not following the inverse square law. This portion, passing thru a region known to be weak in ionization, is feebly absorbed.

During the day time, the sun in some way destroys this layer of electrical discontinuity.

A second portion, entering the middle ionized portion would be refracted back according to the Eccles theory, especially during nocturnal transmission.

A third portion, passing completely thru the middle ionized region would be reflected at the upper, permanently ionized (Heaviside) layer. It would be heavily absorbed by day and less heavily at night.

A fourth portion of the radiation would pass out into space and be lost; and. no doubt, a surface wave travels in the crust of the earth. It is not clear to us, however, that this surface

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[1] "The Electrician," Sept. 27, 1912, and Sept. 19, 1913.
[2] "Proc. I. R. E.," July, 1913.
[3] "Elektrot. Zeitschr.," Heft 16, 1914.
[4] "Jahrbuch der drahtlosen Teleg. und Telephonie," June, 1913.
[5] Royal Inst. lecture on "Photo-electricity." See "Scientific American Supplement," July 10, 1915.
[6] "Astophysical Journal," May, 1912.
[7] A. H. Taylor, "Phys. Rev.," May, 1914.

wave should be considered separately. The application of suitable boundary conditions to the general wave equations causes the appearance of a term interpreted as a surface wave but a mathematical surface cannot contain energy. If the energy transmitted thru the earth is to be reckoned with, it must be as a volume distribution of energy and, as such, it would be rapidly absorbed, so as to be of little moment in long distance work.

If the original wave is split by reflection and refraction into several components of differing phase, it is probable that many of the vagaries of nocturnal transmission are due to interference. This point will be kept in mind in discussing the results to follow.

DESCRIPTION OF APPARATUS

The St. Louis station radiated 7 aerial amperes at 850 meters wave length with a logarithmic decrement of 0.14 and a spark pitch of 700 per second produced by a 2-gap rotary discharger, and 70-cycle transformer. The St. Louis aerial is 30 meters in height, 8.5 meters in breadth, and 100 meters long. It consists of 8 wires, their size being 7 strands of number 20 wire,* with a lead taken 15 meters from the north end.

The St. Louis receiver consisted of an inductive coupler, tuning condensers, 3,200 ohm Brandes telephones, and an audion detector. The shunted telephone method of measuring audibility was used at St. Louis thruout the tests.

Grand Forks used the following waves during the tests:

Wave length	Aerial amps.	Log. Decrement	Height to center of capacity
1500 m	12.0	0.11	25 m
850 m	12.5	0.09	27 m
500 m	11.5	0.18	27 m

The 1,500 meter-wave was radiated from a three-wire antenna 6 meters broad at the far end, and 4 meters broad at the near, or lead-in end. This aerial is 245 meters long, but field tests have so far not detected any marked directivity in the 1,500 meter-wave.

Since the aerial points 15° east of south, the directive effect, if present at all, would be unfavorable for St. Louis. The capacity of this aerial is 0.013 microfarad.

The other two waves were radiated from a 46 meters long, 5 wire aerial, 4 meters in breadth, the lead-in connection being taken from a point 15 meters from the near end.

This antenna has a capacity of approximately 0.0012 micro-

*Diameter of Number 20 wire = 0.032 inch = 0.81 millimeters.

farad. Number 14 stranded phosphor bronze wire is used in both aerials.

A spark pitch of 1,100 per second, from a 60-cycle transformer supply, was obtained by the use of an 8-gap rotary discharger giving a well quenched spark.

The receiving set used for the St. Louis tests comprised an inductive coupler, tuning condensers, 3,200-ohm telephones, and a double audion receiver.[1]

The long aerial, with series condenser was found to be the better for reception of all waves exceeding 750 meters in length.

The Grand Forks observations were rather unsatisfactory, partly on account of the smaller radiation at St. Louis, and partly on account of an attempt to use a special method of quickly getting the audibility with the audion. This method was later found to be unreliable, and the shunted telephone method was used thereafter.

The writers are well aware of the criticisms which have been made to the method of the shunted telephone, [2], [3], but feel that the simple shunt ratio used with the audion receiver gives a fair estimate of the relative strength of signals if the *effective* values, at the received spark frequency, of telephone resistance and inductance be used in computing the shunt ratio. If the spark has harmonics, the tone that fades away last in shunting out a signal should be that of the frequency used in the shunt ratio calculation. At 750 cycles, the effective resistance of a 3,200-ohm "New Navy" telephone is 6,200 ohms, while its inductance is 1.48 henrys. At 1,000 cycles, the figures are 7,250 ohms and 1.36 henrys. We are indebted to Mr. H. S. Sheppard of the Department of Electrical Engineering, University of Michigan, for the graph from which these figures are taken. Individual telephones, even of the same make, will probably show variations, but these values show how widely the telephone impedance may differ from the ohmic resistance.

METHOD OF TESTS

Observations on signal strength were taken hourly thruout the night, with 15-minute periods near sunset and sunrise. Primarily two waves were used by Grand Forks, 1,500 meters and 500 meters, with a 2-minute interval necessary to make the shift in aerials and in transmitter connections, as the transmitter

[1]A. H. Taylor, "Electrical World," March 13, 1915.
[2]F. Braun, "Jahrb. der drahtlosen Teleg. und Telephonie," April, 1914.
[3]L. Israel, "Proc. I. R. E.," June, 1915.

is not located in the same room with the receiving and controlling apparatus. After a two minute rest, the St. Louis wave was observed. The calls were of 3-minute duration. The wave lengths were chosen by both stations of such values as to create a minimum of interference with other stations. During part of one test, a seconds ticker was used at Grand Forks for sending the signals, but irregular and rapid sending, usually with an automatic key, was found more satisfactory and less deceptive.

Observations on general transmission conditions from other stations were made from time to time during the night; and an attempt was made in all cases except one, subsequently mentioned, to have the sensibility of the receiver as nearly uniform as possible by checking the audion with a standard buzzer test, set for the desired wave length, or with a perikon detector of very constant adjustment., It is a matter of regret that the conditions of constant sensibility were not fulfilled at Grand Forks as well as could be desired, inasmuch as two audion bulbs were burned out during the tests, making unavoidable some alterations in sensibility.

The tests were run on the nights of December 23, 1914, January 7, January 28, March 6, April 17, and June 10, 1915. Besides these tests, the two stations had a standing appointment for 9:30 and 10:00 P.M., Central time, on every Monday and Thursday evening until about May 1, when the St. Louis signals were no longer audible at Grand Forks.

RESULTS

Curves I, II, and III of Figure 1 present the data of the first test, on December 23, 1914. The Grand Forks waves show the greatest regularity in this test, but pronounced fading effects were observed in the St. Louis wave, altho it was audible from 7:45 P.M. to 7:40 A.M. Curve I shows the reception of the 500-meter wave, and curve II the 1,500-meter wave at St. Louis. curve III shows the reception of the 850-meter wave at Grand Forks.

Figure 2 shows the weather map* for December 23. Cloudiness evidently prevailed over the larger portion of the region between the stations, and the map for the following day (not shown), indicates that Grand Forks was changing from cloudy to clear, with the reverse true for St. Louis.

The extraordinary regularity of the 1,500-meter wave during

*The writers wish to acknowledge their indebtedness to Prof. H. E. Simpson, of the Special U. S. Weather Bureau, University, N. D., for the loan of the maps.

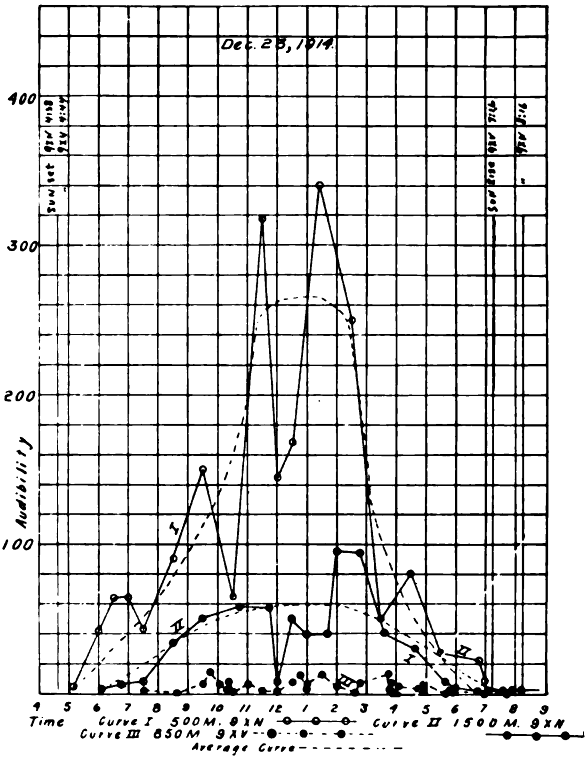

Figure 1

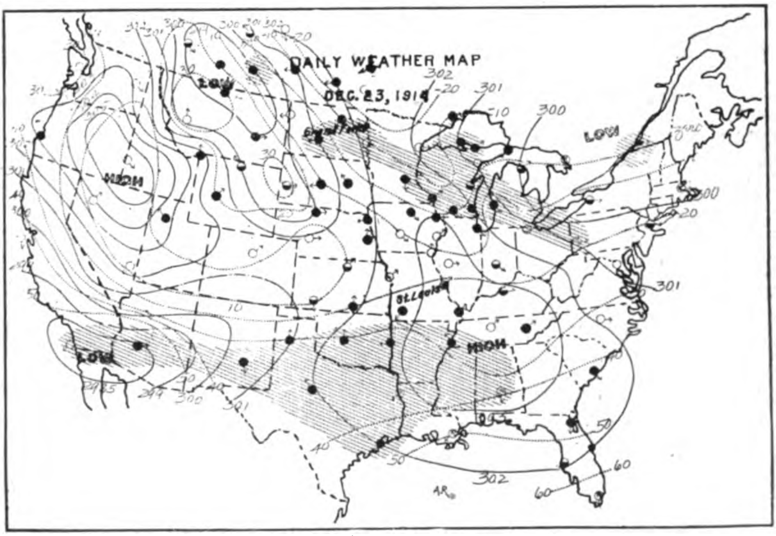

Figure 2

the first half of the night contrasts sharply with the fluctuations of the 500-meter wave. It is doubtful whether the fluctuations of the 850-meter wave are synchronous with those of the 500-meter wave. Experience has shown that fluctuations may occur so rapidly that tests for reciprocity of transmission must be made almost simultaneously to be of any great value. Further special tests are contemplated on this point.

The 1,500-meter wave shows wide variations between 11:45 and 3:40, which are almost exactly duplicated by the 500-meter wave. This was not due to maladjustment of the St. Louis receiver, as the operator reported that other stations to the south and east, sending in this interval, showed little change in intensity.

For neither wave do the curves seem to be symmetrical about the solar midnight. Grand Forks reported good transmission from the south and east, and fair transmission from the west. St. Louis reported transmission in general not better than the average for mid-winter. Good transmission was reported there from south and east.

Enormous variations in the signals of various amateurs in Michigan and Ohio on short waves were reported at Grand Forks and similar results at St. Louis.

The 1,500-meter wave persists well into daylight, being still received at 8:15 A.M. when discontinued. The St. Louis wave, of 850 meters, altho varying widely at Grand Forks, was reported at Memphis as fairly steady. (Memphis is about 250 miles (400 km.) further from Grand Forks than St. Louis and on the same line approximately.)

Taking the audibility of the 1,500-meter wave at 8:00 A. M. as 2, and assuming the midnight audibility of 58 as corresponding to a nearly unabsorbed wave, the absorption coefficient of the Austin-Cohen formula is calculated as 0.00158 at 8:00 A.M. The oversea value given by Austin is 0.0015. Subsequent tests showed this wave to disappear after 10:00 A.M. in mid-winter, and not to be audible at all during the daytime after early spring. Comparing, in a similar way, the midnight audibility of 232, on the average, for the 500-meter wave, with its twilight audibility of unity often observed at St. Louis in mid-winter at 5:00 P.M., the value of the absorption coefficient at twilight would be 0.00155. The value of the expression $i^2 h^2/\lambda^2$, (where i = sender current, h = height to center of capacity, and λ = wave length), is 6.6 times as large for the 500-meter wave as for the 1,500-meter wave. This would give the ratio of audibilities, if unabsorbed

or equally absorbed, received thru equivalent aërial resistance for each wave, as 6.6 in favor of the short wave.

The observed ratio as deduced from the curves of Figure 1 is 4.0. Data on the equivalent aerial resistance at St. Louis is not available, but since a series condenser is used for the 500-meter reception, it is likely that the resistance for that wave is higher than for the 1,500 meter. The ratio 4.0 is therefore probably too small; and we may conclude that the two waves are either not much absorbed in mid-winter at midnight, or that they are equally absorbed. The latter possibility is not to be seriously considered. These speculations are not based upon peak values, but upon average audibilities in the middle of the night.

The absorption at mid-day, even in mid-winter is certainly much higher; as is also the mid-summer midnight value.

Figure 3 shows the data obtained on January 7, 1915, which was a night of very poor transmission between Grand Forks and St. Louis, altho Grand Forks reported excellent transmission from east coast stations. The weather map, Figure 4, shows very cloudy conditions prevailing to the east of Grand Forks, with mixed weather, largely clear, to the south.

The short wave was audible only after 8:30 P.M. The long wave was weak, but persistent at both ends of the night and until 9:30 A.M. The wide fluctuations in the two waves are nearly synchronous.

The St. Louis wave, 850 meters, was audible at 8:42 P.M., but repeatedly inaudible during the night, reaching a maximum about midnight and disappearing just before sunrise at Grand Forks.

Figure 5 and Figure 6 present the data of January 28, which was a night of generally fair transmission. The weather map shows average condition of cloudiness.

All three waves are very irregular, the shortest showing a high value abnormally early, and reaching its best transmission at 1:30 A.M. The 850-meter wave shows its highest maximum and most rapid fluctuation at 4:45 A.M. The 1,500-meter wave is also erratic, and much stronger than the 500-meter between 2:30 and 4:30. This has never been observed before or since except in early morning or evening, when the transmission was poor for both waves. The maximum at 3:40 A.M. is extraordinarily high for this wave.

The variations in the long and short waves seem to be synchronous between 9:00 P.M. and midnight, but at 1:00 A.M. are

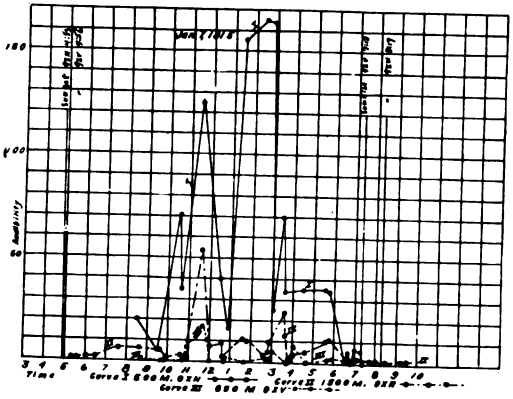

FIGURE 3

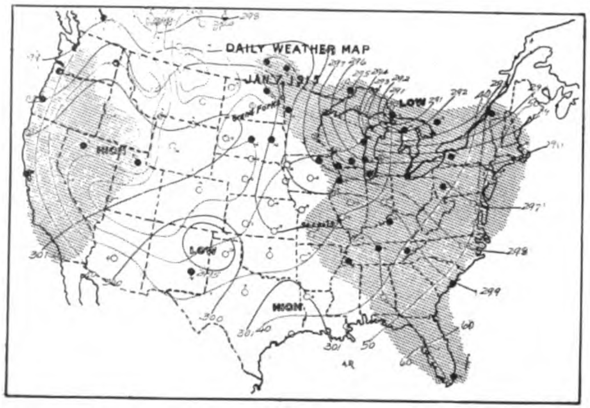

FIGURE 4

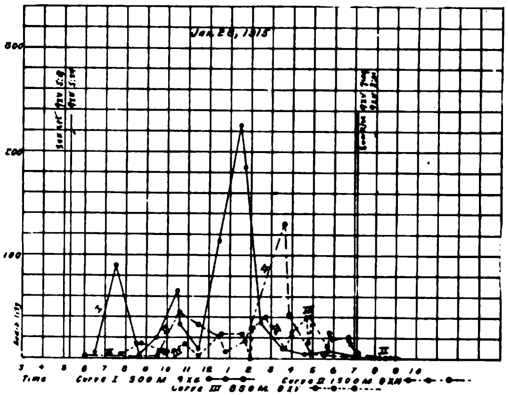

FIGURE 5

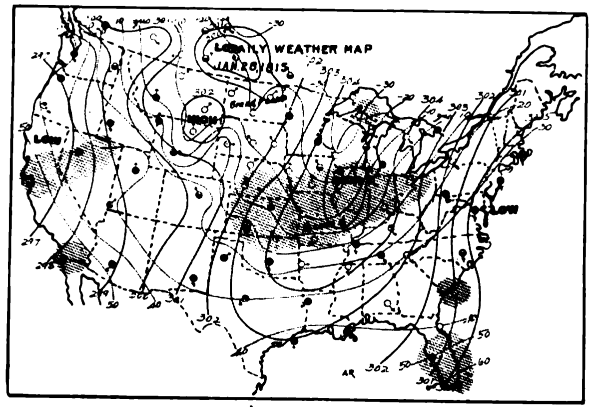

FIGURE 6

in opposite directions. It would seem as tho conditions affecting average transmission are not necessarily the same as those causing fading and "freaks."

Figures 7 and 8 show the results of the test of March 6. The transmission was good for the spring season, but noticeably poorer than in mid-winter. There was general cloudiness between the two stations. In this test Grand Forks used a third wave, of 850 meters, to determine whether fading was reciprocal. A leaky insulator gave the 1,500-meter wave a higher decrement and somewhat smaller radiation than usual, which may partly account for its being so weak at St. Louis. Unfortunately this was not discovered until after the next test and the exact date of the appearance of the leak is not known. This wave was quite steady, showing no trace of the wide variations present in the 500-meter wave at 8:40, 11:40, 1:40, and 3:40. The 850-meter wave of Grand Forks (curve IV), shows variations, but they are not so great as those of the 500-meter wave, nor are they synchronous with them. This wave was not started until 10:00 P.M.

The abnormally high audibility of St. Louis at Grand Forks (curve III), on 850 meters, was due to the use of the double audion there in ultraudion adjustment.* It is not always possible to obtain this adjustment with short waves but it was done on the night of this test with two new bulbs, which subsequently refused to take the adjustment under 2,500-meter wave lengths. It is very doubtful whether the shunted telephone method is reliable with the ultraudion, but since the calculated audibilities will at least show the trend of the transmission, they are here presented.

The steadiness of the St. Louis wave early in the evening is remarkably good, and the general trend of the transmission for both 850-meter waves is the same in the later half of the night, but the variations certainly do not seem to be synchronous. Both 850-meter waves disappear 30 minutes before sunrise at St. Louis; but the 1,500-meter wave, weaker in the night, persists until sunrise at Grand Forks.

About the first of April, 1915, and on a few days later in 1915, a sudden falling off in the transmission was noted. The curves of Figure 9, for the test of April 17, show the 1,500-meter wave to have been inaudible thru the greater part of the night.

There was a very extraordinary clearing up between the stations in the early morning, extending, at St. Louis, to eastern

*See "Electrical World," March 13, 1915; and Armstrong, "Proc. Inst. Radio Engineers," September, 1915, page 224 *et seq.*—EDITOR.

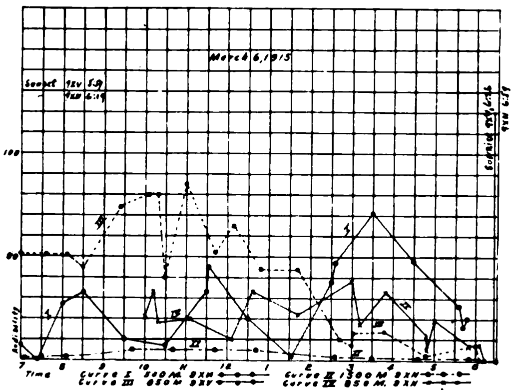

FIGURE 7

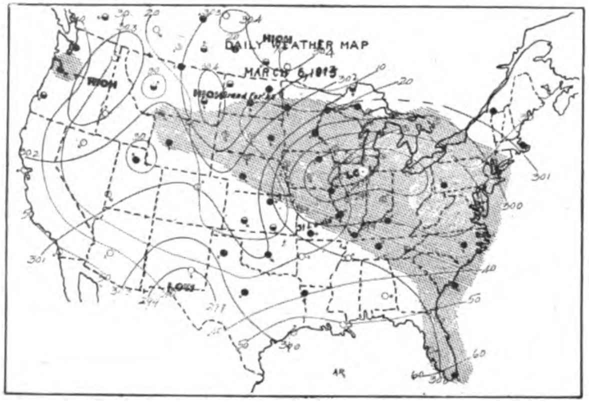

FIGURE 8

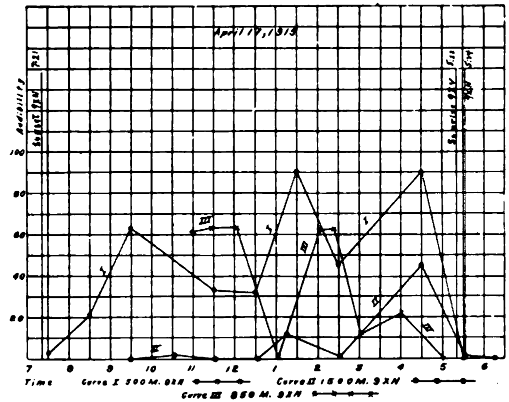

FIGURE 9

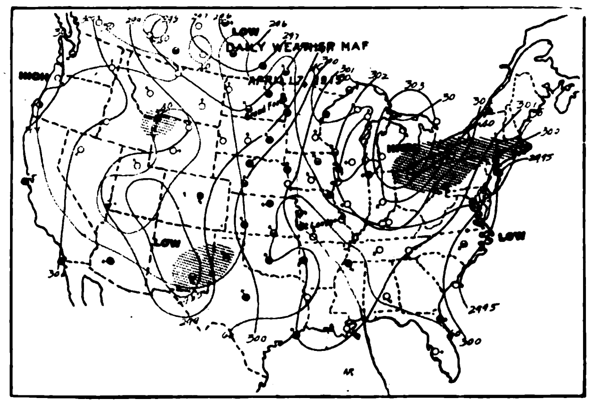

FIGURE 10

and Gulf stations. "NAA" (Arlington, Virginia) was reported at 9:00 P.M. Central time at Grand Forks as unusually strong for the time of year. The St. Louis wave was not picked up until 1:45 A.M., at an audibility of 3, which was its maximum. No curve for this wave is shown.

The 500-meter wave, picked up first at 7:33 P.M. was last heard at 4:46 A.M., while the 1,500-meter wave, picked up first at 10:35 P.M. persisted until 5:36 A.M.

The 850-meter wave of Grand Forks, started at 11:00 P.M., was received with an audibility of 63, and disappeared, as far as can be determined, at about the same time as the 500-meter wave. Both show maxima between 3 and 5 A.M., with disappearance between 5 and 6. The wide fluctuations of these two waves do not appear to be synchronous. On the other hand, the synchronism of the fluctuations of 500-meter and 1,500-meter waves is very striking. The maximum of the 850-meter wave at 2:00 A.M. corresponds roughly to the best reception of the St. Louis wave of the same length at 1:45 A.M.

Figure 10 shows the weather conditions. General cloudiness prevailed between the stations, probably accounting for the fact

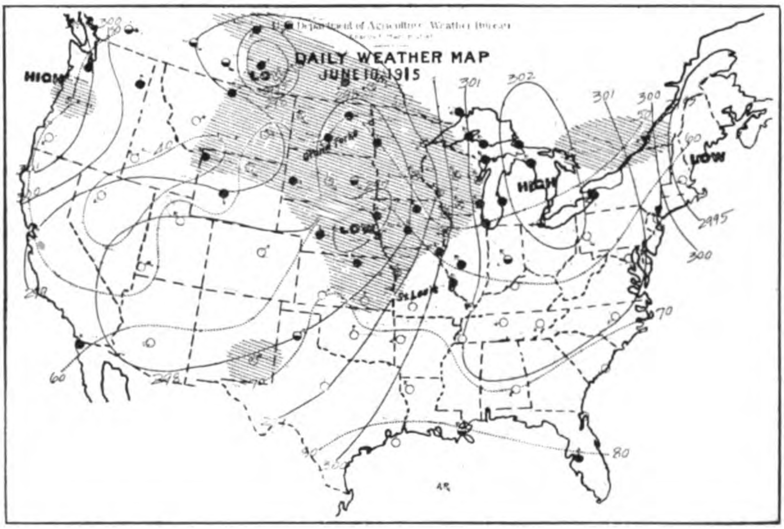

FIGURE 11

that the transmission was unusually good for so late in the spring.

The next test was made on the night of June 10, without

either station picking up any of the waves used. Strays were very numerous, especially at St. Louis; but the signals should have been audible, if not readable, had they come thru.

Figure 11 shows general cloudiness between stations, which, according to previous tests should have produced as favorable conditions as are possible at this time of the year.

Figures 12 and 13, showing the rapid disappearance of the snow and ice sheet between the stations at this time of the year

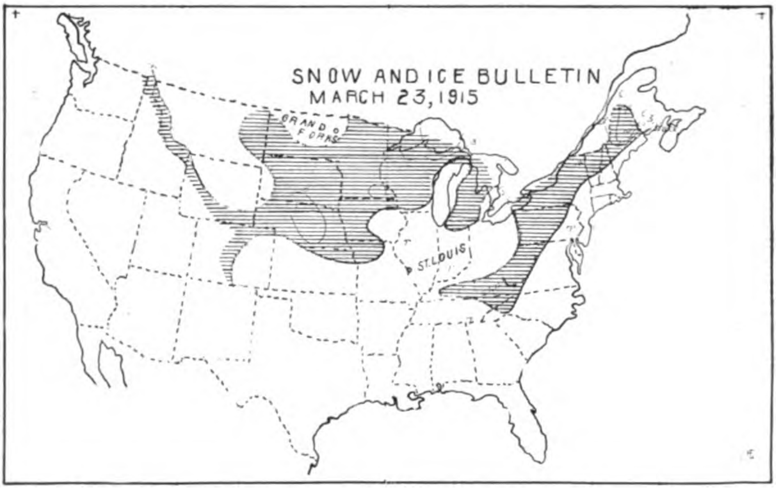

FIGURE 12

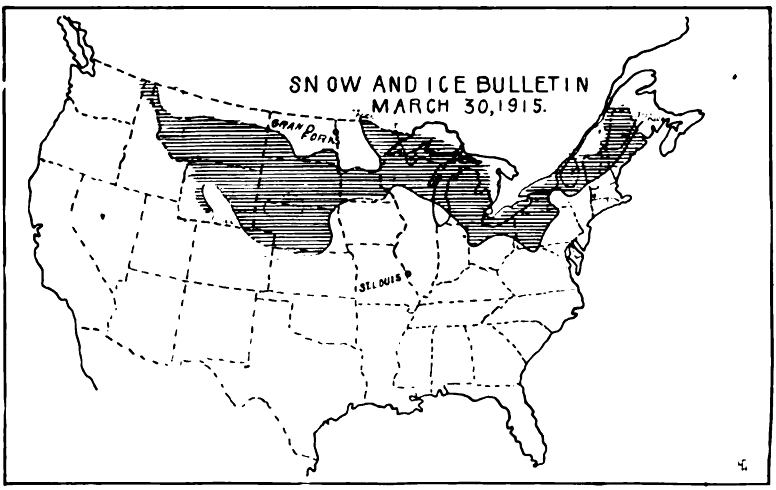

FIGURE 13

or a little earlier, suggest sudden changes in surface conditions which probably have something to do with the falling off of transmission in the spring. This almost sudden variation has also been noted between Grand Forks and Denver.

DISCUSSION OF RESULTS

There seem to be two kinds of fluctuations in nocturnal overland transmission. The first is a rapid fading, and the second is a slow swinging in signal strength. The first may be due to changes, in the nature of interference effects. These could be local at the sender or at the receiver, or they might be caused by rather sharp surfaces of discontinuity almost anywhere between the stations.

The second or slower effect may be due to refracting masses of moving ionized air in the path of transmission, producing at times a lens-like concentration and at other times a dispersive effect. It might be contended that all of the fluctuations can be accounted for on this basis.

The presence or absence of true interference effects has a most important bearing on the transmission theories outlined in the first part of this paper. Fortunately the tests seem to throw a little light upon this point.

It will be noted that nearly all rapid variations in the 1,500-meter wave are duplicated by the 500-meter wave, which is its third harmonic, and which should show interference maxima and minima wherever the 1,500-meter wave does so. On the other hand, very few coincidences in fluctuation are observed between the 850-meter waves and either of the others. A path difference of 250 meters, 1,250 meters, 1,750 meters, etc., or 1, 5, 7, 11 . . . . half wave lengths, would create destructive interference for the 500-meter wave without greatly affecting the 1,500-meter wave. In other words, while every interference maximum or minimum of the 1,500-meter wave should be duplicated in the 500-meter wave, the reverse is not necessarily true. This seems to be in agreement with the results, and presents a strong argument for the presence of true interference effects.

The experiments on reciprocity of transmission are too incomplete to be decisive.

If any speculation be based upon them, it is that while general transmission conditions are reciprocal, rapid fluctuations are not necessarily so.

The experiments emphasize the superior transmission of the

short waves in mid-winter at night, and of the longer waves in the day time. The greater steadiness of the longer wave is noticeable.

It is suggested that the almost sudden spring drop in transmission is connected with the disappearance of the ice sheet and the consequent changes in conductivity and dielectric constant of the earth's surface. Surface ionization due to stirring up of the soil by spring plant growth may play a role.

The generally favorable effect of cloudy days on the following nocturnal transmission is confirmed, thus suggesting that sometimes the conditions causing fluctuations may be at low atmospheric levels.

Simultaneous measurements on the signals of "CWH," a St. Louis amateur, then using a 500-meter wave, made in Fargo and Grand Forks at one time last winter showed his signals to have faded out entirely at Grand Forks while seeming to increase in strength at Fargo, which is 120 kilometers (75 miles) south of Grand Forks. A similar case, involving greater distance between observing stations was reported to one of the writers by "7CE," of Boise, Idaho. In this case the signals faded out at the nearer receiving station but came in strong at the farther one. These and similar cases speak against a general dispersion and in favor of interference.

Surfaces of discontinuity suitable for causing interference effects with short waves should be more numerous, and the discontinuity sharper. This is in accord with the well-known fact that extreme cases of fading and "freaks" happen more often with short waves and over short distances. St. Louis reported a case of fading observed between "5AK" and "5XC," a distance of 56 kilometers, and another regularly observed between St. Louis and "9DB," 160 kilometers. If fading effects over such short distances are due to interference, it probably occurs at the cloud level. In the most striking cases observed at Grand Forks, namely with various amateurs in Ohio, distant 1,300 to 1,500 kilometers, the interfering wave might have come from a very high altitude.

The general trend of the transmission curves shows that the daylight ionization responsible for the general absorption disappears more slowly in the evening than it reappears in the morning: in other words, the transmission is not symmetrical about the solar midnight. This contradicts the statement of W. H. F. Murdock[1] that "at sunset they (the ions) disappear

[1] "The Electrician," Jan. 29, 1915.

as rapidly as they came at sunrise." The transmission curves obtained in Australia by Balsillie show the same asymmetry as those of this paper.[1]

Nipher has observed a change of atmospheric permeability with wind gusts.[2] If this effect were connected in any way with transmission changes it should be present by daylight. Were it possible, by the use of high power and short waves, to examine short wave transmission by daylight over long distances, some daylight fading and swinging might be observed, altho the behavior of longer waves indicates the contrary.

In connection with the somewhat speculative value of the absorption coefficient deduced for twilight transmission it is of interest to note that one of the writers[3] showed that if transmission takes place according to the equation of the form given by Austin, for a given distance there is a critical wave length giving the best transmission.

Cohen has expressed this in a more general form[4] by the equation $\lambda = \frac{\alpha^2 d^2}{4}$, where if $\alpha = 0.00157$ (the average for the 500-meter and 1,500-meter waves), and d = 1,250 kilometers, the optimum wave length would be $\lambda = 960$ meters, which wave should show 28 per cent better audibility than the 1,500-meter wave during the twilight and early morning period. This result has not yet been subjected to experimental test, but mid-day comparisons at Minneapolis (distant 450 kilometers) of the 500-meter, 850-meter, and 1,500-meter waves of "9XN" have decidedly favored the longest wave. Assuming the correctness of the Cohen formula, this result indicates again the high value of the mid-day absorption, as compared with that of early morning.

Since at the most there was but an hour difference in sunset between the two stations, and the general direction is north and south, it was not to be expected that the Marconi-Pickard sunset and sunrise effect would be manifested. Some traces might have been noted had the longest wave been strong enough to warrant good daylight observations.

The sudden drop in the half hour preceding sunrise emphasizes the important role played by the high altitudes which are then being influenced by sunlight.

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[1] "Electrician," Nov. 13, 1914.
[2] Francis E. Nipher, "Trans. St. Louis Acad. Sciences," Vol. XXII, No. 4.
[3] A. S. Blatterman, "Electrical World," Aug. 15, 1914.
[4] Cohen, "Electrical World," Oct. 17, 1914.

The writers realize the incompleteness of this attempt to get systematic data on nocturnal transmission, and venture the following suggestions for further work. First, observations should be almost continuous, in order not to miss the more rapid variations. Second, a greater range of wave length is desirable. Third, sufficient power to extend observations well into daylight should be used. Fourth, thoro tests on reciprocity of transmission are necessary. Finally, coöperation with various stations in different directions would be useful in comparing simultaneous transmission thru regions having widely varying weather.

August 15, 1915.

SUMMARY: Experiments on night transmission, on wave lengths of 500, 850 and 1,500 meters between two inland stations 1,250 kilometers apart are described. The bibliography of wave absorption is first considered critically.

The average transmitting current was 12 amperes and the decrements varied from 0.09 to 0.18. The effective height of the transmitting antenna was 27 meters. Receiving was done with an audion tested against a constant exciting circuit. In making audibility measurements by the shunt-to-telephone method the true impedance at the audio frequency in question was used. Observations were made hourly during night, with fifteen-minute periods near sunset and sunrise.

The curves of signal strength are not symmetrical about the solar midnight. In general, the fluctuations of the 500 meter-wave follow those of the 1,500 meter-wave, but not necessarily *vice versa*. The 850 meter-wave does not seem to fluctuate in marked synchronism with either of these. This is explained on the basis of definite wave interference. Short waves are superior by night, long waves by day, generally.

The disappearance of the ice sheet between the stations and the breaking up of the soil seems to account for a spring drop in transmission. Cloudy days markedly favor the following night transmission. Fading and "freak" effects are more numerous on short waves, thus favoring the interference explanation.

It is suggested that future observations should be almost continuous, of long wave length range, of high power, reciprocal between stations, and in various directions.

DISCUSSION

J. Zenneck: It is well known that the received current depends largely on the decrement of the receiving antenna as well as on the decrement of the transmitting antenna. Both of these decrements are dependent on atmospheric conditions; wet weather, for instance, generally causes a very marked increase in the decrement of the antenna. Therefore, measurements of the received current are only comparable if it is sure that the decrements of the transmitting and receiving antennas have remained constant, or if both of these decrements have been measured and their actual variations taken into account. It is not sufficient to keep the current at the transmitting antenna constant. I should like to ask Professor Taylor if he would kindly tell us whether he has taken into consideration the decrements of the transmitting and receiving antennas and their possible changes caused by atmospheric conditions.

As to the desirability of the co-operation of as many stations as possible with the object of obtaining statistics relative to the transmission of signals, this is perhaps a matter of opinion. It is so very easy to observe changes in the strength of the received signals and so very difficult to make sure that these changes are really due to atmospheric conditions. I should therefore advise that great care should be exercised in selecting stations with which to carry on such work. Uncertain and unreliable material is worse than no material at all.

J. Zenneck (communicated): In connection with the experiments made by Professor Taylor, I should strongly recommend that all measurements on the variation in strength of radio telegraphic signals, be made by using a *galvanometer*, as Dr. Louis W. Austin did in his experiments (PROCEEDINGS OF THE INSTITUTE OF RADIO ENGINEERS, Volume III (1915), page 103). Results obtained by the telephonic method of measuring the strength of the received signals may always have been influenced by physiological or psychological effects (R. H. Marriott, PROCEEDINGS OF THE INSTITUTE OF RADIO ENGINEERS, Volume II (1914), page 37), and therefore never possess the same degree of reliability as those obtained by the galvanometer.

A. Hoyt Taylor: I would like to point out to Professor Zenneck that, altho there are in the neighborhood of several hundred stations within our range, there are only two with

whom we have co-operated. Professor Zenneck has referred to a very interesting point; that is, the study of the decrement. For instance, we discovered at "9 X N" a very slight leakage of an insulator by noting that the decrement of the 1,500 meter wave jumped from 0.11 to 0.13, indicating that something was wrong. In reply to Professor Zenneck's suggestion that variation in the decrement might account for the apparent variations in transmission, I would like to point out that in wet weather where one gets abnormally high decrements and a lowering of energy radiated, we invariably get an improvement in the transmission. In other words, the change of decrement is in the wrong direction to explain our results. Very large variations in decrement might be shown on an amateur set with wooden insulators but, with a good sending and receiving set, it would be entirely impossible to get decrement variations of a sufficient magnitude to account for even a small percentage of the changes which we have observed. At "9 X N" we observed the decrements of both sending and receiving circuits, altho at St. Louis we observed only the decrement of the sending circuit. I fully realize the value of Professor Zenneck's criticisms of the co-operation idea and regret that there are not more stations in my part of the country with whom I can confidently co-operate and know that their observations will be reliable.

W. H. G. Bullard: Whereas I have not had much personal experience along the lines which constitute the subject of this paper, I have received considerable information thru official reports that bear on the general subject, and have formed some theories as to the conditions which give rise to the phenomenon of falling away and increasing ot radio signals on the approach or recession of sunrise or sunset. This is particularly noticeable in localities where a rapid change of temperature accompanies either sunset or sunrise, and noticeably so in Alaska where most marked phenomena occur. Such theory as I have formed also to some extent accounts for some ot the static disturbances noticed, particularly on high, large capacity antennas.

I have preferred to think of a large conducting surface, if such may be imagined, at some distance above the earth, and with which it is associated as the conducting surface of a high condenser with the intervening atmosphere as the dielectric. The height of this conducting surface will depend, considerably, on the temperature; the higher this is, the higher above the

earth this conducting surface and the less the capacity of the condenser. Consequently the electric waves are held between two surfaces of different distances apart, dependent upon the temperature. With the higher temperature of daylight, the signals should be correspondingly weaker than when the conducting surfaces are nearer together, as during night when the temperature falls. Under the condition of night when the temperature is lowered, the conducting surfaces are nearer together, the energy is concentrated in a smaller space, with the result that signals at a given station are of increased intensity. The phenomenon of increased strength of signals due to fading of daylight is marked in Alaska, and it is in such localities also that the most marked changes of temperature exist, and that the temperature rapidly falls immediately on the disappearance of the sun. The reverse takes place just after daylight, when there is a rapid increase of temperature after sunrise and signals immediately fall away. This seems to indicate that in places where the rate of change of temperature is greatest, there should be the greatest change noted in intensity of received signals.

On the surface of this conducting medium at some distance above the earth (which surface may be compared to the surface of the ocean), there are zones of static electric disturbances of greater or less amplitude, precisely as there are waves of greater or less height on the ocean's surface. These zones or regions dip below the surface on which they exist and may give up their charges to an antenna that is high enough to receive them and produce the characteristic sounds of static. Many of these zones will be small and cannot reach the antenna, but when an extra large zone comes along, it deposits an extra large charge. Static disturbances are most generally marked along about the time of sunset and sunrise and their effects might similarly be traced to the raising or lowering of the conducting area due to the rapid change of temperature at those times. After sunrise, the conducting area is higher and not so many static regions can reach the antenna while on the lowering of the area due to the lowering of temperature, these waves are within the reach of high antenna and so give up their charges.

It will take considerable study and observation to test such a theory, but I think those who are investigating the changes in intensity of radio signals under various meteorological conditions would do well to approach this subject from the point of view of changes of temperature, from whatever causes these may be due.

Speaking of what Professor Taylor says relative to co-operation with observing stations, I have the pleasure of controlling to a certain extent, the activities of a considerable chain of radio stations, fifty-five at least, from high to low and medium powered, and I can assure Professor Taylor that any of these stations which will be of use to him will gladly co-operate at any time of the day or night, and will guarantee to have observers in those stations who will in all respects be competent to take such measurements as he may desire. Further, I should like to announce publicly that it is the policy of the Naval Radio Service at all times actively to co-operate along any lines that will tend to the betterment of the art, and would like to have it known, that altho in some quarters, the government by certain of its activities, has been accused of hindering the art, yet it always stands ready to place its stations, as far as compatible with the public interests, at the disposition of those who may have any ideas they may wish to develop, and will gladly help thru its personnel or its material.

V. Ford Greaves: I have been very much interested in Professor Taylor's paper. It illustrates some of the difficulties of the radio inspection service of the Government in determining whether or not a certain radio station will transmit 100 miles (160 km.).

I have noted what Professor Taylor said in regard to the operation of unlicensed stations in the vicinity of St. Louis. On account of the limited appropriation and small inspection force, it has not been possible to devote much time to the inspection of inland stations. The radio service will appreciate reports of the operation of such stations from those who are interested in reducing unnecessary interference.

I read in the newspapers that the Marconi Company is to equip a considerable number of barges plying the Mississippi between New Orleans and Minneapolis, with radio apparatus to be used for commercial business. Undoubtedly this will result in more thoro inspection service in the central States, provided Congress will make the necessary appropriation.

Roy A. Weagant: I do not really think that I can add anything to the information of the evening. We are all under very serious obligations to Professor Taylor for tackling this problem. Those of us who attempted to do anything with it, found that the combination of an infinite number of variables is a very serious proposition to work out on a definite basis. My own

observations have been that almost all the phenomena that we get together at various times seem to fit in with almost any of the various theories which have been advanced to account for them. Personally I do not know how we can hope to get any information in this particular line of investigation, except as Professor Taylor has suggested; namely, making a really tremendous number ot observations over a very long period of time. Professor Zenneck, speaking from a world of actual experience, pointed out the difficulty of making measurements really mean anything. My own experience is that variations, due to variations of the apparatus alone, are often thousands of times greater than anything happening in the intervening medium. I do not know of any commercial form of radio detector, of sufficient reliability, combined with sufficient sensitiveness to make readings at all quantitative. The audion, with which we are all familiar, generally changes its sensitiveness from the minute you close the switch on it. It is not constant in any two consecutive instants, hardly, and not at all over any considerable period of time. More or less similar remarks are true of any form of detector in use.

A. Hoyt Taylor: I can assure Mr. Weagant that there is a method of checking the sensitiveness of audions and will be glad to show him the details of it if he so desires.

R. H. Langley: Professor Taylor has discussed some very interesting phases of the effect of weather conditions on radio telegraphic transmission distances. It is interesting to note that complete weather records have been kept, in the United States for forty-five years. The Signal Corps of the Army organized a weather service in 1871. Temperature records were kept at the Army Posts as far back as 1820. But in spite of this vast quantity of data, weather predictions are still more or less inaccurate and unsatisfactory. If we are to rely, therefore, upon such predictions for predetermining the possibility of communications between any two radio stations at any given time, the value of such predeterminations must be small. It is also apparent that radio stations could not be rated in terms of the maximum distance which could be guaranteed under the worst conditions, since this would be almost nothing. And again, how shall we specify what shall be considered to be worse or best, or average conditions?

The possibility of predicting weather conditions by their effect on radio transmission seems also to be very small, since the

actual change in radio range occurs almost without exception, hours and even days after the meteorological conditions which produced that change have come into existence. This, of course, is not true of the effects of sunlight, sunrise, sunset, moonlight, and so forth, but these require no prediction.

A. S. Blatterman: Professor Zenneck's remarks on the possibility of antenna decrements seriously affecting the reported results of our experiments may be misleading. It is true, as Professor Zenneck says, that received antenna current is affected to an extent by the antenna decrements at receiver and sender. This statement must however be taken with caution; it may not at once be apparent just what is the order of magnitude of the effect. Among the factors affecting antenna decrement are wave length, rain, snow and ice and anything acting to alter the conductivity of the earth in the immediate neighborhood of the station, that is, the effective resistance of the antenna and its earth connection. Of the total resistance of the antenna a part, the radiation resistance, represents useful radiant energy and this part of the resistance term is independent of ordinary changes in atmospheric conditions. It depends on the geometric configuration of the aerial system, as is well known, and on the effective height of the aerial and the wave length. For a given set of these conditions, it is therefore only that part of the decrement (and resistance) representing Joulean losses which can affect the aerial current. In a well designed sending antenna the constant radiation resistance may be many times the variable ohmic resistance of the wires, leaks and earth plates, and therefore the small changes which occur in the latter due to atmospheric changes only slightly affect the whole effective value of antenna resistance and hence the currents.

Admitting, however, that the decrement has a noticeable effect this certainly cannot be taken to explain the very sudden and extraordinary changes in signal strength which have so often been observed; for, while it is well known that the decrement of a given aerial varies from instant to instant thruout the day there are no records, at least to my knowledge, which show the sudden decided variations in this quantity which would be necessary to explain such rapid fading effects as are exemplified by curve 11, Figure 1.

One might entertain for a moment the notion that changes in the average of the ordinates of the transmission curves for different days were indications of lower or higher decrements

at the antennas, were it not, as Professor Taylor has already pointed out, that in wet weather when one would expect weaker signals we have always found that transmission actually improved. But it certainly cannot explain the *sudden* fading which we are studying.

I must take exception to Mr. Weagant's remarks in which he would dismiss all idea of swinging and fading in transmission and lay it to irregular detector action. We are quite sure that our curves are not plots between detector sensibility and time. Personally I have observed the fading effects for about seven years with a great many different detectors, the electrolytic, the crystals and the audion, and if intelligently handled and tested from time to time, it is a practical certainty that variations in sensibility are totally inadequate to obscure the changing conditions between stations. Moreover, one frequently hears the signals from one station getting stronger while those of another are fading out, which shows pretty positively that the detector is not to blame. If Mr. Weagant really believes that irregular detector sensibility overshadows the changing meteorological conditions between stations, I can only say that he cannot have directed his attention to transmission at night over land, especially on the shorter wave lengths.

I might say that since preparing the paper we have been studying more in detail the question of reciprocity in transmission and hope to present some material on this in the near future.

THE THEORY AND DESIGN OF RADIO-TELEGRAPHIC TRANSFORMERS *

By

Fulton Cutting

(Radio Engineer)

The conventional radio plant charges the condenser of the oscillating circuit by means of a transformer whose primary is connected to an alternating current source. The character of the phenomena is well shown in Figure 1, which is a picture

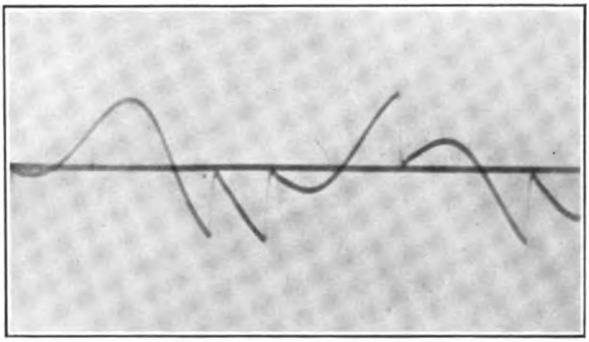

Figure 1—Voltage Across a Sparking Condenser Taken with an Electrostatic Oscillograph. (No Residual Condenser Charge.) 60 Cycles

taken with an electrostatic oscillograph of the voltage across a condenser charged thru a transformer by a 60 cycle a. c. source and discharging across a gap. The voltage of the condenser rises as current flows into it until the sparking voltage is reached and discharge takes place. The radio (high) frequency oscillation across the gap is over so quickly and the energy of the condenser dissipated that the phenomena of sparking appear, from an audio (low) frequency standpoint, simply as if the charge

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had been suddenly conjured away. The condenser immediately starts filling up until the sparking voltage is again reached and another charge is spilt into the oscillating circuit.

From a mathematical standpoint we may treat this condition of affairs as a special case of coupled circuits acting under an impressed electromotive force.

The primary of the two circuits contains resistance, inductance, and an alternating impressed e.m.f., and the secondary contains resistance, inductance, and capacity. Sparking is represented simply as a discontinuity in the condenser charge.

Such a discontinuity sets up transient terms in the two circuits. For each fresh condenser discharge, new transient terms are set up; so that if the phenomenon is recurrent, the currents in the two circuits will be made up of the usual forced term plus a string of transient terms in different stages of damping. The present treatment assumes that the oldest of these transient terms has dwindled to a size that can be neglected. That is, the phenomenon of continued charging and discharging of the condenser is assumed to have reached a steady state. Under this assumption it is proposed to find expressions from which the output of the transformer may be computed.

Figure 2 represents the circuits. The primary and secondary currents are i_1 and i_2. The rest of the letters have their conventional meanings.

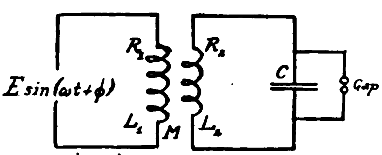


FIGURE 2—Diagram of Circuits

The differential equations of the two circuits are:

$$(1) \qquad L_1 \frac{d\,i_1}{d\,t} + M \frac{d\,i_2}{d\,t} + R_1\,i_1 = E\,sin\,(\omega t + \phi_o)$$

$$(2) \qquad L_2 \frac{d\,i_2}{dt} + M \frac{d\,i_1}{dt} + R_2\,i_2 + \frac{1}{C}\int i_2\,d\,t = 0$$

Eliminating the i_1's:

$$(L_1 L_2 - M^2)\frac{d^3\,i_2}{d\,t^3} + (R_1 L_2 + R_2 L_1)\frac{d^2\,i_2}{d\,t^2} + \left(R_1\,R_2 + \frac{L_1}{C}\right)\frac{d\,i_2}{d\,t} + \frac{R_1}{C}\,i_2 = E\,M\,\omega^2\,sin\,(\omega\,t + \phi_o)$$

or:

$$(3)\qquad \frac{d^3 i_2}{dt^3}+\omega(\gamma_1+\gamma_2)\frac{d^2 i_2}{dt^2}+\omega^2(a\gamma_1\gamma_2+\theta^2)\frac{d i_2}{d t}+\omega^3 a\gamma_1\theta^2 i_2 = \frac{E M \omega^2}{a L_1 L_2} sin(\omega t+\phi_o)$$

where:

$$a=1-\frac{M^2}{L_1 L_2}$$

$$\gamma_1=\frac{R_1}{a L_1 \omega}$$

$$\gamma_2=\frac{R_2}{a L_2 \omega}$$

$$\theta^2=\frac{1}{a L_2 C \omega^2}$$

These four constants have zero dimensions. a depends on the coefficient of coupling; that is, on the geometry of the transformer and is independent of the values of the inductances. γ_1 and γ_2 are of the nature of modified damping terms. θ is the most important quantity in the whole investigation. It turns out that it is the ratio of the natural period of the two circuits to the impressed period.

Equation (3) is a linear differential equation with constant coefficients; so that to find the free solution, we proceed in the usual way, and put:

$$i_2'=B\,\varepsilon^{kt}$$

which on substitution in the left-hand member of (3) yields the cubic:

$$(4)\qquad k^3+\omega(\gamma_1+\gamma_2)k^2+\omega^2(a\gamma_1\gamma_2+\theta^2)k+\omega^3 a\gamma_1\theta^2=0$$

for the determination of k. We shall not attempt to solve this equation, but we shall simply designate its three roots by:

$$k_1=k_1$$

$$k_2=-\Delta+j\omega'$$

$$k_3=--\Delta j\omega'.$$

Now as these satisfy (4) we must have:

$$(k-k_1)(k-k_2)(k-k_3)=0.$$

Expanding this expression and equating its coefficients with those of (4), we get the relations:

$$(5)\qquad -k_1+2\Delta=\omega(\gamma_1+\gamma_2)$$

$$(6)\qquad -2a\omega\gamma_1 k_1+\Omega^2=\omega^2(a\gamma_1\gamma_2+\theta^2)$$

(7) $$-k_1\,\Omega^2 = a\,\omega^3\,\gamma_1\,\theta^2$$
where:

$$\Omega^2 = \Delta^2 + \omega'^2$$

We shall now be able to get approximate roots for the cubic (4). Dividing (6) by (7):

$$\frac{1}{-k_1}\left(1 - \frac{2\,a\,\gamma_1\,k_1/\omega}{\Omega^2/\omega^2}\right) = \frac{1}{a\,\omega\,\gamma_1}\left(1 + \frac{a\,\gamma_1\,\gamma_2}{\theta^2}\right)$$

Neglect $\frac{2\,a\,\gamma_1\,k_1/\omega}{\Omega^2/\omega^2}$ and $\frac{a\,\gamma_1\,\gamma_2}{\theta^2}$ in comparison with unity. This we shall see later is an excellent approximation. It gives us:

(8) $$k_1 = -a\,\omega\,\gamma_1$$

(9) $$\Omega^2 = \omega^2\,\theta^2$$

(10) $$\Delta = \frac{\omega}{2}\left(\gamma_1\,(1-a) + \gamma_2\right)$$

These results, however, are not necessary for the present and we shall write the free solution simply as:

$$i_2' = B_1\,\varepsilon^{k_1 t} + B_2\,\varepsilon^{k_2 t} + B_3\,\varepsilon^{k_3 t}$$

By routine methods, we may without much labor find the "forced" solution of (3), which is:

$$i_2'' = -I_2\cos(\omega t + \phi_2)$$

where:

(11) $$I_2 = \frac{E\,M\,\omega}{a L_1\,L_2\,\omega^2\sqrt{(\gamma_1+\gamma_2 - a\,\gamma_1\,\theta^2)^2 + (\theta^2 + a\,\gamma_1\,\gamma_2 - 1)^2}}$$

(12) $$\phi_2 = \phi_o - \tan^{-1}\frac{\gamma_1+\gamma_2 - a\,\gamma_1\,\theta^2}{\theta^2 + a\,\gamma_1\,\gamma_2 - 1}$$

So the complete solution of (3) is:

(13) $$i_2 = -I_2\cos(\omega t + \phi_2) + B_1\,\varepsilon^{k_1 t} + B_2\,\varepsilon^{k_2 t} + B_3\,\varepsilon^{k_3 t}$$

Turning now to the primary and proceeding in the same manner, we get by eliminating the i_2's from (1) and (2):

(14) $$\frac{d^3 i_1}{dt^3} + \omega\,(\gamma_1+\gamma_2)\frac{d^2 i_1}{dt^2} + \omega^2\,(a\,\gamma_1\,\gamma_2 + \theta^2)\frac{d\,i_1}{dt} + \omega^3\,a\,\gamma_1\,\theta^2\,i_1$$
$$= \frac{E\,\omega}{a L_1 L_2}\left\{R_2\cos(\omega t + \phi_o) - \left(L_2\,\omega - \frac{1}{C\,\omega}\right)\sin(\omega t + \phi_o)\right\}$$
$$= -\frac{E\,\omega\,Z_2}{a L_1 L_2}\sin\left(\omega t + \phi_o - \tan^{-1}\frac{R_2}{L_2\omega - \frac{1}{C\,\omega}}\right)$$

where:
$$Z_2=\sqrt{R_2^2+\left(L_2\,\omega-\frac{1}{C\,\omega}\right)^2}$$
$$=L_2\,\omega\sqrt{a^2\,\gamma_2^2+(1-a\,\theta^2)^2}$$
$$\frac{R_2}{L_2\,\omega-\frac{1}{C\,\omega}}=\frac{a\,\gamma_2}{1-a\,\theta^2}$$

The coefficients of the left-hand member of (14) are the same as those of (3). The free solution of the primary will, therefore, be of the same form as that of the secondary and we can write at once as the complete solution of (14):

(15) $$i_1=I_1\cos(\omega t+\phi_1)+A_1\,\varepsilon^{k_1 t}+A_2\,\varepsilon^{k_2 t}+A_3\,\varepsilon^{k_3 t}$$

where:
$$I_1=\frac{E\,Z_2}{a L_1 L_2\,\omega^2\sqrt{(\gamma_1+\gamma_2-a\,\gamma_1\,\theta^2)^2+(\theta^2+a\,\gamma_1\,\gamma_2-1)^2}}$$
$$\phi_1=\phi_2-\tan^{-1}\frac{a\,\gamma_2}{1-a\,\theta^2}$$

In equations (13) and (15) we have written in all six undetermined constants. The differential equations, however, admit of only three independent constants, and so that there must be relations between the A's and the B's. These relations are found by substituting (13) and (15) in (1) and (2). Leaving out the forced terms since these satisfy the differential equations identically, we get:
$$\sum \varepsilon^{k_i t}\left(A_i\,(k_i+\Delta)+k_i\frac{M}{L_1}B_i\right)=0$$
$$\sum \varepsilon^{k_i t}\left(B_i\,(k_i+a\,\omega\,\gamma_2+a\,\omega^2\,\theta^2)+k_i\frac{M}{L_2}A_i\right)=0$$
$$i=1,\ 2,\ 3$$

These equations must be satisfied for all values of t. The coefficients of the ε^{kt}'s do not involve t, so they must, therefore, each be identically zero. This gives:

(16) $$A_i\left(1+\frac{a\,\omega\,\gamma_1}{k_i}\right)=-\frac{M}{L_1}B_i$$

(17) $$B_i\left(1+\frac{a\,\omega\,\gamma_2}{k_i}+\frac{a\,\omega^2\,\theta^2}{k_i^{\,2}}\right)=-\frac{M}{L_2}A_i \qquad i=1,\ 2,\ 3$$

It must be noted that these two equations are not independent. One is easily transformed into the other by means of the cubic (4). Having found the relations between the A's and the B's, we are now in a position to evaluate them in terms of

the constants of the circuits by means of the terminal conditions.

As mentioned above, the problem to be considered is that of a condenser charged thru a transformer and periodically discharged. We shall only consider the cases of discharge occurring every integral number of half cycles. It is necessary to assume that all the phenomena concerned are regular and symmetrical. That is, we assume that what goes on between any two successive sparks is similar to what goes on between any other successive sparks. Also if sparking occurs in different phases, no magnitudes will be altered but only signs. These restrictions are made so as to rule out anything in the way of a situation where sparking during the positive phase takes place at a different voltage or phase from sparking during the negative phase. Our conditions are, therefore, those of uniform operation under a steady state, and one mathematical expression covering the interval between sparks will tell the whole story.

Time is measured from the moment after sparking. That is, we start the interval with the condenser discharged, the residual charge being an experimental constant. The duration of the condenser discharge is extremely small. In this short time the magnetic field cannot appreciably change, so the currents are taken as continuous (this being verified by experiment). Remembering that the phenomena between sparks are identical, we see that the currents at the beginning of the interval between sparks must be equal in magnitude to the currents at the end. Let us say that the condenser discharges every n half cycles of the impressed e.m.f. and we get:

(18) $$i_{1\,(t=0)} = (-1)^n\, i_{1\,(t=n\pi/\omega)}$$

(19) $$i_{2\,(t=0)} = (-1)^n\, i_{2\,(t=n\pi/\omega)}$$

(20) $$q_{(t=0)} = -Q_o$$

where $-Q_o$ is the residual charge. q the charge in the condenser is given by:

(21) $$q = \int i_2\,dt = -\frac{I_2}{\omega}\sin(\omega t+\phi_2) + \frac{B_1}{k_1}\varepsilon^{k_1 t} + \frac{B_2}{k_2}\varepsilon^{k_2 t} + \frac{B_3}{k_3}\varepsilon^{k_3 t}$$

Applying (18), (19), and (20) to (15), (13), and (21) respectively:

(22) $$\begin{cases} \sum A_i\left(1-(-1)^n\varepsilon^{\frac{n\pi k_i}{\omega}}\right) = 0 \\ \sum B_i\left(1-(-1)^n\varepsilon^{\frac{n\pi k_i}{\omega}}\right) = 0 \\ \sum \dfrac{B_i}{k_i} = \dfrac{I_2}{\omega}\sin\phi_2 - Q_o \qquad i=1,\,2,\,3 \end{cases}$$

Together with (16) and (17) these equations enable us to get the A's and B's explicitly in terms of the constants of the circuits. For convenience put:

$$l_1 = 1 - (-1)^n \varepsilon^{\frac{n \pi k_1^2}{\omega}}$$

$$m_1 = -\frac{L_1}{M}\left(1 + \frac{a\,\omega\,\gamma_1}{k_i}\right) = \frac{B_i}{A_i}$$

$$p = \frac{I_2}{\omega} \sin \phi_2 - Q_o$$

$$k_i = 1/k_i \qquad\qquad i = 1,\ 2,\ 3$$

Eliminating the B's, equations (22) become:

$$l_1 A_1 \quad + l_2 A_2 \quad + l_3 A_3 = 0$$

$$m_1 l_1 A_1 + m_2 l_2 A_2 + m_3 l_3 A_3 = 0$$

$$m_1 k_1 A_1 + m_2 k_2 A_2 + m_3 k_3 A_3 = P.$$

Solving:

$$A_1 \begin{vmatrix} l_1 & l_2 & l_3 \\ m_1 l_1 & m_2 l_2 & m_3 l_3 \\ m_1 k_1 & m_2 k_2 & m_3 k_3 \end{vmatrix} = \begin{vmatrix} 0 & l_2 & l_3 \\ 0 & m_2 l_2 & m_3 l_3 \\ P & m_2 k_2 & m_3 k_3 \end{vmatrix}$$

Similarly for A_2 and A_3. Calling the left-hand determinant H we have:

$$A_1 = l_2 l_3 (m_3 - m_2) \frac{P}{H}$$

$$A_2 = l_1 l_3 (m_1 - m_3) \frac{P}{H}$$

$$A_3 = l_1 l_2 (m_2 - m_1) \frac{P}{H}$$

Expanding:

$$A_1 = -j\,\frac{2\,\alpha\,\omega\,\gamma_1\,\omega'\,L_1\,\varepsilon^{-\frac{n\pi\Delta}{\omega}}}{\Omega^2\,H}\left(\frac{I_1}{Z_2}\sin\phi_2-\frac{Q_o}{M}\right)\left(\cosh\frac{n\pi\Delta}{\omega}-(-1)^n\cos\frac{n\pi\omega'}{\omega}\right)$$

$$A_2=\frac{L_1}{H}\left(\frac{I_1}{Z_2}\sin\phi_2-\frac{Q_o}{M}\right)\left(1-(-1)^n e^{\frac{n\pi k_1}{\omega}}\right)\left[1-(-1)^n\varepsilon^{-\frac{n\pi\Delta}{\omega}}\left(\cos\frac{n\pi\omega'}{\omega}-j\sin\frac{n\pi\omega'}{\omega}\right)\right]\left(1+m_1-\frac{\alpha\,\omega\,\gamma_1\,(\Delta-j\,\omega')}{\Omega^2}\right)$$

$$A_3=-\frac{L_1}{H}\left(\frac{I_2}{Z_2}\sin\phi_2-\frac{Q_o}{M}\right)\left(1-(-1)^n\varepsilon^{\frac{n\pi k_1}{\omega}}\right)\left[1-(-1)^n\varepsilon^{-\frac{n\pi\Delta}{\omega}}\left(\cos\frac{n\pi\omega'}{\omega}+j\sin\frac{n\pi\omega'}{\omega}\right)\right]\left(1-m_1-\frac{\alpha\,\omega\,\gamma_1\,(\Delta+j\,\omega')}{\Omega^2}\right)$$

$$H=j\,\frac{2L_1^2}{\Omega\,M}\Bigg[\left(1-(-1)^n\varepsilon^{\frac{n\pi k_1}{\omega}}\right)\left(1-\frac{2\,\alpha\,\omega\,\gamma_1\,\Delta}{\Omega^2}+\frac{\alpha^2\,\gamma_1^2\,\omega^2}{\Omega^2}\right)\left\{\frac{\omega'}{\Omega}-(-1)^n\varepsilon^{-\frac{n\pi\Delta}{\omega}}\cos\left(\frac{n\pi\omega'}{\omega}+\tan^{-1}\frac{\Delta}{\omega'}.\right)\right\}$$

$$-m_1\left(1-(-1)^n\varepsilon^{\frac{n\pi k_1}{\omega}}\right)\left\{\frac{\omega'}{\Omega}\left(1-\frac{2\,\alpha\,\omega\,\gamma_1\,\Delta}{\Omega^2}\right)-(-1)^n\frac{\varepsilon^{-\frac{n\pi\Delta}{\omega}}}{\Omega}\sqrt{\omega'^2\left(1-\frac{2\,\alpha\,\omega\,\gamma_1\,\Delta}{\Omega^2}\right)^2+(\alpha\,\omega\,\gamma_1+\Delta)^2\left(1-\frac{\alpha\,\omega\,\gamma_1\,\Delta}{\Omega^2}\right)^2}\,.\right.$$

$$\left.\cos\left(\frac{n\pi\omega'}{\omega}+\tan^{-1}\frac{(\alpha\,\omega\,\gamma_1+\Delta)\left(1-\frac{\alpha\,\omega\,\gamma_1\,\Delta}{\Omega^2}\right)}{\omega\left(1-\frac{2\,\alpha\,\omega\,\gamma_1\,\Delta}{\Omega_2}\right)}\right)\right\}+\frac{2\,\alpha\,\omega\,\gamma_1\,\omega'\,m_1}{k_1\,\Omega}\,\varepsilon^{-\frac{n\pi\Delta}{\omega}}\left\{\cosh\frac{n\pi\Delta}{\omega}-(-1)^n\cos\frac{n\pi\omega'}{\omega}\right\}\Bigg]$$

Before proceeding further, it will be well to make some approximations. These approximations would ordinarily be made at a later stage of the investigation, but their introduction here will not affect the final result. The gain in simplicity is worth the sacrifice of elegance.

Terms of the order of Δ^2/Ω^2 will be neglected in comparison with unity. m_1 is of this order, and ω' may be replaced by Ω with the same degree of accuracy. We have then for the approximate value of the constants:

$$
(23)\begin{cases}
A_1 = -2\,a\,\omega\,\gamma_1 L_2 (1-\alpha)\left(\dfrac{I_1}{Z_2}\sin\phi_2 - \dfrac{Q_o}{M}\right)\times \\
\qquad\dfrac{\cosh\dfrac{n\pi\Delta}{\omega} - (-1)^n\cos n\pi\theta}{\left(1-(-1)^n\varepsilon^{\frac{n\pi k_1}{\omega}}\right)\left(\varepsilon^{\frac{n\pi\Delta}{\omega}} - (-1)^n\cos n\pi\theta + \dfrac{\Delta}{\omega\theta}\sin n\pi\theta\right)} \\
A_2 = \dfrac{L_2\,\omega\,\theta}{2}(1-\alpha)\left(\dfrac{I_1}{Z_2}\sin\phi_2 - \dfrac{Q_o}{M}\right)(a-jb) \\
A_3 = \dfrac{L_2\,\omega\,\theta}{2}(1-\alpha)\left(\dfrac{I_1}{Z_2}\sin\phi_2 - \dfrac{Q_o}{M}\right)(a+jb)
\end{cases}
$$

where

$$
a = \frac{\dfrac{a\gamma_1}{\theta}\varepsilon^{\frac{n\pi\Delta}{\omega}} + (-1)^n\sin n\pi\theta - (-1)^n\dfrac{a\gamma_1}{\theta}\cos n\pi\theta}{\varepsilon^{\frac{n\pi\Delta}{\omega}} - (-1)^n\cos n\pi\theta + (-1)^n\dfrac{\Delta}{\omega\theta}\sin n\pi\theta}
$$

$$
b = \frac{\varepsilon^{\frac{n\pi\Delta}{\omega}} - (-1)^n\cos n\pi\theta - (-1)^n\dfrac{a\gamma_1}{\theta}\sin n\pi\theta}{\varepsilon^{\frac{n\pi\Delta}{\omega}} - (-1)^n\cos n\pi\theta + (-1)^n\dfrac{\Delta}{\omega\theta}\sin n\pi\theta},
$$

and from these we get:

$$
(24)\begin{cases}
B_1 = m_1 A_1 = negligible \\
B_2 = -\dfrac{\omega\theta}{2}\left(\dfrac{I_2}{\omega}\sin\phi_2 - Q_o\right)\times \\
\qquad\dfrac{(-1)^n\sin n\pi\theta - j\left(\varepsilon^{\frac{n\pi\Delta}{\omega}} - (-1)^n\cos n\pi\theta\right)}{\varepsilon^{\frac{n\pi\Delta}{\omega}} - (-1)^n\cos n\pi\theta + (-1)^n\dfrac{\Delta}{\omega\theta}\sin n\pi\theta} \\
B_3 = -\dfrac{\omega\theta}{2}\left(\dfrac{I_2}{\omega}\sin\phi_2 - Q_o\right)\times \\
\qquad\dfrac{(-1)^n\sin n\pi\theta + j\left(\varepsilon^{\frac{n\pi\Delta}{\omega}} - (-1)^n\cos n\pi\theta\right)}{\varepsilon^{\frac{n\pi\Delta}{\omega}} - (-1)^n\cos n\pi\theta + (-1)^n\dfrac{\Delta}{\omega\theta}\sin n\pi\theta}
\end{cases}
$$

ϕ_2 which depends on ϕ_o, the phase of the e.m.f. when $t=0$, is still undetermined. We may get light on this point by a consideration of the final charge in the condenser as a function of ϕ_2.

Putting (24) into (21) and letting $t=\frac{n\pi}{\omega}$, we get the final charge in the condenser, Q, that is, the charge at the moment of sparking.

$$(25)\quad Q=(-1)^n\frac{2I_2}{\omega}\sin\phi_2\left[\frac{(-1)^n\cosh\frac{n\pi\Delta}{\omega}-\cos n\pi\theta}{(-1)^n\varepsilon^{\frac{n\pi\Delta}{\omega}}-\cos n\pi\theta+\frac{\Delta}{\omega\theta}\sin n\pi\theta}\right]$$
$$-(-1)^nQ_o\left[\frac{(-1)^n\varepsilon^{-\frac{n\pi\Delta}{\omega}}-\cos n\pi\theta-\frac{\Delta}{\omega\theta}\sin n\pi\theta}{(-1)^n\varepsilon^{\frac{n\pi\Delta}{\omega}}-\cos n\pi\theta+\frac{\Delta}{\omega\theta}\sin n\pi\theta}\right]$$

Let us for the moment keep resistance, inductance and capacity constant, and fix our attention on the spark length and the phase ϕ_2. Mathematically Q is now a function of ϕ_2 only. Changing the gap length changes the discharge voltage, and consequently the final condenser charge, so that physically, Q, depends only on the length of the gap. Since we have only one independent variable from the mathematical point of view, and similarly only one from the physical point of view, these two must coincide. Best adjustment of spark length, therefore, will mean the ϕ_2 given by:

$$\frac{\partial Q}{\partial\phi_2}=0.$$

The differentiation is not necessary, since we see at once that for maximum Q we must have:

$$\phi_2=\pi/2,$$

which gives from (12):

$$\phi_o=\pi/2+\tan^{-1}\frac{\gamma_1+\gamma_2-a\gamma_1\theta^2}{\theta^2+a\gamma_1\gamma_2-1}$$

Q becomes:

$$(26)\quad Q=(-1)^n\frac{2I_2}{\omega}\cdot\frac{(-1)^n\cosh\frac{n\pi\Delta}{\omega}-\cos n\pi\theta}{\left[\begin{array}{l}(-1)^n\varepsilon^{\frac{n\pi\Delta}{\omega}}-\cos n\pi\theta+\frac{\Delta}{\omega\theta}\sin n\pi\theta\\ +(-1)^n\frac{Q_o}{Q}\Big((-1)^n\varepsilon^{-\frac{n\pi\Delta}{\omega}}-\cos n\pi\theta\\ \qquad\qquad-\frac{\Delta}{\omega\theta}\sin n\pi\theta\Big)\end{array}\right]}$$

This form is chosen as $\frac{Q_o}{Q}$ is a constant whose value depends on the quenching of the gap.

We are now in a position to write down the expression for output of the system.

This will be taken as the energy, W_2, lost by the condenser thru discharge.

$$W_2 = \frac{1}{2C}(Q^2 - Q_o^2) = \frac{Q^2}{2C}\left(1 - \frac{Q_o^2}{Q^2}\right)$$

$$(27) \qquad W_2 = \frac{2E^2\theta^2(1-\alpha)(1-Q_o^2/Q^2)}{\alpha L_1\omega^2[(\gamma_1+\gamma_2-\alpha\gamma_1\theta^2)^2+(\theta^2+\alpha\gamma_1\gamma_2-1)^2]} \cdot \left[\frac{(-1)^n\cosh\frac{n\pi\Delta}{\omega} - \cos n\pi\theta}{(-1)^n\varepsilon^{\frac{n\pi\Delta}{\omega}} - \cos n\pi\theta + \frac{\Delta}{\omega\theta}\sin n\pi\theta + \frac{Q_o}{Q}\left(\varepsilon^{-\frac{n\pi\Delta}{\omega}} - (-1)^n\cos n\pi\theta - (-1)^n\frac{\Delta}{\omega\theta}\sin n\pi\theta\right)}\right]^2$$

Q_o is ordinarily zero. The oscillograms show that the voltage across the condenser is suddenly reduced to zero at the moment of sparking. This is to be expected since the spark presumably goes out when the energy of the closed circuit is all transferred to the antenna. This may happen after one or more beats have occurred in the oscillating circuits. The number of beats that are executed before the spark goes out depends, of course, on the quenching properties of the gap. We care very little, however, about this part of the whole phenomenon. As long as the gap does not arc, the duration of the spark is infinitesimal as compared to the periods of time with which we are dealing. The value of Q_0 is all we require, and the physical evidence is that it is zero. Putting $Q_o = 0$ we have two important cases:
(a) $n=1$, or two sparks per cycle:

$$(28) \qquad W_2 = \frac{2E^2\theta^2(1-\alpha)}{\alpha L_1\omega^2[(\gamma_1+\gamma_2-\alpha\gamma_1\theta^2)^2+(\theta^2+\alpha\gamma_1\gamma_2-1)^2]} \times \left[\frac{\cosh\frac{\pi\Delta}{\omega} + \cos\pi\theta}{\varepsilon^{\frac{\pi\Delta}{\omega}} + \cos\pi\theta - \frac{\Delta}{\omega\theta}\sin\pi\theta}\right]^2$$

(b) $n=2$, or one spark per cycle:

$$(29) \quad W_2 = \frac{2E^2\,\theta\,(1-a)}{aL_1\omega^2[(\gamma_1+\gamma_2-a\gamma_1\theta^2)^2+(\theta^2+a\gamma_1\gamma_2-1)^2]} \times \left[\frac{\cosh\frac{2\pi\Delta}{\omega}-\cos 2\pi\theta}{\varepsilon^{\frac{2\pi\Delta}{\omega}}-\cos 2\pi\theta+\frac{\Delta}{\omega\theta}\sin 2\pi\theta}\right]^2$$

The latter case is the better of the two for experimental purposes and is plotted in Figure 3. The curves shown illustrate the effect of tuning on output. a, L_1, E, and ω are held constant and are therefore grouped on the left-hand side of the equation as a multiplier of the output, W_2. In this way it is possible to plot actual values so that W_2 may be found numerically from the curves by simply assigning values to the constants and the independent variable, θ. $\gamma_1(1-a)+\gamma_2$ represents the damping and is taken as a parameter. $\theta\left(=\sqrt{\frac{1}{aL_2C\,\omega^2}}\right)$, it should be remembered, is the ratio of the free period of the system to the impressed, and its variation may best be regarded as due to the variation of the capacity, C, since this is the only quantity which changes θ without also changing γ_1 and γ_2.

The salient feature of the output curves is that on either side of resonance, $\theta=1$, when the free period is equal to the impressed, there are maxima. The positions of these maxima are only slightly affected by the damping. That is, tuning is more or less independent of the resistances. Tuning, moreover, is not affected by the primary inductance, but the output is inversely proportional thereto. The output, also, as might be expected, is proportional to the square of the impressed voltage. The secondary inductance and the capacity are directly connected with tuning (that is, the value of θ), but have no influence outside of this on the magnitude of the output.

The coupling factor a (one minus the square of the coefficient of coupling) enters explicitly in numerator and denominator, and implicitly as well, since θ, γ_1, and γ_2 all depend on a.

The curves in Figure 5 show the relation between coupling and output. In this case, θ is fixed and the output multiplied by a constant term is plotted against a. For brevity a new letter is introduced.

$$D = a\,(\gamma_1\,(1-a)+\gamma_2) = \frac{R_1}{L_1\,\omega}(1-a)+\frac{R_2}{L_2\,\omega}$$

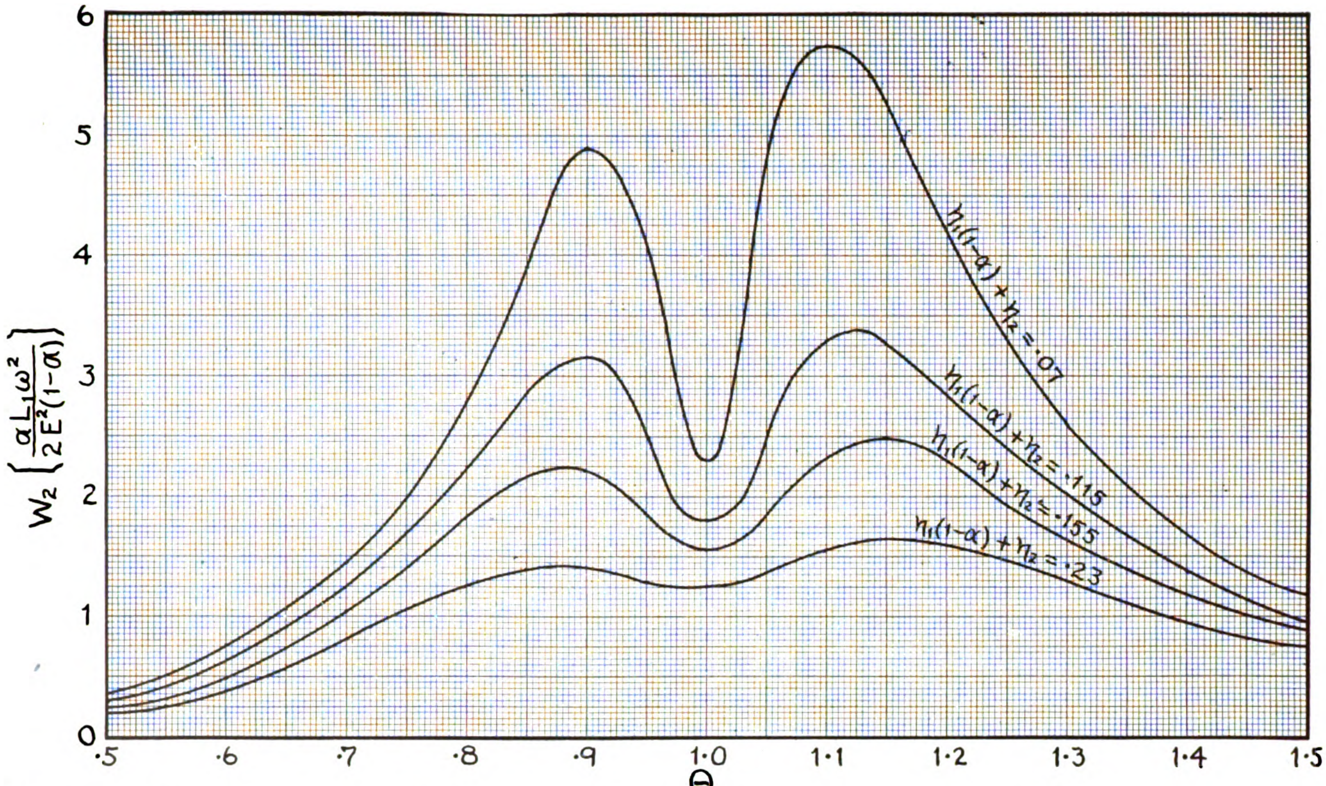


Fig. 3 Transformer Characteristics One Spark per Cycle

Plot of $$W_2\left\{\frac{\alpha L_1\omega^2}{2E^2(1-\alpha)}\right\} = \frac{\Theta^2}{\{\eta_1(1-\alpha)+\eta_2\}^2+\{\Theta^2-1\}^2}\left\{\frac{\cosh\frac{2\pi\Delta}{\omega}-\cos 2\pi\Theta}{e^{\frac{2\pi\Delta}{\omega}}-\cos 2\pi\Theta+\frac{\Delta}{\omega\Theta}\sin 2\pi\Theta}\right\}^2$$

This term is taken as constant, in spite of its dependence on α, in order to facilitate plotting. The error due to this is not great along the useful portions of the curve,—that is, in the region

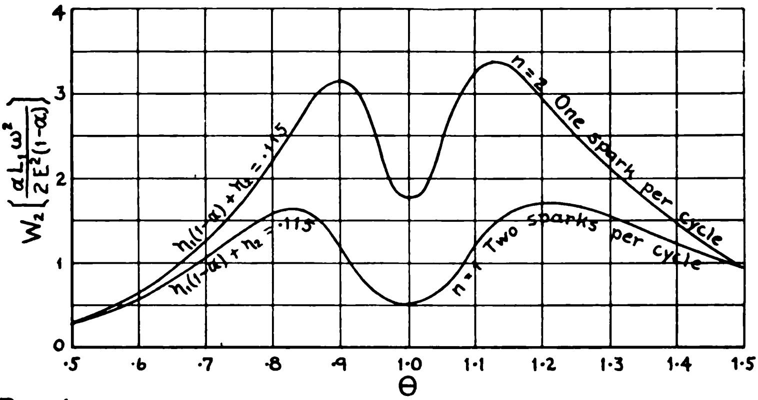


Fig. 4 Transformer Characteristics for One and Two Sparks per Cycle

where α is small. At first sight it will appear singular that for $\alpha=0$, (unity coupling), there is no output. The reason for this is that when α approaches zero the capacity must be increased indefinitely in order to keep θ constant. At $\alpha=0$, therefore, the

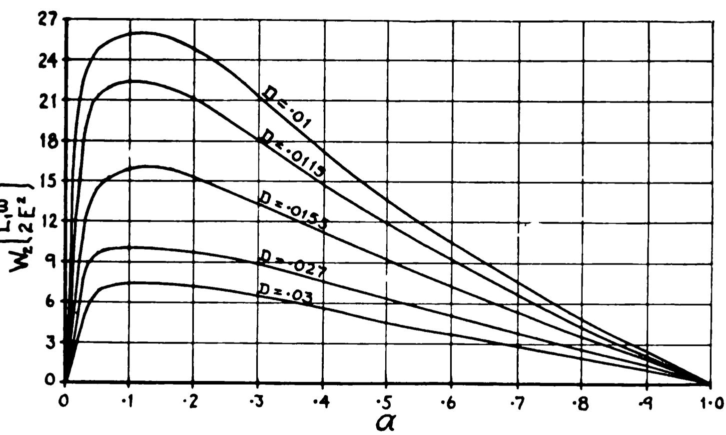


Fig. 5 Effect of α on Output $\theta = 0{\cdot}9$ $n=2$

$$W_2\left(\frac{L_1\omega^2}{2E^2}\right) = \frac{\left(\frac{1}{\alpha}-1\right)\theta^2}{\left(\frac{1}{\alpha}D\right)^2+\left(\theta^2-1\right)^2}\left\{\frac{\cosh\frac{\pi}{2}D - \cos 2\pi\theta}{e^{\frac{\pi}{2}D}-\cos 2\pi\theta+\frac{D}{2\theta\alpha}\sin 2\pi\theta}\right\}^2$$

condenser is infinitely large and its potential cannot be raised without an infinite supply of power. If, however, instead of keeping θ constant, C and L_2 are held fast, the output approaches a definite limit as α approaches zero. This limit is:

$$\lim_{\alpha \doteq 0} W_2 = \frac{1}{2} C E^2 \frac{L_2}{L_1} \left(\frac{1}{1+(DL_2 C \omega^2)^2} \right) \tag{30}$$

The expressions for the condenser charge, and the primary and secondary currents at any moment are:

(*a*) Condenser charge, q:

$$q = -\frac{I_2}{\omega} \left[\cos(\omega t + \phi_2 - \pi/2) - \varepsilon^{-\Delta t} \cos(\phi_2 - \pi/2) \frac{\sqrt{2 \varepsilon^{\frac{n\pi\Delta}{\omega}} \left(\cosh \frac{n\pi\Delta}{\omega} - (-1)^n \cos n\pi\theta \right)}}{\varepsilon^{\frac{n\pi\Delta}{\omega}} - (-1)^n \cos n\pi\theta + (-1)^n \frac{\Delta}{\omega\theta} \sin n\pi\theta} \times \cos\left(\omega\theta t - \tan^{-1} \frac{\Delta}{\omega\theta} + \tan^{-1} \frac{\sin n\pi\theta}{(-1)^n \varepsilon^{\frac{n\pi\Delta}{\omega}} - \cos n\pi\theta} \right) \right] \tag{31}$$

(*b*) Secondary current, i_2:

$$i_2 = I_2 \left[\sin(\omega t + \phi_2 - \pi/2) - \theta \varepsilon^{-\Delta t} \sin \phi_2 \frac{\sqrt{2 \varepsilon^{\frac{n\pi\Delta}{\omega}} \left(\cosh \frac{n\pi\Delta}{\omega} - (-1)^n \cos n\pi\theta \right)}}{\varepsilon^{\frac{n\pi\Delta}{\omega}} - (-1)^n \cos n\pi\theta + (-1)^n \frac{\Delta}{\omega\theta} \sin n\pi\theta} \times \sin\left(\omega\theta t + \tan^{-1} \frac{\sin n\pi\theta}{(-1)^n \varepsilon^{\frac{n\pi\Delta}{\omega}} - \cos n\pi\theta} \right) \right] \tag{32}$$

(*c*) Primary current, i_1:

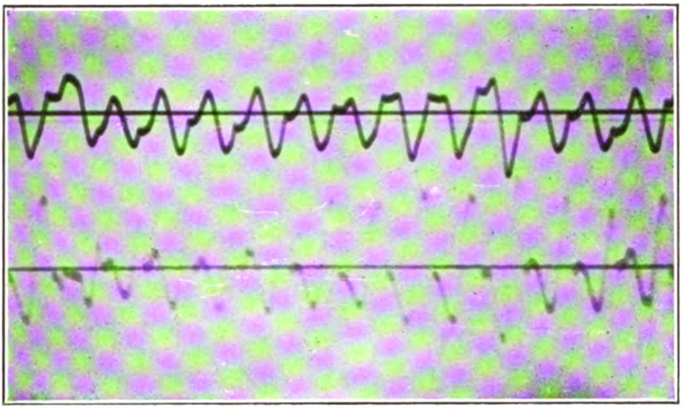
$$
(33) \quad i_1 = -I_1 \Bigg[\sin\left(\omega t + \phi_2 - \pi/2 - \tan^{-1}\frac{a\gamma_2}{1-a\theta^2}\right) + \frac{(1-a)\,\theta \sin\phi_2}{\left(\varepsilon^{\frac{n\pi\Delta}{\omega}} - (-1)^n \cos n\pi\theta + (-1)^n \frac{a\gamma_1}{\theta}\sin n\pi\theta\right)\sqrt{(1-a\theta^2)^2 + a^2\gamma_2^2}} \times
$$

$$
\left\{ \frac{2a\gamma_1}{\theta}\left(\frac{\cosh\frac{n\pi\Delta}{\omega} - (-1)^n \cos n\pi\theta}{1-(-1)^n \varepsilon^{n\pi a\eta_1}}\right)\varepsilon^{a\eta_1\omega t} \right.
$$

$$
- \varepsilon^{-\Delta t}\sqrt{2\varepsilon^{\frac{n\pi\Delta}{\omega}}\left(\cosh\frac{n\pi\Delta}{\omega} - (-1)^n \cos n\pi\theta + (-1)^n \frac{a\gamma_1}{\theta}\sin n\pi\theta\right)}
$$

$$
\left. \sin\left(\omega\theta t + \tan^{-1}\frac{\frac{a\gamma_1}{\theta}\varepsilon^{\frac{n\pi\Delta}{\omega}} + (-1)^n \sin n\pi\theta - (-1)^n \frac{a\gamma_1}{\theta}\cos n\pi\theta}{\varepsilon^{\frac{n\pi\Delta}{\omega}} - (-1)^n \cos n\pi\theta - (-1)^n \frac{a\gamma_1}{\theta}\sin n\pi\theta}\right) \right\} \Bigg]
$$

ϕ_2, it should be remembered, is equal to $\pi/2$ for the condition of maximum output,—that is, ideal adjustment of the spark gap. In practice, this condition is never realized on account of the irregular action of the gap as shown in Figure 6. At resonance ϕ_2 is nearly $\pi/2$ but off resonance it may be considerably different



Time→

FIGURE 6—Condenser Voltage (bottom) and Secondary Current (top) of 500 Cycle Transformer Circuit Showing Irregularity of Phase and Length of Spark

from this valve. Figure 7 is plotted from (31) and (32) on the assumption that $\phi_2 = 2\pi/3$. The dashed curves represent the forced components of current and condenser charge, and the dotted curves represent the free components. The sum of these give the actual current and condenser charge. These computed curves closely resemble the curves shown in the oscillogram, Figure 8, which was taken for the same circuit constants. It will be noticed that both the calculated and observed curves of voltage (or condenser charge which is proportional thereto) start concave downwards after the sudden drop from maximum voltage to zero which represents the sparking point. In other respects as well, such as the rapid rise before sparking, the curves are similar. At the moment of sparking there is a discontinuity in the first derivative of the current. In fact the cur-

rent, which before sparking was decreasing, actually increases slightly after sparking. It should be noticed also that in both the computed and observed cases the current is partly rectified.

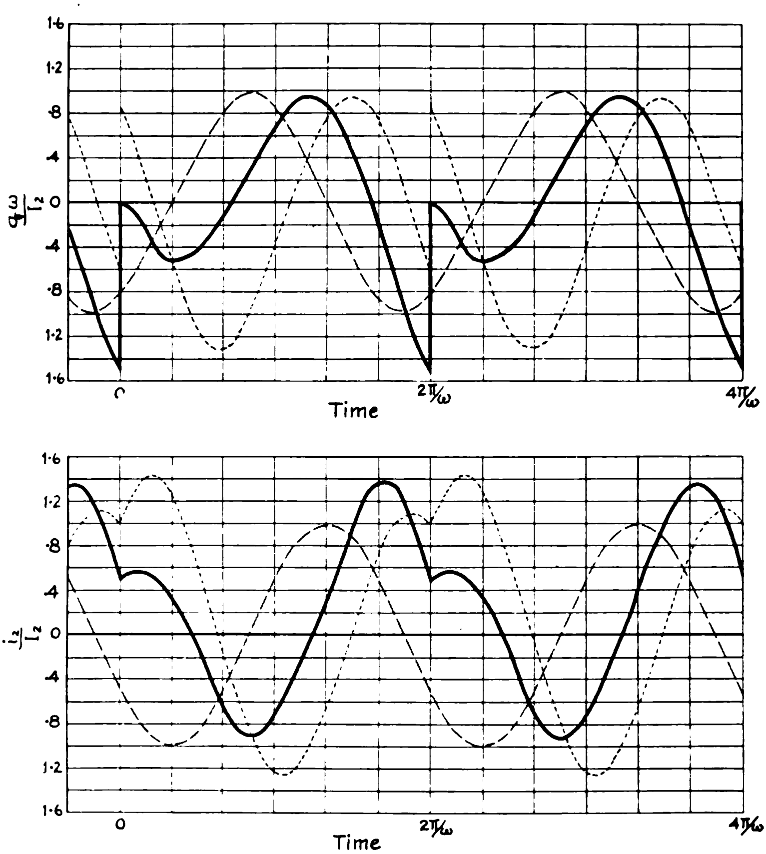


Fig. 7 (Top) Calculated Voltage Wave, (Bottom) Calculated Current Wave, for $\Theta = 1.2$, $\eta_1(1-\alpha)+\eta_2 = 0.09$, $\varphi_2 = 120°$

--------- free component

———- forced component

The energy input, W_1, for the period between discharges, that is from $t=0$ to $t=\frac{n\pi}{\omega}$, is:

$$W_1 = \int_o^{\frac{n\pi}{\omega}} i_1 E \sin(\omega t + \phi_o)\, dt$$

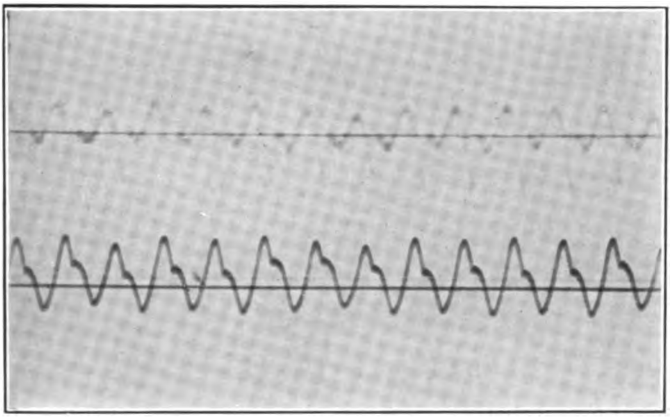
$$(34)\quad W_1=\frac{2E^2}{aL_1\,\omega^2\left[(\gamma_1+\gamma_2-a\gamma_1\theta^2)^2+(\theta^2+a\gamma_1\gamma_2-1)^2\right]}\times$$

$$\left[\frac{n\pi}{4}\left\{(\gamma_1+\gamma_2-a\gamma_1\theta^2)\,(1-a\theta^2)+a\gamma_2\,(\theta^2+a\gamma_1\gamma_2-1)\right\}\right.$$

$$+\left\{\frac{(-1)^n\cosh\dfrac{n\pi\Delta}{\omega}-\cos n\pi\theta}{(-1)^n\varepsilon^{\frac{n\pi\Delta}{\omega}}-\cos n\pi\theta+\dfrac{a\gamma_1}{\theta}\sin n\pi\theta}\right\}\times$$

$$\left\{\theta^2\,(1-a)\frac{\left(\theta^2+\dfrac{\Delta^2}{\omega^2}-1\right)\left(\theta^2-1-a\gamma_1^2(1-a\theta^2)\right)-\dfrac{2\Delta}{\omega}(\gamma_1+\gamma_2-a\gamma_1\theta^2)}{\dfrac{2\Delta^2}{\omega^2}(1+\theta^2)+(1-\theta^2)^2}\right.$$

$$\left.\left.-a\gamma_1\,(1-a)\,\left(a\gamma_1\,(\theta^2+a\gamma_1\gamma_2-1)-(\gamma_1+\gamma_2-a\gamma_1\,\theta^2\right)\right\}\right]$$



Time→

FIGURE 8—Condenser Voltage (top) and Secondary Current (bottom of 500 Cycle Transformer Circuit for $\theta=1.2$, $\gamma_1\,(1-a)+\gamma_2=.09$. One Spark per Cycle

MEASUREMENT OF CONSTANTS

The constants that enter in the theory are L_1, L_2, a, C, γ_1, $_2$, E, and ω (Δ and θ are simple functions of these). Of these C and ω may be measured with ease and accuracy. A voltmeter measurement of E is accurate only when the generator is sinusoidal. This will be taken up later. What especially ab-

sorbs our attention, however, is the measurement of L_1, L_2, and a, since it is principally on these that tuning depends.

In measuring the constants of an electric circuit it is desirable, if possible, to use methods which will involve only the use of ordinary voltmeters, ammeters, and wattmeters. In doing this, however, we must always be on our guard to see whether or not the readings of the instruments have the same meaning as the quantities used in the theoretical relations. If, for instance, a transformer is operated close to resonance the currents will be sinusoidal, but the voltage will be characteristic of the generator. A voltmeter across the line, therefore, may read much too high, for the reading is due to the fundamental plus all the harmonics which in the particular case exist only in the voltmeter circuit and contribute nothing to the main circuit The oscillogram shown in Figure 9 is an example of sinusoidal currents driven by an electromotive force abounding in harmonics.

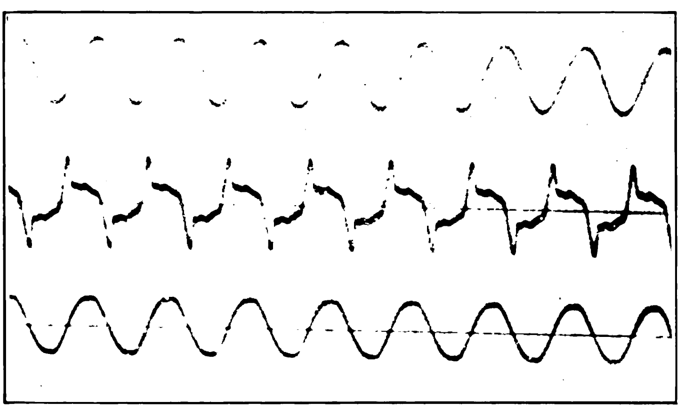


FIGURE 9—500 Cycle Transformer Circuit Resonated to Fundamental. Primary Current (bottom), Primary Voltage (middle), Secondary Current (top). Non-Sparking

Measurements to determine the constants are made under steady state, non-sparking conditions: Equations (13) and (15) afford the expressions we want. Taking I_1, I_2 and E as the root-mean-square values of sinusoidal currents and electromotive force, we have:

$$(35) \qquad I_1 = \frac{E L_2 \omega \sqrt{(1 - a \theta^2)^2 + a^2 \gamma_2^2}}{a L_1 L_2 \omega^2 \sqrt{(\gamma_1 + \gamma_2 - a \gamma_1 \theta^2)^2 + (\theta^2 + a \gamma_1 \gamma_2 - 1)^2}}$$

$$(36) \qquad I_2 = \frac{E\,M\,\omega}{\alpha L_1 L_2 \omega^2 \sqrt{(\eta_1+\eta_2-\alpha\eta_1\theta^2)^2+(\theta^2+\alpha\eta_1\eta_2-1)^2}}$$

Resonance we shall define as that condition where the natural period of the two circuits is the same as the impressed period, that is:

$$\theta = 1,$$

or

$$(37) \qquad L_1\,\omega\left(L_2\,\omega - \frac{1}{C\,\omega}\right) = M^2\,\omega^2$$

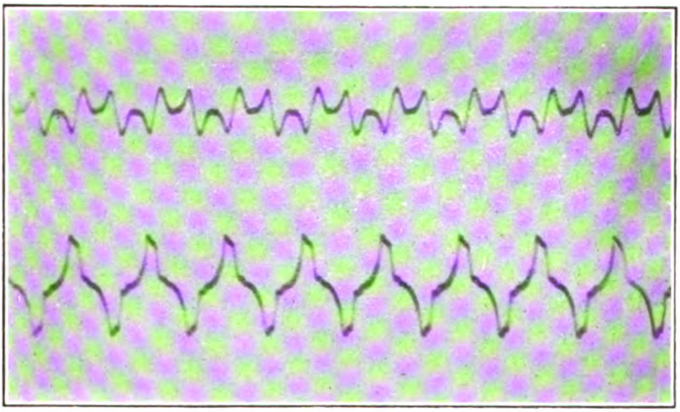


FIGURE 10—Primary Voltage (bottom) and Secondary Current (top). Result of Small Capacity, Illustrating Necessity of Operating Close to Resonance. Non-Sparking

At this adjustment the primary and secondary currents are very nearly a maximum.

When the voltage is not sinusoidal, great care must be taken to screen out the harmonics from all the currents. This is done by taking all measurements at resonance. Owing to the sharpness of tuning of radio circuits, adjustment for resonance to the fundamental is so bad an adjustment for the harmonics that the currents are practically sinusoidal, as in Figure 9. The following procedure will give the various constants:

(1) L_2:

Connect the secondary of the transformer in series with a condenser to the source of e.m.f. Resonate, and we have:

$$L_2 = 1/C'\,\omega'^2$$

This method is especially adapted to air transformers. For iron transformers, it is not so good, as the large impedance of the secondary makes it difficult to supply even the power required

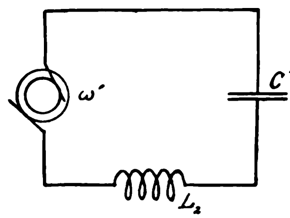


ILLUSTRATION A

for the iron loses, with the result that the magnetization of the iron is far below normal. This leads to incorrect results.

(2) L_1:

If large capacity is available repeat (1) replacing L_2 by L_1. This is the only direct method of measuring primary inductance since the volt-ampere, and the three voltmeter methods are inappropriate if the voltage contains harmonics to any marked degree. For iron transformers, an exploring coil should be used to ensure this test being made at the normal magnetization of the iron.

(3) a and L_1:

Resonate the transformer and measure primary current, $I_{1,r}$, and secondary current, $I_{2,r}$. Then from (35) and (36), putting $\theta = 1$:

$$a = \frac{1}{L_2 C_r \omega^2}$$

$$\frac{M}{L_1} = \frac{I_{1,r}}{I_{2,r}} = \frac{L_2}{M}(1-a)$$

$$L_1 = \left(\frac{I_{2,r}}{I_{1,r}}\right)^2 L_2 (1-a)$$

This last relation solved for L_2 is:

$$L_2 = L_1 \left(\frac{I_{1,r}}{I_{2,r}}\right)^2 + \frac{1}{C_r \omega^2}$$

At this point it is interesting to note that the degree of magnetization of the iron has hardly any effect on the position of resonance. That is, the transformer will resonate on the same condenser for weak as well as for strong magnetization of the iron. The reason for this will be understood from an examination of the condition for resonance—(37). As usual the resistance

terms have been neglected. There is another point, however, in which the formula is not strictly accurate. From the very beginning of the investigation, altho no reference was made to it at the time, it was assumed that the mutual inductance of the primary with respect to the secondary was the same as the mutual inductance of the secondary with respect to the primary. On account of the vagaries of the iron this assumption is not strictly accurate. We should have written M_1 and M_2 instead of simply M, giving:

$$L_1 \omega \left(L_2 \omega - \frac{1}{C \omega} \right) = M_1 M_2 \omega^2$$

This done, however, we are not so badly off as might be supposed, for if M_1 is a certain complicated function of the time, L_1 is a multiple of that same function, and if M_2 is another different function of the time, L_2 is a multiple of that different function. If coupling is close, as is ordinarily the case with iron transformers, $\frac{1}{C \omega}$ will be small as compared to $L_2 \omega$, so we see that altho L_1, L_2 and M_1, M_2 change with the time, their ratios do not change and the resonance condition is always fulfilled.

The method for measuring γ_1 and γ_2 depends on a measurement of voltage. If this voltage contains harmonics which contribute nothing to the resonant circuits, the ordinary voltmeter will read too high. Harmonics which are excluded from the transformer will get thru the voltmeter circuit. The wattmeter will read correctly, for altho harmonics may get thru the voltage coil, their integral effect over a whole period will be zero unless the same harmonics appear in the current. The power factor, therefore, will always be found too low if the voltage wave is distorted.

To correct this error, it is necessary to make a "resonant" voltmeter. This is accomplished by putting in series across the line an inductance, a capacity, and a milliammeter. The circuit is tuned to resonance so that the milliammeter reads the ohmic drop across the line. If the circuit is exactly a thousand ohms resistance, the milliammeter will be direct reading. Before each reading, however, it is essential to tune the circuit with precision or the ammeter will read too low. Tuning is effected either by varying the condenser or by using an inductance of the variometer type. Maximum reading of the ammeter will represent the actual value of the fundamental component of the voltage.

Equipped with such an instrument, we may proceed to measure damping coefficients:

(4) Δ, γ_1 and γ_2:

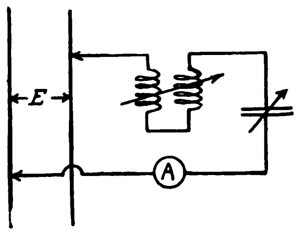


ILLUSTRATION B

Combining (35) and (36) with $\theta=1$,

$$\Delta = E\,C_r\,\omega^2 \frac{I_{1,r}}{2\,I_{2,r}}$$

$$\gamma_2 = \frac{1-a}{a}\sqrt{2\,(1-P.F.)}$$

where $P.F.$ is the power factor.

$$\gamma_1 = \frac{\frac{2\,\Delta}{\omega} - \gamma_2}{1-a}$$

The expression for γ_2 is very inaccurate if a is small. But if a is small we may approximate and write:

$$\gamma_1 + \gamma_2 - a\,\gamma_1\,\theta^2 = \gamma_1\,(1-a) + \gamma_2 = \frac{2\,\Delta}{\omega}$$

Using this substitution it is unnecessary for ordinary purposes to know either γ_1 or γ_2 explicitly.

The methods for a all include the inductance of the generator in the inductance, L_1, of the primary. If a for the transformer alone is desired the following method is convenient.

(5) a for transformer alone:

Repeat (1):

$$L_2\,C'\,\omega'^2 = 1.$$

Short-circuit the primary, and resonate again:

$$a\,L_2\,C\,\omega^2 = 1$$

So that we get:

$$a = \frac{C'\,\omega'^2}{C\,\omega^2}$$

The accuracy of this method depends on the fact that the inductance of the generator is very small as compared to the inductance of the secondary of the transformer; so that including the generator in the secondary does not have an appreciable effect on the resonance. This method is suitable only for air transformers.

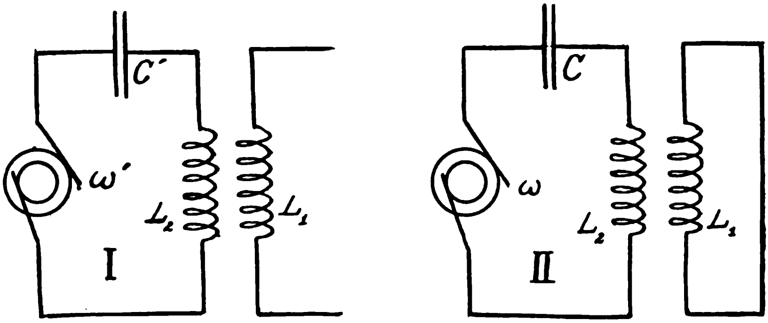


ILLUSTRATION C

There are a few points about the technique of resonance measurements that it is well to mention before leaving this subject. Resonance, it will be remembered, is defined as the condition where $\theta=1$. At this point both primary and secondary currents are approximately a maximum, and the resonant point may be found by the use of an ammeter. Maximum secondary current will give maximum condenser voltage. This leads to another method of detecting resonance. An electrostatic voltmeter shunted across the condenser will give maximum reading at resonance. For radio work, this method is very sensitive. The condensers ordinarily used are small, and a few milliamperes will often raise the condenser to a potential of several hundred volts.

In most resonance measurements it is necessary to apply very low voltages. If the full voltage of a generator be applied to a resonated transformer, the result is a bigger load than putting a dead short-circuit across the terminals of the generator. A dead short-circuit is somewhat controlled by the reactance of the armature, but even this is neutralized when the transformer is resonated. What saves the generator from destruction is the breaking down of the insulation of the secondary and the establishment of an arc which short-circuits the condenser and destroys the resonance.

When resonating an unknown inductance care should be taken that a harmonic in the electromotive force is not resonated instead of the fundamental. The accompanying oscillogram, Figure 11, shows the result of using a capacity resonant to the third harmonic. A large current is flowing in both primary and

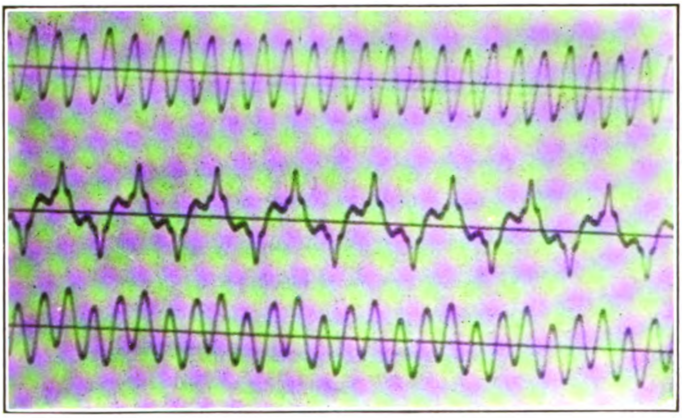


FIGURE 11—500 Cycle Transformer Circuit Resonated to Third Harmonic. Primary Current (bottom), Primary Voltage (middle), Secondary Current (top)

secondary, but the fundamental has almost entirely disappeared. Tuning is sharp. In one case two kilowatts of third harmonic power was the output of a 500 v. 5 k.w. Holtzer-Cabot generator. In another case, thru accidental adjustment of capacity, the third harmonic current attained such a value as to cause an arc to pass over the top of the Leyden jars.

Harmonic currents may be distinguished from the fundamental in several different ways. The oscillograph, of course, shows immediately just what is going on. Secondly, if considerable harmonic current is flowing into Leyden jars, the jars will sing with a note higher than that of the fundamental.

Lastly the nth harmonic will raise the condenser to a potential only $1/n$th as large as a fundamental current of the same value.

EFFECT OF E.M.F. WAVE SHAPE

After the emphasis that has been laid on measuring transformer constants at resonance, the question naturally arises, of what purpose this serves if the transformer is not operated at

resonance. Leaving aside the fact that this method alone gives us true values of inductances, coupling, etc., the conditions at resonance closely approximate the conditions at actual operation. Transformers are usually operated with a condenser about three-quarters or one and a quarter the resonant condenser. At both these points the currents are still close to pure sine waves. Above resonance this is very much the case. Even at twice resonant condenser value the currents are nearly sine waves. Below resonance, the third harmonic begins to appear, but at one-half resonant condenser value it does not yet appear in the secondary current strongly enough to have much influence on tuning (see Figure 12 and Figure 13). Sparking has the effect

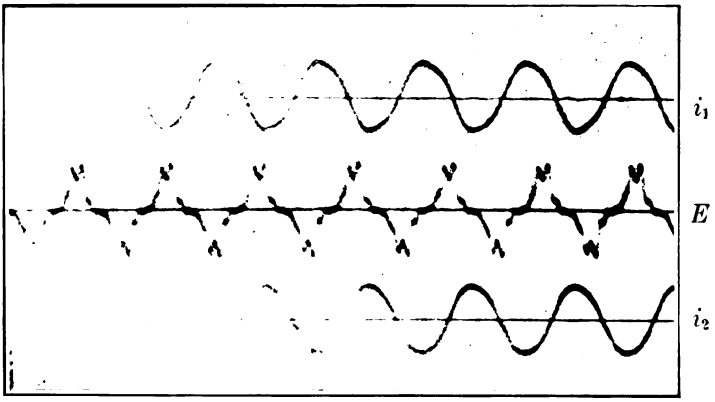


FIGURE 12—Voltage and Currents at Twice Resonant Condenser. Non-Sparking

of increasing the apparent size of the condenser and will consequently tend to diminish the harmonics. At both these points, the currents are still close to pure sine waves. We arrive, then, at the important conclusion that the shape of the e.m.f. wave has no effect on the shape of the current wave. All voltage waves possessing the same fundamental will act alike—they deliver only fundamental current.

GENERATOR DESIGN

Since harmonics are excluded from the transformer circuits, we see that the shape of the generator e.m.f. wave has no influence on the phenomena of charging the condenser, provided

the system is somewhere near resonance. A flat-topped e.m.f. wave will give the same result as a peaked wave provided the fundamentals of both are equal. It becomes unnecessary, therefore, in designing radio generators to make any effort to obtain flat-topped or peaked waves, or even sinusoidal waves. No advantage will be gained by any special wave form except perhaps the sinusoidal, for in that case alone will a voltmeter across the generator terminals read effective voltage.

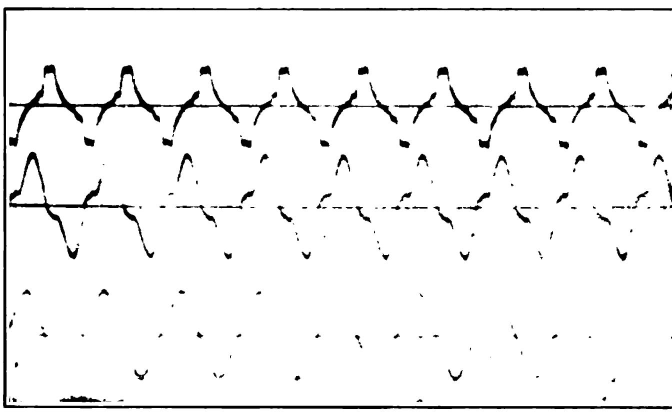


FIGURE 13—Voltage and Currents at Half Resonant Condenser. Non-Sparking

SUPPLEMENTARY METHOD FOR a

In view of the fact that at twice resonant condenser the currents are sinusoidal (Figure 12), the following method may be used for determining a.

Resonate the transformer and measure primary current I_1, and secondary current, I_2. This applies to (43); that is:

$$\frac{I_{1,r}}{I_{2,r}} = \frac{L_2}{M}(1-a)$$

Adjust the condenser to twice resonant value, that is $\theta^2 = 1/2$, and measure primary current $I_{1,2r}$ and secondary current $I_{2,2r}$. Then combining (35) and (36) with $\theta^2 = 1/2$, we get:

$$\frac{I_{1,2r}}{I_{2,2r}} = \frac{L_2}{M}(1-a/2)$$

Solving for α:

$$\alpha = \frac{\dfrac{I_{1,r}}{I_{2,r}} - \dfrac{I_{1,2r}}{I_{2,2r}}}{\dfrac{I_{1,r}}{I_{2,r}} - \dfrac{1}{2}\dfrac{I_{1,2r}}{I_{2,2r}}}$$

This method is insensitive for close coupling.

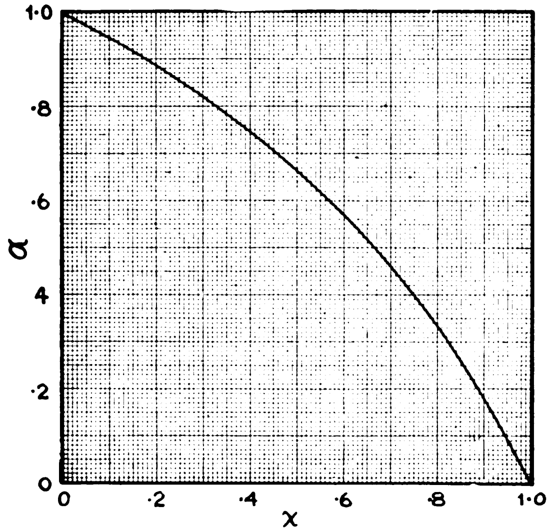


Fig. 14 Curve for finding α

$$\alpha = \frac{1-x}{1-\frac{x}{2}} \qquad x = \frac{I_{1,2r}/I_{2,2r}}{I_{1,r}/I_{2,r}}$$

If the coupling is close, put an exterior inductance in the primary. Thus loosened, measure the new coupling factor α'. If C_r' is the new resonant condenser, the real α is given by:

$$\alpha = \alpha' \frac{C'_r\,\omega'^2}{C_r\,\omega^2}$$

EXPERIMENTAL VERIFICATION OF THEORY

OUTPUT OF A SPARKING CONDENSER

Object: Given a condenser charged by an alternating current source thru a transformer, and discharging regularly, to find the energy lost by the condenser during a discharge. This is to be repeated for various condenser values.

General Method: Oscillograph the condenser voltage determining, V, the voltage at the instant of sparking, and V_o, the residual voltage after sparking. The energy lost per discharge is then given by:

$$W_2 = \frac{1}{2} C (V^2 - V_o^2).$$

Apparatus: The alternating current source is a 5 K. W., 250 volt, 500 cycle Holtzer-Cabot motor-generator set. This feeds the primary of a step up, closed core transformer. The secondary of the transformer is connected across a battery of Leyden jars. The jars discharge thru a rotary, synchronous gap, the energy of the radio frequency oscillation being absorbed by an auxiliary circuit in place of an antenna.

The rotary synchronous gap is simply an insulated copper arm carried on a projection of the generator shaft. This arm revolves inside a brass ring split into two halves. Set screws project thru the ring forming spark points. The spark passes from one-half of the ring thru the revolving arm and out the other half. A rubbing contact on the arm provides a means of using one air gap alone instead of two in series. The length of the gap is adjustable by giving an angular displacement to the split ring. Adjustment should be made so that the spark occurs before the spark points are opposite each other. This makes up for irregularities of voltage and insures the occurrence of a spark each time the points pass. Adjustment for sparking after the points have passed each other is a manifestly unstable condition for regular sparking.

A stroboscope is connected to the auxiliary oscillating circuit to observe the regularity of sparking. (This is not shown on the wiring diagram, Figure 15.)

The condenser voltage is measured by means of an oscillograph of the electromagnetic type. The high-potential side of a current transformer is connected across the Leyden jars, in series with a running water resistance. The water resistance is simply a glass tube, water entering the grounded end and leaving the high tension end in a broken stream. The secondary of the current transformer is short-circuited thru the vibrator of the oscillograph.

The current transformer is a small toroidal core with the primary and secondary wound on top of each other. The electrical theory of this transformation is taken up in another paper.

Measurement of sparking voltage is accomplished in the fol-

lowing manner. The spot of light from the vibrator is focused on a transparent screen. A scale is marked on the screen so that the maximum displacement of the spot may be easily observed. The spot is first brought to the zero of the scale, or its position simply noted. Then, when current is turned on, the spot becomes a band and the extreme excursion of the spot is easily

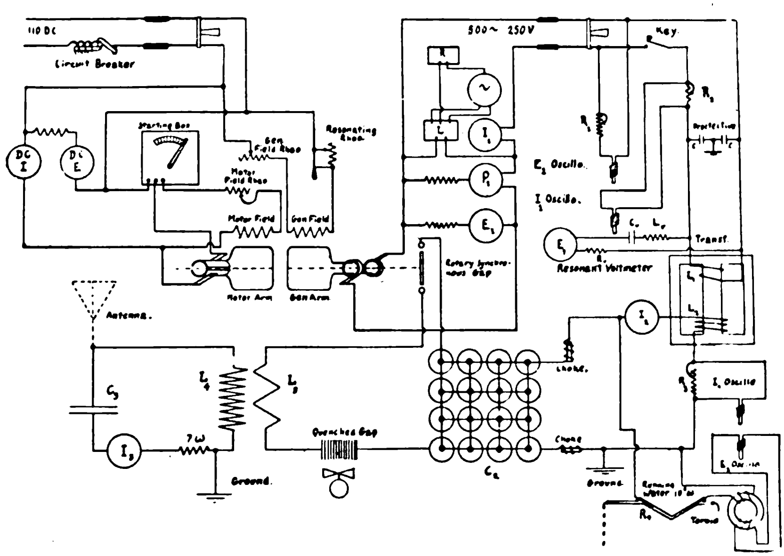


FIGURE 15—Wiring Diagram of 500 Cycle Radio Set

observed. This simple method is available because maximum potential is always sparking potential. There are exceptions to this, but only for "freak" adjustments of the rotary synchronous gap (as in Figure 23). The residual potential in the case of the rotary synchronous gap is found to have the convenient value of zero. This is shown by numerous oscillograms.

Besides the ordinary electrical instruments a "resonant" voltmeter is shunted across the terminals of the generator in order to determine the value of the 500 cycle fundamental.

Calibration: To calibrate the oscillograph, resonate the transformer cutting down the generator field so that there shall be no sparking. Read the secondary current, I_2, and the deflection of the oscillograph, **D**, in centimeters. Now:

$$\mathbf{D} \propto V.$$

where V is the maximum secondary voltage. Also,

$$V=\sqrt{2}\frac{I_2}{C_r\,\omega}$$

The oscillograph, therefore, reads $\dfrac{\sqrt{2}\,I_2}{C_r\,\omega\,\mathrm{D}}$ volts per centimeter.

Details of the Method: During a run, keep all the transformer constants fixed. Also keep the frequency constant and similarly the impressed voltage as read on the "resonant" voltmeter. In order to leave the generator armature out of the calculations, keep these last two constant for all conditions of load. Starting at resonant condenser and proceeding above and below resonance by increments or decrements of one Leyden jar, read the primary wattmeter and the maximum condenser voltage as given by the oscillograph. At each point adjust the rotary synchronous gap so as to give the greatest condenser voltage without spoiling the regularity of sparking.

This adjustment consists simply in lengthening the gap until there is a change to a lower spark frequency. Proper adjustment is at the point just preceding break down.

During the test, keep the flow thru the water resistance constant. Re-calibrate the oscillograph at frequent intervals to make sure that the water resistance does not vary.

Transformer Constants.

In order to determine the constants of the transformer alone without the generator, the following procedure is necessary:

(1) Primary inductance—$L_{1,t}$:

(*a*) Put a paper condenser in the primary circuit, and resonate with open circuited secondary, maintaining normal magnetization of the iron by the use of an exploring coil.

$$L_o+L_{1,t}=\frac{1}{C_r'\,\omega'^2}$$

where $L_{1,t}$ is the inductance of the transformer primary and L_o the inductance of the generator armature.

To get L_o, short-circuit the generator thru an ammeter. Then if E_o is the open circuit voltage—

$$L_o=\frac{E_o}{I\,\omega}\qquad \text{or:}$$

(*b*) Take open circuit voltage and current readings at normal saturation. Then if E_1 is the value of the fundamental voltage and E_3 the value of the third harmonic,

$$L_{1,t}=\frac{E_1+E_3/3}{I_{1,o}\,\omega}$$

The higher harmonics are too small to contribute an appreciable amount of current.

(2) Secondary inductance—L_2:

Owing to the symmetry of the transformer we can take—

$$L_2 = \frac{n_2^2}{n_1^2} L_1$$

where n_1 and n_2 are the respective number of turns of primary and secondary.

(3) Over-all coupling—a:

Resonate the transformer:

$$a = \frac{1}{L_2 C_r \omega^2}$$

This a is for the whole system, generator included.

(4) Transformer coupling—a:

$$a_t = 1 - (1 - a) \frac{L_o + L_{1,t}}{L_{1,t}}$$

(5) Resonant condenser for transformer alone—$C_{r,t}$:

$$C_{r,t} = \frac{1}{a_t L_2 \omega_t}$$

where ω_t is the frequency at which the set is to be operated.

(6) Damping—$\frac{2\Delta}{\omega}$:

Adjust the condenser to the value $C_{r,t}$ and the frequency to ω_t. Read E, the fundamental of the impressed voltage, and the primary and secondary currents, $I_{1,r}$ and $I_{2,r}$:

$$\frac{2\Delta}{\omega_t} = E\, C_{r,t}\, \omega_t \frac{I_{1,r}}{I_{2,r}^2}$$

From these readings we also get:

$$\frac{1 - a_t}{a_t L_{1,t} \omega_t^2} = \frac{I_{1,r}^2}{I_{2,r}^2} C_{r,t}$$

or:

$$a_t = \frac{1}{\frac{I_{1,r}^2}{I_{2,r}^2} C_{r,t} L_{1,t} \omega_t^2 + 1}$$

This value of a_t should check up with the value of a_t already obtained.

Numerical Values for Constants.

Closed Core Transformer.

$n_1 = 30$ turns; $n_2 = 1600$ turns; $R_1 = 0.0151$ ohms; $R_2 = 34$ ohms.

(1) $L_{1,t}$:

(*a*) Resonance at—$C_r{}^1 = 9.2\ \mu f$; 473 cycles.

$$L_o + L_{1,t} = 0.0123 \text{ henrys.}$$

Short circuit:

$E_o = 47.5$ volts; $I_o = 29.3$ amp.; 535 cycles.

$$L_o = 0.00048 \text{ henrys.}$$

$$\therefore \qquad L_{1,t} = 0.0118 \quad \text{''}$$

(*b*) Open circuited secondary—Curve plotted for different magnetizations of iron (Figure 16). At "550," normal magnetization.

$$L_{1,t} = 0.0114 \text{ henrys.}$$

Average of (*a*) and (*b*):

$$L_{1,t} = 0.0116 \text{ henrys.}$$

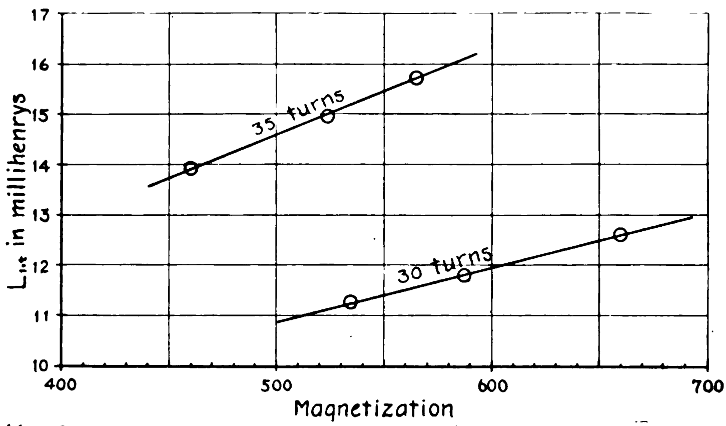


Fig. 16 Inductance of Transformer Primary vs. Magnetization of Iron

(2) L_2:

$$L_2 = \left(\frac{1600}{30}\right)^2 0.0116$$
$$= 33.0 \text{ henrys.}$$

(3) "Over-all" a:

Resonance at:

$$C_r = 0.018\ \mu f;\ 500 \text{ cycles.}$$

(4) Transformer a_t:

$$a_t = 1 - 0.83\,\frac{0.0121}{0.0116} = 0.134$$

(5) Resonant transformer condenser: $C_{r,t}$ at 530 cycles;

$$C_{r,t} = \frac{1}{0.134 \cdot 33 \cdot (2\pi \cdot 530)^2}$$
$$= 0.0204\ \mu f.$$

(6) Damping—$\frac{2\Delta}{\omega}$:

$$C_{r,t}=0.02\ \mu f;\ E=13 \text{ volts};\ 530 \text{ cycles};$$
$$I_1=24.5 \text{ amp.};\ I_2=0.485 \text{ amp.}$$

$$\frac{2\Delta}{\omega}=0.090$$

Also:

$$\frac{1-d_t}{a_t L_{1,t}\,\omega_t^2}=0.51\cdot 10^{-4}$$

The check for a_t gives:

$$a_t=0.132.$$

This agreement is rather better than we have a right to expect.

Theoretical Output of Transformer.

The expression for the output of the transformer in terms of the constants just determined is:

$$W_2=\frac{2E^2\,0.51\cdot 10^{-4}\,\theta^2}{(0.09)^2+(\theta^2-1)^2}\left[\frac{\cosh 0.283-\cos 2\pi\theta}{\varepsilon^{0.283}-\cos 2\pi\theta+\frac{0.045}{\theta}\sin 2\pi\theta}\right]^2$$

This is for one spark per cycle with impressed voltage E. The whole expression involving θ may be found from Figure 3 by means of the accompanying interpolating graph, Figure 17.

Experimental Results.

Calibration of oscillograph.

$\mathfrak{D}=13/32''$ $C=0.02\ \mu f$, cyclage $=530$ $I_2=0.730$.

$\therefore$ Deflection is equivalent to 1190 volts per $1/32''$.

Voltage Readings:

The sparking voltages for different condenser values in the vicinity of resonance are given in Figure 18. The energy output per spark discharge, W_2, is plotted (Figure 19) against the square root of the ratio of calculated resonant capacity to actual capacity—that is the ratio of the impressed period of the circuit to its free period.

Discussion of Results.

The theoretical and experimental curves plotted in Figure 19 show substantially that the theory is correct. In both cases there is a marked depression for $\theta=1$; that is, resonance adjustment. It should be noted that this resonance adjustment with the impressed e.m.f. on the transformer held constant, is not the same resonant adjustment as that which includes the generator. Including the generator, increases a and conse-

quently decreases the product $C\omega^2$ for resonance. The two maxima of the experimental curve come at the same value of θ, as do those of the theoretical curve. That these maxima are smaller than the theoretical may be explained by the irregularity of the spark length. The accompanying oscillogram, Figure 20, altho without zero lines, gives an idea of the sort of surge that runs thru both current and voltage. This surge appears to have some regularity. It is not, however, sufficiently

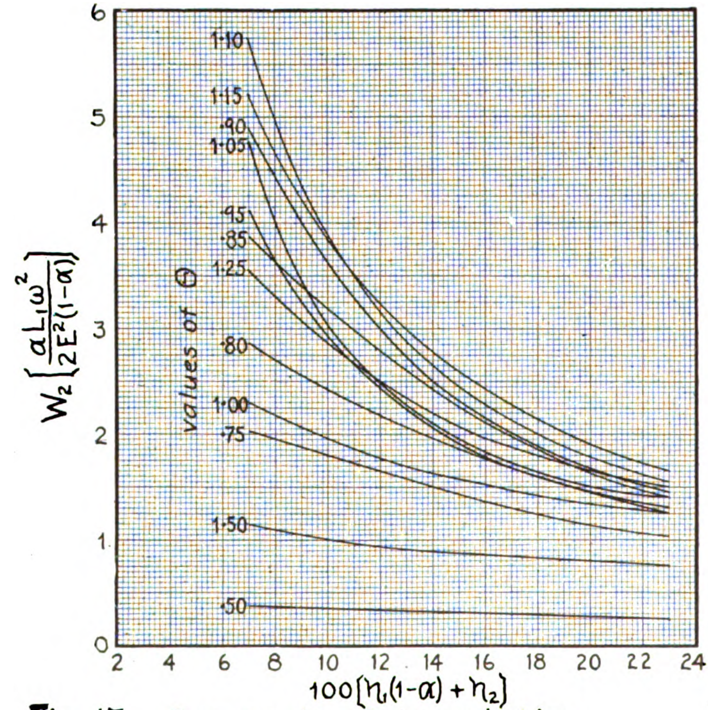


Fig. 17 Curves for Interpolation

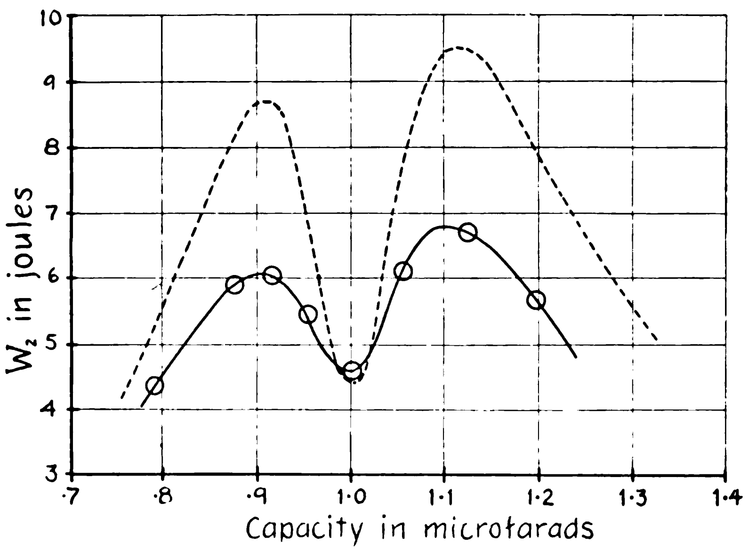
regular to be due to some slow electrical period, and its cause has not been traced out. This irregularity of spark length necessitates setting the gap for a lower sparking voltage than that which would correspond to a regular phenomenon. If this is not done the gap begins to spark intermittently. The result is that the average sparking voltage (which is the quantity observed) is lower than what it would be for uniform sparking. At the resonant position, the variation of the spark length is

C in μf	E in volts	P in K.W.	D in inches	V in volts
.014	146	3.8	24/32	28,600
.016	146	4.3	24½/32	29,200
.018	146	4.5	22/32	26,200
.020	146	4.0	18/32	21,400
.022	146	4.1	19/32	22,600
.024	146	4.0	19/32	22,600
.026	146	4.0	17/32	20,200
.032	146	3.8	14/32	16,700

FIGURE 18—Sparking Voltages for Different Capacities

very much smaller; so that a longer adjustment of the gap is possible; and a correspondingly greater output in proportion is reached.

For two sparks per cycle, it was not possible to get a complete output curve. At resonant adjustment the phenomena



Fig· 19 Energy Output per Spark Discharge

----- Computed —— Observed

went over from two to one spark per cycle. The rotary synchronous gap could not be shortened sufficiently to obtain two sparks per cycle. The reason for this may be seen from the theoretical curve for two sparks per cycle, Figure 4. The output at resonance is very small so that the necessary gap length must be small. If the gap is too long the discharge will stop completely, or under proper conditions go over into one of fewer sparks per cycle.

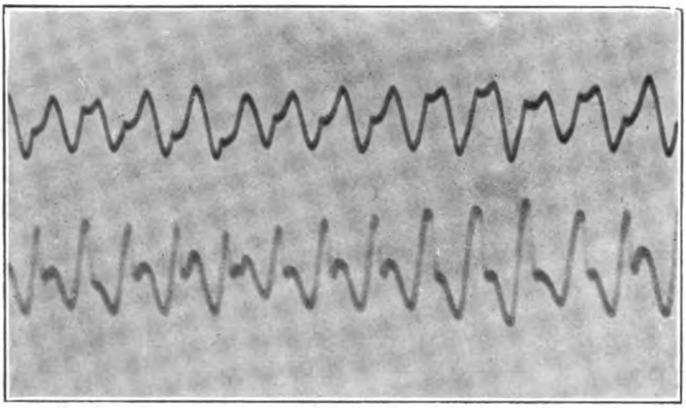


FIGURE 20—Condenser Voltage (bottom) and Secondary Current (top) Showing Surging of Spark Length. One Spark per Cycle

There is also a practical disadvantage in operating at two sparks per cycle. The maxima come further from the resonant point than in the case of one spark per cycle. The result is that it is difficult to get the sparking condition started. On closing the switch the voltage will not build up so high as for a position closer to resonance, and sparking may not start. If the gap is shortened to a length which will allow sparking to start, the phenomenon is very apt to pass over from two sparks a cycle to four or more sparks per cycle. Once started, however, the condition of two sparks per cycle will persist with a gap length longer than that required for starting.

Transformer Design.

The question of transformer design is usually one of designing a transformer for a given condenser, power, frequency, and efficiency. The problem is solved by starting at the condenser end. Let us say, for instance, that we wish a spark once a

cycle, the frequency being 500 cycles per second. Let us, moreover, choose to operate with a condenser smaller than the resonant condenser, since this is a point of better efficiency than for a capacity larger than resonant.

Taking $\theta = 1.1$, we must make $a L_2 = \dfrac{1}{1.21\, C\, \omega^2}$. The output is about the same for a between 0.05 and 0.2. The power factor increases as a decreases so it is well to take a small. Of course, this means a large L_2 and greater expense for winding, but it has the advantage of making $\dfrac{2\Delta}{\omega}$ large with the result that the tuning of the transformer is not so sharp. a, however, should be large enough to make the transformer self-controlling; that is, the current with short-circuited secondary should be within bounds. The value of the short-circuit primary current is: $I_{1,s} = \dfrac{E}{a L_1 \omega}$. Having chosen a, we get immediately the value of the secondary inductance, L_2. The actual realization of a given a for any particular type of iron transformer can only be obtained by a careful calculation of leakage fluxes.

There remains the calculation of the primary inductance, L_1. This is inversely proportional to the power output. From the curves L_1 may be calculated provided we know $\gamma_1 (1-a) + \gamma_2$. Now at resonance:

$$E/I_1 = R_1 (1-a) + R_2 \frac{L_1}{L_2}$$

The right-hand member is the apparent resistance of the whole circuit. If P is the power we may write approximately:

$$P\,(1 - \mathit{Effic.}) = I_1^2 \left[R_1 (1-a) + R_2 \frac{L_1}{L_2} \right]$$

where I_1 is the full load current. This will give

$$R_1 (1-a) + R_2 \frac{L_1}{L_2}.$$

So we have:

$$\gamma_1 (1-a) + \gamma_2 = \left[R_1 (1-a) + R_2 \frac{L_1}{L_2} \right] \frac{1}{a L_1 \omega}.$$

L_1 may now be found by use of the curves by a simple trial and error method. The result, however, will not be very trustworthy. The actual working output cannot be expected to come up to the theoretical output, and so the calculated value of L_1 will probably be too large. The character of the gap, in

so far as it affects the phase of sparking, has considerable bearing on the output; so that L_1 can never be a quantity determined with the same accuracy as L_2. It is easy, however, to wind the primary with taps so that the output may be adjustable.

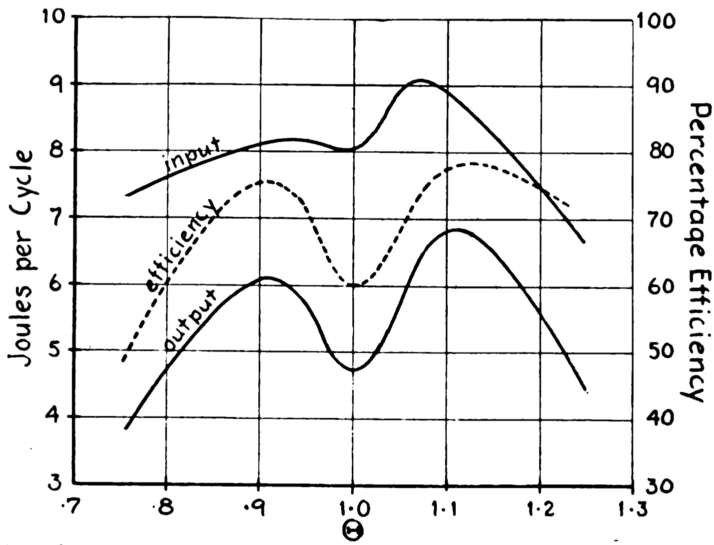


Fig. 21 Experimental Curves of Input, Output, % Efficiency

The desired efficiency is attained by the ordinary calculations for the quantity of copper and iron necessary.

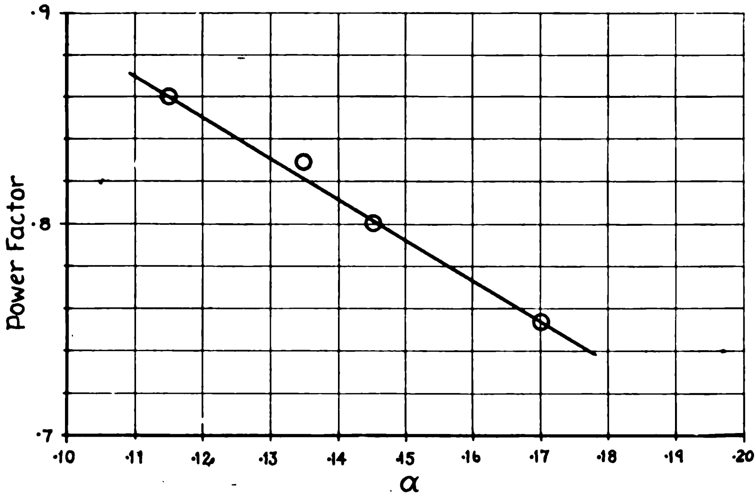
At this point it is well to mention that a proper gap is essential to the operation of any transformer. Too much stress cannot be laid on this. With the condenser fixed the output depends on the length of the gap only. In what has gone before it was, of course, assumed that the gap was such as to get the best possible results out of the rest of the system.

Figure 23 shows the effect of a very poor adjustment of the synchronous gap used in the preceding experiment. Sparking occurs at a low voltage but a large current is flowing in the secondary which causes a useless rise of voltage and waste of power.

Air Transformers.

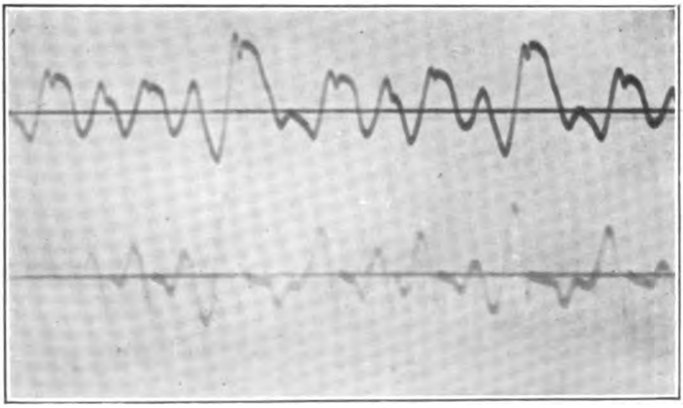
In all our equations, the primary and secondary inductances enter always as the products $L_1\omega$ and $L_2\omega$. At 500 cycles

($\omega = 3{,}000$) the inductances required are small compared to those used at commercial frequencies. The omission of iron in the transformer circuit immediately suggests itself, and is, in



Fig·22 Experimental Curve of Power Factor vs· α

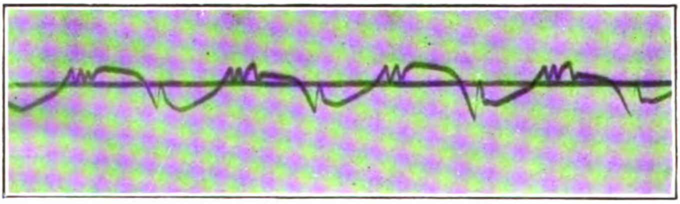
fact, entirely practical. For the last four months, a small 2 K.W. air transformer has been in constant operation at the Cruft Laboratory with entirely satisfactory results.



Time →

FIGURE 23—Condenser Voltage (bottom) and Secondary Current (top) for Short Adjustment of Rotary. Gap Showing Early Spark and Occasional Arcing. One Spark per Cycle

The problem in the design of air transformers is: given the efficiency, to get the proper inductance in primary and secondary and at the same time reasonably close coupling. Loose coupling decreases the power factor and is therefore undesirable.



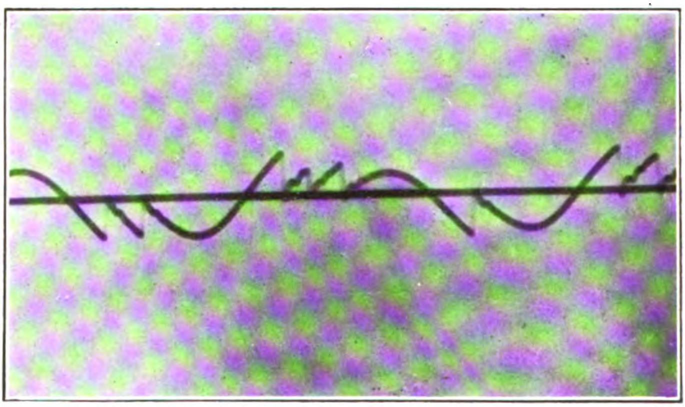


FIGURE 24—Types of Irregular Sparking. Condenser Voltage Taken with an Electrostatic Oscillograph. 60 Cycles

How to get enough inductance is the principal question. The Maxwellian form for coils immediately suggests itself as being economical of copper. Two coils of this shape placed side by side make a good transformer, efficient and easy to build, but with rather loose coupling, and consequently not too good a power factor. Moreover, if α is large, the output tends to be low, and L_1 must be made small to counteract this effect. If L_1 is small the inductance of the generator armature has a greater loosening effect on the coupling than if L_1 is large. α is therefore further increased by a small L_1 and we go from bad to worse. It is, in fact, so important not to get the coupling

too loose that much effort should be expended in this direction.

Sandwiching a set of coils always increases the coupling. With extensive sandwiching we may get the effect of the primary and secondary practically occupying the same space and consequently almost unity coupling. This cannot be pushed too far in practice as it involves too much insulation and a poor space factor. A poor space factor necessitates larger over-all dimensions and a consequent increase in copper. A desirable form for an air transformer for 500 cycles is one with a secondary winding between two primary windings in series, the over-all dimensions of both coils set up and including the insulation between coils being Maxwellian. That is, the cross sections of the two windings side by side should be a square of which the length of the side should be about four times the mean diameter of the coils.

Air transformers are increasingly easy to build as the frequency is raised. At 1,000 cycles the secondary inductance need be only one quarter as great, for the same condenser, as at 500 cycles. This indicates that it is desirable to replace the 500 cycle set sparking twice a cycle by a 1,000 cycle set sparking once a cycle. A cheap and efficient air transformer having a good power factor may then be built. There is, moreover, the added advantage already stated that we may operate closer to resonance for one spark per cycle with a resulting improvement in the regularity of the spark frequency. The arguments used for changing from 1,000 cycles may equally well be applied to the proposition of increasing the frequency above 1,000 cycles and sparking still fewer times per cycle so as to keep the spark frequency at about 1,000 sparks per second. Progress in this direction depends on the question of generator design.

The essential difference between iron and air transformers is that the efficiency of the former is handicapped by a fixed iron loss, whereas the efficiency of the latter may be made as close to one as we please. Of course the iron loss of a transformer may be reduced by decreasing the quantity of iron, but pushing this to extremes means passing over into the domain of the air core.

CRUFT LABORATORY,

May 29, 1915.

SUMMARY: Starting with the general solution of the differential equation of the circuit and a steady sparking state, expressions for input, output, currents, and condenser charge are calculated by means of the boundary conditions of the interval. The boundary conditions are continuity of currents at sparking and a residual condenser charge experimentally determined. Calculated current and voltage waves for a special case are compared with an oscillogram taken for the same transformer constants. A number of curves illustrating the quantities studied (and also curves facilitating their computation) are given; and, on comparison, are found to agree closely with oscillograms taken during actual operation.

On the basis of the derived theory, a complete set of transformer measurements are explained, and numerical illustrations given. The effect of harmonic components in the e.m.f., and the avoidance of their effects are explained, together with the effect of the wave form of the e.m.f.

In connection with the experimental verification of the theory, there are described the necessary measuring apparatus, including the oscillographs.

The design of the transformer to meet any desired requirements is then treated in detail from the theoretical and practical standpoints. The use of air core power transformers at frequencies of 500 cycles and over is considered and found to be feasible. An interleaved set of coils of "Maxwellian" cross section is recommended for the "sandwiched" primary and secondary. An actual satisfactory transformer of this type for 2 K. W. is described.

DISCUSSION

J. H. Morecroft (communicated): Mr. Cutting's paper on the radio transformer deals with a very complex phenomenon, as one might well believe by glancing at some of the equations given therein.

Every time the switch of an alternating current circuit is closed, there may exist for several cycles after the closing, currents and voltages much higher (or lower) than the values which exist during the so-called steady state, which values are given by the ordinary formulas—connecting voltage, current and impedance. These irregular values are due to the so-called transient current, a current which may be oscillatory or exponential and which enables the circuit to "settle down" to its normal operating condition.

The time during which the transient term lasts depends upon the damping of the circuit, of course; in radio circuits it may perhaps be 10 or 20 cycles but when a radio set is sparking once per cycle, a transient turn is introduced at each and every cycle. Every time a spark occurs a new transient turn is added to those already in the circuit; thus at a given instant the actual current in the secondary circuit is made up of a great many currents, mathematically speaking; one of these is the steady state term, another is the first cycle of the transient term introduced at the last spark, another the second cycle of the transient term introduced by the previous spark, etc. If the damping of the circuit is such that the twentieth cycle of a transient is the last to be of any appreciable value, the above summation of terms must be carried out to the twentieth cycle of the transient introduced by the spark occurring twenty sparks before the one considered. (Natural frequency = impressed frequency, assumed.)

The brief analysis given above holds also for the form of the voltage wave across the condenser in the secondary circuit; thus every time a spark occurs, the condenser voltage is reduced nearly to zero; between the times of successive sparks the voltage wave will not be a sine wave but a complex form as indicated for the current.

In my classroom I have called this phenomenon the "steady state of transients" because, altho the form of wave may be very complex, it recurs periodically. I have never been able to solve the problem completely and so feel that we owe Mr. Cutting our thanks for the arduous work which is apparent in his deriva-

tions. It is inconvenient that the phenomenon does not lend itself to more simple treatment than the author has found possible; thus the average radio engineer will not care to calculate the input to his transformer from an expression as formidable as that given in equation 34 or some of its predecessors. However, such equations may be entirely the fault of the phenomenon and not that of the author.

I found some difficulty in following some of the author's derivations; thus in the part headed "Details of Method" occurs the expression

$$L_{1,t} = \frac{E_1 + E_3/3}{I_{1,o}\,\omega}$$

This equation, I take it, presupposes a knowledge of the equation of the voltage wave but what is the meaning of I_1? If it is the reading of an ordinary ammeter in the circuit, the expression seems to me to be incorrect.

The question of armature reaction on the voltage generated in an alternator also seems to receive no attention by the author; thus the value of L_o obtained by the method given may be much too large; using an ordinary alternator, the inductance so obtained may be twice the actual value of inductance. In the case of a radio alternator having relatively high armature reactions, the error might be much greater.

In the experimental results given to confirm the theory, the terminal voltage of the alternator was held constant, so that in so far as the steady relation between I and E are concerned, the armature inductance is neglected. The transient terms, however, cannot be made independent of the armature inductance as this helps to determine their period, amplitude, etc. It seems to me that in this particular the experimental results do not apply to the theory.

The actual experimental data given in tabular form is very disappointing. Even when the phenomenon being investigated is very steady, the measurement (as carried out by the author) of the width of an oscillograph beam to 1/32 inch (0.8 mm.) is somewhat questionable. When the irregularity of the sparking of the ordinary gap is considered, it seems to me doubtful whether the recording of a measurement of the beam to 1/32 inch means very much; the conditions of sparking must have been very irregular, otherwise the power input could surely have been measured more accurately.

Assuming, however, that the sparking voltages are measured to ± 1/64 inch (0.04 cm.) (this means that the position of the outer

extremity of a beam could be located to about 1/100 inch (0.025 cm.)), I calculated the value of W_2 from the author's data: thus the reading given as 14/32 inch was considered as something between 14.5/32 inch and 13.5/32 inch. The author's Figure 19, then takes the form of my Figure 1. The curve for W_2 may be

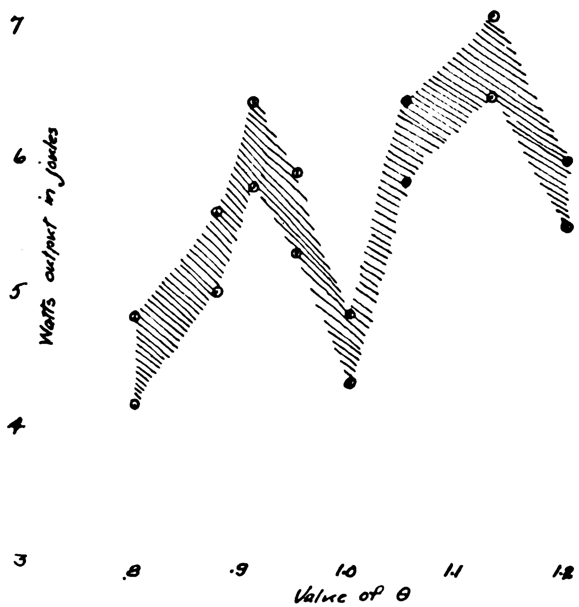
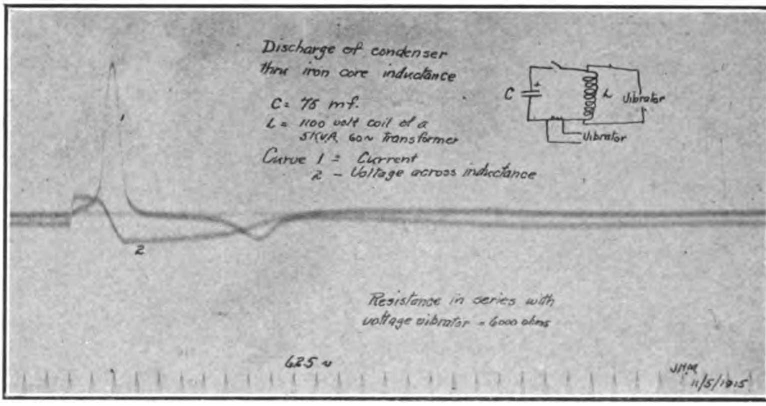


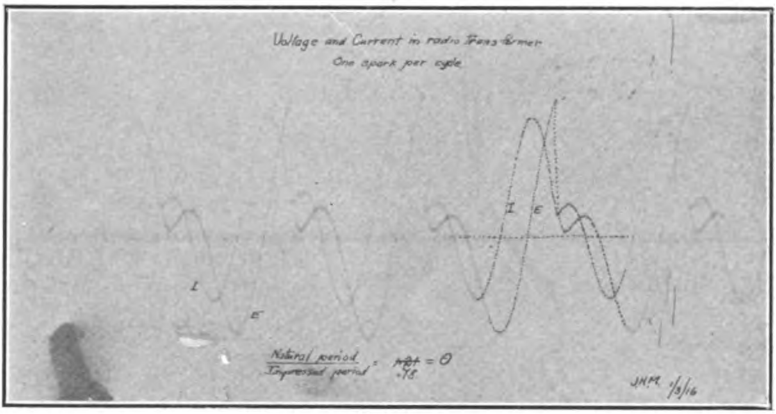
FIGURE 1

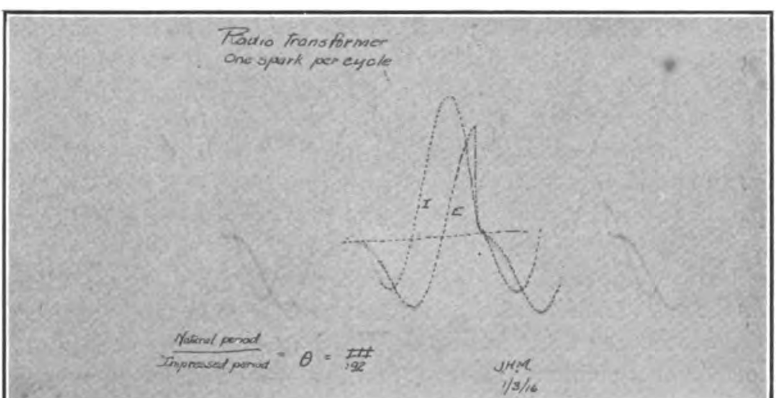
drawn *anywhere thru the shaded area with equal justification.* It will be noticed that the form of the curve for θ less than 1, is different from the author's. This is because the value of W_2 was incorrectly calculated by Mr. Cutting. For $\theta = 0.87$, he should have had 5.3 joules, but in the figure 5.9 joules is plotted.

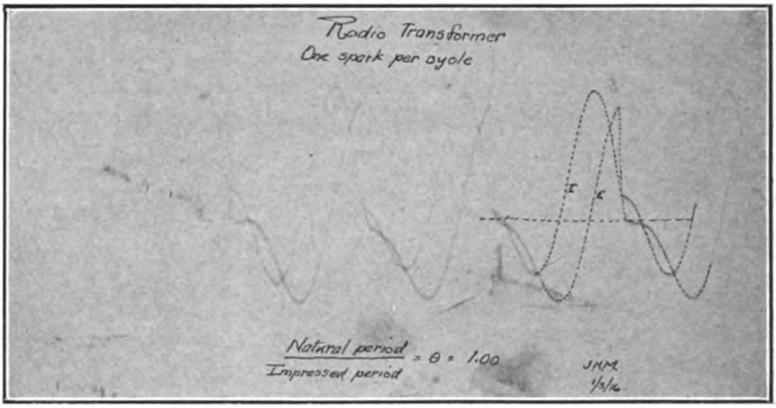
The curve for efficiency in Figure 21 is still more doubtful because it depends not only upon Figure 19, but also on the idea that the sparks took place uniformly, one per cycle, and that the voltage of each spark was the same.

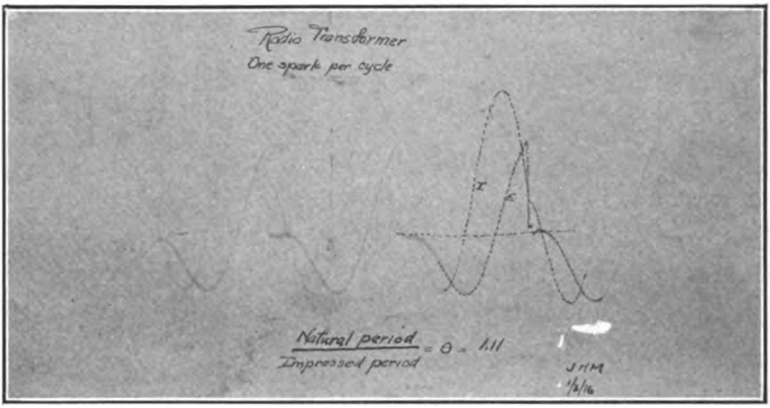
Under the heading of "Transformer Design," the author recommends the use of a condenser smaller than that required to give resonance. It is well known, however, that a radio set must have a condenser larger than that required for resonance to give uniform sparking.

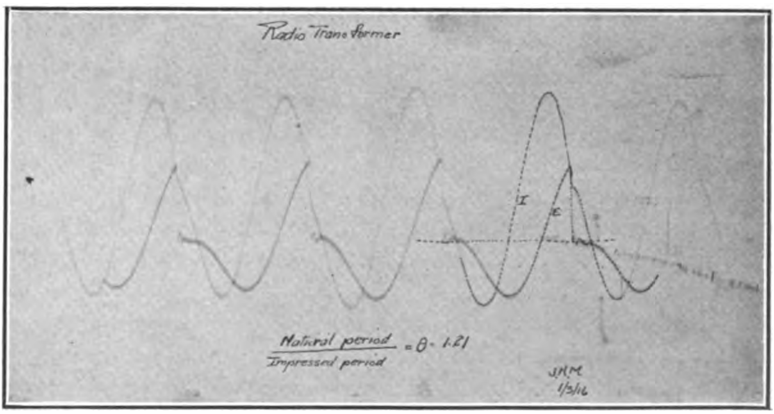

Discharge of condenser
thru iron core inductance
C = 75 mf.
L = 1100 volt coil of a
5KVA 60~ transformer
Curve 1 = Current
2 - Voltage across inductance
C
L
Vibrator
Vibrator
1
2
Resistance in series with
voltage vibrator = 6000 ohms
625 ~
JHM
11/5/1915


Voltage and Current in radio Transformer
One spark per cycle
I
E
I
E
Natural period / Impressed period
JHM 1/3/16


Radio Transformer
One spark per cycle
I
E
Natural period / Impressed period
JHM
1/3/16


Radio Transformer
One spark per cycle
I
E
Natural period / Impressed period = θ = 1.00
JHM
1/3/16


Radio Transformer
One spark per cycle
I
E
Natural period / Impressed period = θ = 1.11
JHM
1/3/16


Radio Transformer
I
E
Natural period / Impressed period = θ = 1.21
JHM
1/3/16

Why did the author use a quenched spark in series with a rotating spark gap?

I thought it possible to obtain a set of readings perhaps more accurate than those of Mr. Cutting and more in accordance with the premises of his theory; thus in his experiment he assumed there was one spark per cycle; I made certain that such was the case. He had deflections of the order of 1/2 inch (1.2 cm.), whereas I got deflections of 2 inches (5 cm.), and moreover was able to measure this larger quantity with less error than he did in his smaller measurements. It must be remembered in this connection that the efficiency, etc., at which the theory arrives, depend upon the squared value of this measurement.

I performed a series of tests on a 150-cycle alternator. The armature was equipped with a rotating switch (a fiber disk with steel inserts arranged 360 degrees (electrical) apart), and this switch was connected across the condensers; thus forming a short circuit on the condensers, once per cycle. The resistance of this discharge path was so chosen that the condenser could practically discharge during the time of short circuit. This time was about 15 degrees (electrical). The insulated brush bearing on the fiber disk was movable over 360 degrees of arc, so that the condensers could be short circuited on any portion of the cycle desired.

An oscillograph vibrator in series with 1,200 ohms resistance was connected across the condensers and the image from this vibrator was thrown upon the upper transparent screen of the oscillograph. The oscillograph motor (which actuates the oscillating mirror) was driven from another source of power so as not to disturb anything in the test circuit. For each value of capacity in the secondary circuit the position of the short circuiting brush was varied until the short circuit occurred at the maximum voltage it was possible to obtain.

A second vibrator of the oscillograph was connected to a variable source of continuous voltage and adjusted to the same deflection per volt as was the first. Then as the short circuit brush was moved this "measuring vibrator" was made to follow the peak of the voltage wave; thus making it possible to ascertain when the peak was highest and at the same time give the "spark voltage" by the indication of a voltmeter connected to the terminals of the vibrator.

The oscillograph motor was run at 1/2 synchronous speed so that two complete cycles were thus seen on the transparent screen and the uniformity and regularity of the wave noted.

As the whole adjustment was carried out in the dark, narrow beams of light could be used making it possible to read the spark voltage to $\pm$ 1 volt.

The alternator used had a smooth core armature with an inductance less than 1 per cent. of that of the outside circuit; the armature reaction was negligibly small so that the voltage acting in the circuit was practically the same for all adjustments of secondary condenser. The transformer used had an air core so that the variable effect of the iron core on the resistance and inductance of the circuit might be eliminated.

The form of the oscillatory current in a circuit having a closed iron core (as I judge had the transformer used by Mr. Cutting) is not of such a shape as can be written in the simple form used by the theory in question. To illustrate this phase

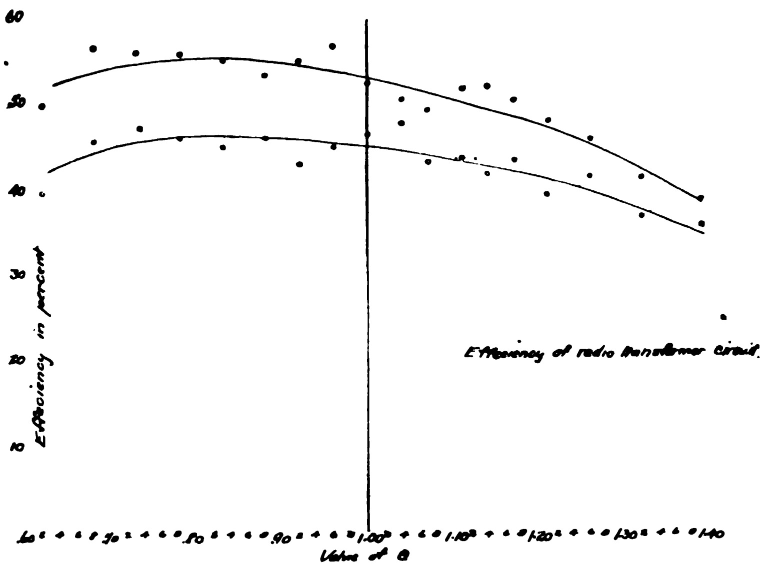


FIGURE 2

of the problem I give an oscillogram showing the form of oscillotory current and condenser voltage in such a circuit. While I cannot say to what extent this effect existed in Mr. Cutting's circuit, I am almost sure that the free oscillations were by no means of the simple form he has assumed.

The values of constants used in my test were as follows:

Inductance of alternator armature	0.000276 henrys
Inductance of transformer primary	0.0295 henrys
Resistance of transformer primary	1.87 ohms
Inductance of transformer secondary	0.0400 henrys
Resistance of transformer secondary	2.27 ohms
Mutual inductance of transformer	0.0253 henrys
Coupling	73.5 per cent.

Secondary capacity varied from 20 microfarads to 115 microfarads.

Several runs were made and efficiency curves were determined. They were all of the same form. Two of them are given in my Figure 2. It seems as tho the points obtained must be more accurate than those given by Mr. Cutting and yet I could find no depression in the curve at the resonance point in any of the curves I obtained. I am giving five oscillograms of the current and voltage curves in the circuit I tested; they serve to show the type of curves on which my measurements were made.

It seems to me, therefore, that more experimental confirmation of the formulas given is called for before we can regard the theory proved.

Fulton Cutting: I am much gratified to see the interest that Professor Morecroft has shown in my solution of the problem which he has properly called "The Steady State of Transients."

The expressions involved have, to be sure, a somewhat forbidding appearance; but to me appear relatively simple on account of the struggle I had to reduce them to their present form.

Equations (28) and (29) for instance, were, before approximations were introduced, composed of expressions like those appearing on page 164, and I consider it very fortunate that it was possible to boil down these expressions to the form obtained.

Professor Morecroft gives the results of some experiments which he claims are not in accordance with mine, and which do not fit the theory. He uses a method which should give consistent results, and I am surprised that his observations are not more uniform. If he can measure potentials to one volt, his points, I should think, would lie more closely on a smooth curve. The efficiency curve he obtains has no depression at the point $\theta = 1$, whereas, the curve I obtained has a marked depression at this point. Now this has no bearing on the theory,

as I did not derive theoretically an expression for efficiency. The reason for that is that the expression obtained by dividing output by input becomes practically indeterminate for the point $\theta=1$. In order to get something reliable at $\theta=1$, the principal point of interest, it might be necessary to keep all the terms which were negligible in the expressions of output and input. This, I am sorry to say, I did not have the patience to do, much less plot the results, if I had done it. In any event, it is very improbable that the theoretical curve of efficiency corresponding to Professor Morecroft's experimental case should have any marked depression at $\theta=1$. The constants used give $n_1 \ (1-\alpha)+n_2=0.21$. This is a very dull circuit. Its output curve is almost coincident with the bottom curve of Figure 3, and so we can only expect that the efficiency curve is also dull, and should show little, if any, depression at $\theta=1$. It is a pity Professor Morecroft did not plot on output instead of efficiency, for in that case his results might be compared directly with the curves of Figure 3.

Professor Morecroft seems to think, that I had no method of determining whether or not sparks were occurring once a cycle, but that I simply "assumed" that they were. In the first place, the ear detects "missing" in a rotary gap, almost as easily as in an automobile motor. Secondly, if the rotating mirror of the oscillograph is driven non-synchronously, as was the case, any irregularity of sparking is at once seen, as the eye registers the instantaneous tracing of the light spot.

It seems as if Professor Morecroft were unduly exercised over the presence of iron in my transformer. He shows an oscillogram labeled "Discharge of condenser thru iron core inductance." The circuit here is almost aperiodic, so that I cannot see that it has much bearing on the problem. The simplicity of the free oscillation of the transformer circuit is well shown by the close agreement between the oscillogram of Figure 8, and the corresponding calculated curves of Figure 7. Close to resonance I have been unable to detect, oscillographically or otherwise, the difference between air and iron transformers.

It would seem as if free periods and resistance terms should vary cyclically when iron is present. Perhaps they do, but with the transformer I used (which has a low flux density), I never found any evidence of much irregularities. Measurements of damping factors and inductances are, of course, much more easily measured for air transformers than for iron. They are advantageous for experiments on that account, but I did not find that there is any other difference between them.

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ONE HUNDRED AND ELEVEN BROADWAY

NEW YORK, N. Y.

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PROCEEDINGS OF THE SECTIONS OF THE INSTITUTE OF RADIO ENGINEERS

WASHINGTON SECTION

On the evening of January 26, 1916, the annual meeting of the Washington Section was held at the Commercial Club in that city. The pleasantly informal speeches were preceded by a dinner. Lieutenant Louis M. Evans (Secretary of the Washington Section for 1915), presided in the absence from Washington of Lieutenant S. C. Hooper (Chairman of the Washington Section for 1915). The toastmaster of the evening was Captain W. H. G. Bullard. He introduced in succession the following speakers: Brigadier-General Scriven, U. S. A., Rear-Admiral Griffen, U. S. N., Mr. V. Ford Greaves (on behalf of Secretary of Commerce Chamberlain), Captain Gibbs, U. S. A., Professor Alfred N. Goldsmith, and Mr. George H. Clark (Secretary of the Washington Section for 1916). The subjects of the various addresses were as follows: "Radio Work of the Signal Corps," "Radio in the Naval Service," "Radio Regulation," "The Radio Stations at Honolulu," "The Institute and Advance in Radio," and "Review of the Activities of 1915." The Committee on Arrangements, which was largely responsible for the highly successful character of the evening, were Messrs. T. Lincoln Townsend and Charles J. Pannill. The following toast, by Mr. Townsend, was felt to voice clearly one aim of the Institute:

TO THE ENGINEER OF TO-MORROW

Here is a toast that we want to drink to a fellow we'll never know—
The fellow who's going to take our place when it's time for us to go.
We wonder what kind of a chap he'll be and we wish we could take his hand,
Just to whisper, "We wish you well, old man," in a way that he'd understand.
We'd like to give him the cheering word that we've longed at times to hear;
We'd like to give him the warm handclasp when never a friend seems near.
We've gained our knowledge by sheer hard work, and we wish we could pass it on
To the fellow who'll come to take our place some day when we are gone.

On the evening of February twenty-ninth, a joint meeting of the American Institute of Electrical Engineers and the Washington Section of The Institute of Radio Engineers was held at

the Cosmos Club, Washington. A paper by Captain W. H. G. Bullard, Superintendent of the Naval Radio Service, on "Recent Developments in the Naval Radio Service" was presented. It was fully illustrated by slides, and was widely discussed.

On the evening of March twenty-ninth, a meeting of the Washington Sections was held at the laboratory of Mr. J. H. Rogers, Hyattsville, Md. The meeting was devoted to the consideration of certain current radio problems.

On the evening of April twenty-eighth, a meeting of the Washington Section was held in the State, War, and Navy Building, Washington. An informal address on certain recent radio litigation was made by Mr. Frederick A. Kolster of the Bureau ot Standards. This was followed by a number of informal speeches and general discussion.

BOSTON SECTION

On the evening of January 27, 1916, a meeting of the Boston Section was held in the Cruft High Tension Laboratory, Harvard University, Cambridge, Mass. Professor Hidetsugu Yagi presented a paper on "Arc Oscillations in Coupled Circuits," this being followed by a Braun tube demonstration of impact excitation by Dr. E. L. Chaffee. There were ninety-one present. The chairman was Professor G. W. Pierce, Past Vice-President of the Institute. The discussion was carried on by Professor J. E. Ives, Mr. Fulton Cutting, Dr. E. L. Chaffee, and Mr. Melville Eastham, Secretary of the Boston Section.

On the evening of March thirtieth, a meeting was held at the Cruft Laboratory. Professor George W. Pierce presented a paper on "The Radiation Characteristics of Flat-Top Antennas." This was followed by a discussion in which, among others, Professor J. E. Ives and Mr. Fulton Cutting participated. The attendance was seventy-eight.

On the evening of April twenty-eighth, a meeting was held at the Cruft Laboratory. A paper by Professor A. E. Kennelly (President of the Institute), and Mr. H. A. Affel on "Skin Effect in Conductors at Radio Frequencies Up to 100,000 Cycles per Second" was delivered. The evening being inclement, fifty-nine were present. The paper was discussed by Professor J. C. Hubbard and Mr. Melville Eastham.

On the evening of January eighth, a dinner of the Seattle Section was given, Mr. Robert H. Marriott (chairman of the section), presiding. The attendance was nineteen. Financial matters were considered after the dinner, and there was a discussion on the question of the control of radio transmitters and receivers on vessels lying in harbors.

On the evening of March fourth, a meeting of the section was held in Denny Hall, Washington University. Messrs. T. M. Libby and F. M. Ryan conducted a number of demonstrations of experiments dealing with the physical aspects of radio communication.

On the evening of March eighteenth, a dinner was given at the Butler Hotel, Seattle, to the following visiting members of the Institute and guests: Messrs. L. F. H. Betts, Lieutenant, E. J. Blankenship, D. N. Cosgrove, Philip Farnsworth, A. H. Ginman, V. Ford Greaves, Frederick A. Kolster, Dr. Carl E. Magnusson, Judge Jere Neterer, Dr. Frederick A. Osborne, Greenleaf W. Pickard, Frank N. Waterman, and Roy A. Weagant. The dinner, which was largely attended, was a most pleasant and instructive occasion.

RADIO IN ALASKA*

By

A. H. Ginman

(General Superintendent, Pacific Coast Division, Marconi Wireless Telegraph Company of America)

In the development of all new countries, the telegraph has always been one of the most important factors of progress. In Alaska, the progress of colonization has been very slow; short seasons and lack of transportation facilities having retarded general progress and also, perhaps, the conservative policy of those who have hitherto governed the territory. The advent of the Marconi Company into Alaska gave rise to the now familiar spectacle of radio communication successfully competing with a submarine cable by giving fast and accurate service at reduced rates; and this in face of the fact that radio telegraphy has, until recently, been considered by the public as the most expensive telegraph service of all.

Besides its mineral resources, which have only been partially developed, Alaska has agricultural possibilities, which have been computed by experts as capable of supporting a population of five million engaged solely in dairying and grazing. When these various potential resources are developed, and this will probably be within a few years—for our Government is at present engaged in the construction of hundreds of miles of railroad in that country—Alaska will indeed be able to compare favorably with her sister states. It is in this development that the Marconi Company proposes to play its part.

In Alaska can be found almost every topographical formation and geographical arrangement known, and it can readily be understood what tremendous difficulties are encountered in the building of telegraph lines and the laying of submarine cables. In that country, this method of communication was primarily intended for military purposes, but it has been utilized to a far greater extent by the Alaskans for commercial purposes, as it was their only means of outside communication.

Volcanic disturbances are of frequent occurrence along the

* Received by the Editor, December 24, 1915.

Coast; and while only of a mild character, are a continual source of trouble to cable authorities. The only cable repair vessel assigned to this duty is stationed at Seattle, and repairs are consequently delayed until that vessel can locate the trouble, which may take two days or two weeks. Thus it would seem that not only is radio a valuable asset to Alaska, but the only logical and efficient method of communication. The Government, thru the Navy and Military Departments, has established radio stations at convenient points thruout the North, and localities hitherto unserved are now connected by an organized system. Numerous canneries and mining companies have installed Marconi equipments, thus enabling them to keep in touch with the outside world, which, in the pre-radio days, they were unable to do without great loss of time, it being not unusual for them to send messages by boat from a mine to the nearest cable or radio office.

Two years ago, the Marconi Company decided to extend its activities in Alaska, and to-day it has semi-high-power stations at Ketchikan, the first port of entry into Alaska, at Juneau, the capital city, and at Astoria, Oregon, U. S. A., thus forming the nucleus of a system that will eventually extend along the Alaskan peninsula when the volume of business justifies the step. These stations were put into commission in July of this year and are maintaining a fast and reliable day and night service. As an instance of this may be mentioned the experience of a mine owner in Juneau, who dispatched a message from there to Los Angeles, California, and received a reply forty minutes after filing the original message at Juneau.

The country in the vicinity of Ketchikan is of volcanic origin, with a sub-soil from eighteen to thirty-six inches (45 to 90 cm.) deep, of a very soft character, (the underlying rock being of shaly composition), and found to be easily excavated by the construction engineers. Facing a strip of water known as the Tongass Narrows, four towers of the self-supporting type have been erected to carry the antenna, these towers being so located that they outline a rectangle three hundred by six hundred feet (91 by 182 meters). One of these is shown in Figure 1. The long axis is in the true direction of Astoria, Oregon. This was so arranged in order to obtain maximum radiating and receptive qualities, for it has been found that such an aerial has directional radiating power. Each tower is three hundred feet (91 meters) high and surmounted by a wooden top mast projecting fourteen feet (4.3 meters) above the head of the steel

portion. Upon these are mounted 80,000 volt, triple petticoat insulators, carrying the antenna made of two silicon bronze wires, each having seven strands of number 18 wire,* the tension of which can be adapted to meet any conditions of abnormal strain, due, for example, to high wind storms, sleet or ice. This antenna serves a double purpose, being used as a transmitting aerial for the

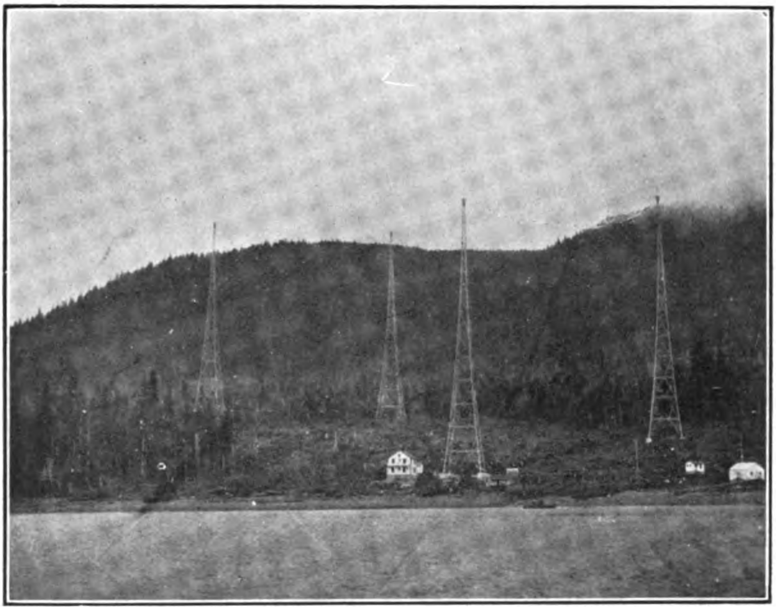


FIGURE 1—Ketchikan, Alaska Radio Station

marine service and a receiving aerial for the 5 and 25 kilowatt sets. A twenty wire antenna is suspended on triatics† between the towers, and leads to the reinforced steel concrete power house, located approximately three hundred feet (91 meters) from the two lower towers, where it is connected to the 25 kilowatt transmitter. A two mile transmission line carrying 2,200 volts, single phase current, at a frequency of 60 cycles connects the Ketchikan city power house with the radio station, and about three hundred feet (91 meters) from the power plant it is brought into the building by means of an underground conduit, a precaution deemed

* Diameter of number 18 wire = 0.0403 inch = 0.102 cm.

† (A triatic stay consists of two pendant portions attached to the two masts, their other ends being usually provided with thimbles reeved in place; thru these thimbles the main wires between the masts pass—EDITOR.)

necessary to avoid trouble which might arise from the proximity of the antenna to the overhead transmission line. Here it is connected to the high tension switchboard and thence distributed to the various units: transmitting apparatus, and light-

FIGURE 2—20 wire aerial terminal, showing high tension porcelain insulation, 20″ wood strain insulation, and 2 Bradfields

ing and heating transformers. A synchronous rotary converter is used for furnishing 70 volt direct current for the operation of the solenoid keys and side disc motors and by means of an extended shaft drives a rotary discharger, which controls the number and duration of the spark discharges. The disc discharger embodies the latest improvements found desirable for handling such currents. The disc is thirty inches (76 cm.) in diameter and rotates at 1790 revolutions per minute. Close to its periphery are inserted brass studs equally spaced around the disc, which, in rotating, pass between two side discs set to give a clearance of 1/32 of an inch (0.8 mm.). The discharge takes place across this small air gap, that is, between the side discs and each of the revolving studs in turn. The discharger is shown in operation in Figure 7. As the discharger is rotated

synchronously with the alternator which supplies energy to the condenser which is discharged, it is necessary to time the discharge to occur at or near the peak of the voltage wave in the alternator. The correct point on the voltage curve depends on

FIGURE 3—"Tower C," Ketchikan Radio Station

FIGURE 4—Tower safety stops, showing complete arrangement, (winch or crab, and the galvanized iron nailed to top of stump to preserve the same. Note steel halyard slack; Manila links shackled to a short piece of steel halyard which in turn is clamped to the halyard; ship cleat bolted to stump with 12″ lags. Ketchikan.

several factors and arrangements are provided to permit adjustments being made of the interval between the time at which the revolving stud passes the side discs and the time when the machine voltage is at its highest, thus obtaining maximum effects.

The condensers consist of glass and zinc plates placed in earthenware containers, filled with oil, a bank of thirty units being used in the circuit. Copper bus bars of ample dimensions lead from the condenser bank to the inductive coupler and to the disc discharger. The coils of this coupler are wound

with a specially designed cable, and so arranged that each strand carries the same amount of current, thus decreasing heat losses.

The receiving office is seventy-five feet (23 meters) from the power house and contains the operating key and the usual equip-

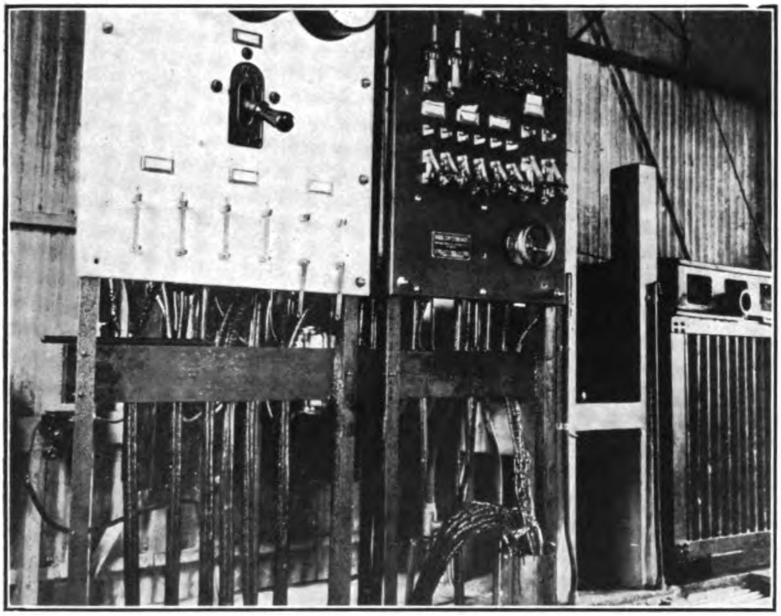


FIGURE 5—Ketchikan Station Switchboards

ment, supplemented by two loose-coupled receivers (fitted for crystal and valve detectors), having ranges from one hundred to four thousand meters and from one hundred to seven thousand meters, respectively. A difficulty met with, even in the use of power as low as 25 kilowatts, for radio telegraphic purposes, is that of controlling the current so that speedy manipulation is possible. For satisfactory service, the operator must be able to handle the key as if it were controlling no more power than is usually required to operate a land line key. This means that every time the key is depressed, power to the extent of 25 kilowatts must be supplied to the radio circuits and the delivery of power must cease as soon as the key is lifted; and, moreover, the starting and stopping of this flow of energy must be as nearly as possible instantaneous. This is accomplished by means of a solenoid key in the 22,000 volt leads, which is actuated

by the operator's key, the solenoid key being provided with an air blast for blowing out arcs which might be formed. Thus it will be seen that the operator can control the apparatus while far removed from the disturbing noise of the power house.

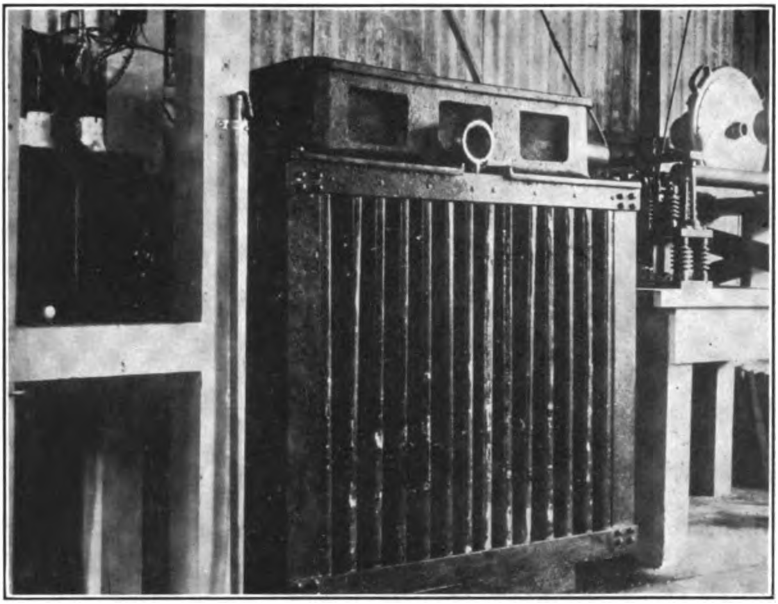


FIGURE 6—On right, blowers, high tension solenoid key; center, K. W. transformer; left, remote control oil switch

The living quarters for the staff contain all modern conveniences, and are furnished in a liberal manner. The water supply is obtained from a 12,000 gallon reservoir, located on high ground to the rear of the station, fed by springs the source ot which is in the nearby hills and virgin forests that surround the station. The forests added greatly to the difficulties of our construction engineers, when coupled with the fact that the rainfall in that vicinity averages one hundred and sixty-eight inches (4.3 m.) yearly. However, this excessive rainfall is of great advantage as regards the ground system which consists of three thousand pounds (1,400 kg.) of zinc plates buried in a circle around the power house, supplementing which are a number of four feet (1.2 meter) strips running out on the beach to mean low tide level, thus insuring at all times a good electrical ground.

To the north two hundred and fifty miles (400 km.) is the Juneau Station which, when completed, will be a counterpart of the one at Ketchikan, with the exception that it will have only two three hundred foot (91 meter) towers. In the interim, the old station has been remodeled with a 10 kilowatt plant.

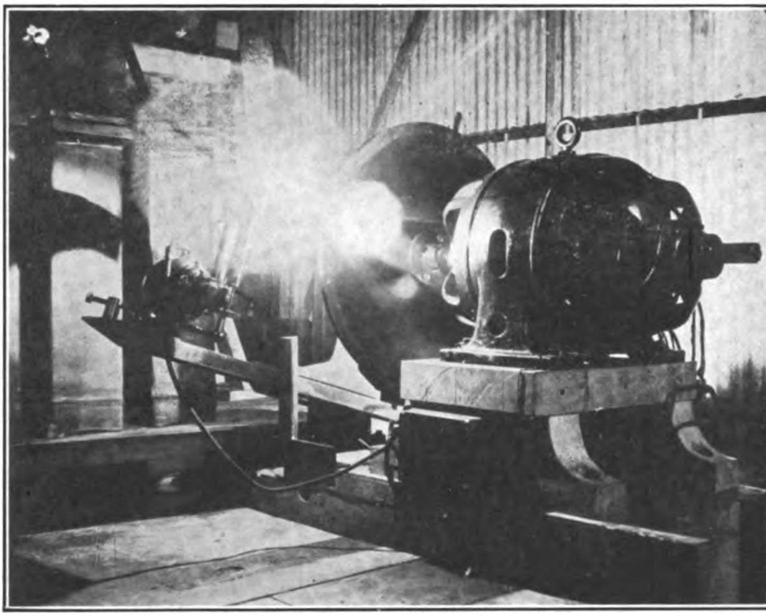


FIGURE 7—25 K.W. spark; note synchronism of disc indicated by the stud near top of disc. Exposure 20 seconds during which time disc made 597 revolutions. Ketchikan

Radio conditions existing in Alaska are not to be found, I believe, in any other part of the globe. This is due possibly to the geographical situation of Alaska, and the continuous daylight during the summer months. It frequently happens that a vessel in Alaskan waters, while in communication with another station seven hundred miles (1,100 km.) distant, is unable to communicate with a station in the opposite direction more than twenty miles (30 km.), even tho the latter is at the same time able to communicate with a station seven hundred miles (1,100 km.) distant from it. Another phenomenon worthy of comment was noticed during the first few weeks of our operations. At Astoria (where strays are much more intense than at either of the Alaskan stations) the atmosphere would be quite

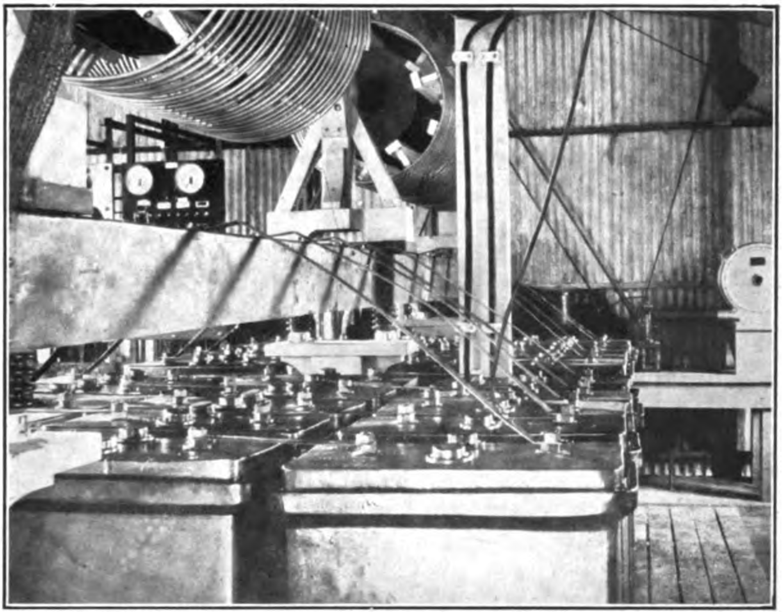


FIGURE 8—25 K.W. condenser bank; in upper left corner can be seen the secondary reactance between 25 K.W. transformer and condenser.

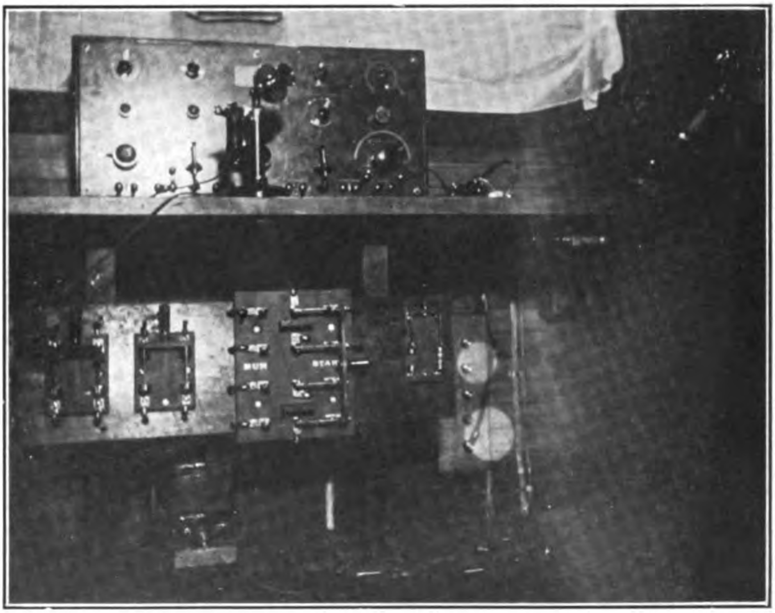


FIGURE 9—Receiving room at Ketchikan. On top of table center is the 4000 meter marine tuner; on extreme right in corner is change over switch; underneath table is a wooden switchboard, mounted on which are: (starting at left,) a D.P. switch acting as two S.P. switches for disc and blower motors, 220 volt mains for converter, 4 pole starting switch, 5 K.W. rotary switch.

clear of strays until noon; in fact so much so that one obtained the impression that the antenna was disconnected or that the receivers were out of adjustment. However, at noon each day, strays put in an appearance and gradually increased in intensity

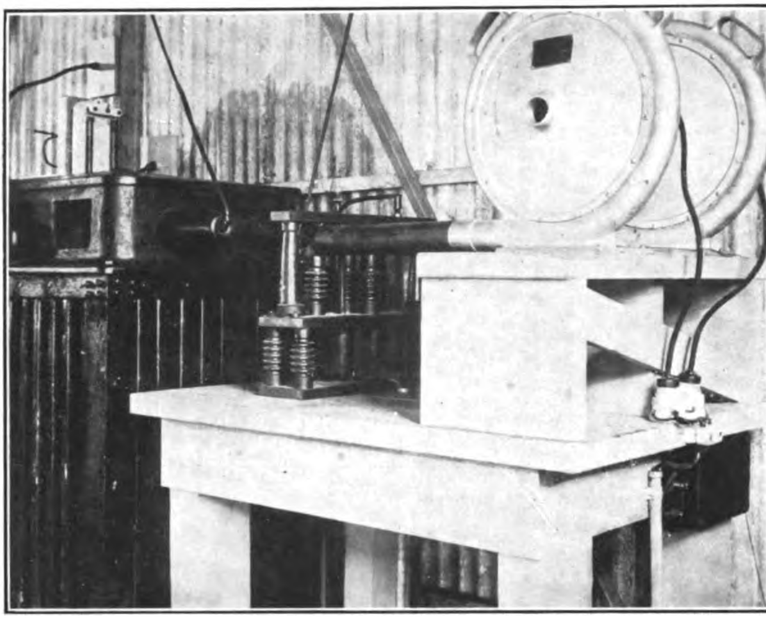


FIGURE 10—Blowers (A.C.) on right; center, solenoid key; left, 25 K.W. transformer. Ketchikan

until 1 P. M., when they reached a maximum, remaining so until 5 P. M. Thence until midnight, atmospheric conditions were those usually prevailing at similar stations. Altho the evening period of decrease varied from time to time, the mid-day rise in the stray curve appeared to be constant until the season had passed, as it suddenly did on August 28th.

On short wave lengths the results were not favorable, but on working up to twelve hundred meters communication was easily maintained, and the longer the wave length the better were the results at the distant stations. Reception of signals at Ketchikan has been remarkably good, messages having been received intact from the west, from Siberia, Japan and Honolulu; from the north, all Signal Corps stations; from the south,

Darien (in the Canal Zone), and all Pacific Coast stations; from the east, from Tuckerton, Sayville, Arlington, Key West, and at times messages from the Hanover Station, in Germany.*

December 24, 1915.

SUMMARY: The chain of semi-high-power stations (25 K.W.) established by the Marconi Company at Ketchikan, Juneau,.and Astoria in Alaska are described. The masts, insulation, antenna, power supply, rotary converter, discharger, and relay key of these stations are considered in detail. The climatic and radio stray conditions are shown to be unusual. Remarkable directive absorption is encountered. The favorable receiving conditions are illustrated.

* (The distance from Eilvese, Hanover to Ketchikan, Alaska is approximately 4,600 miles or 7,400 km.—EDITOR.)

DISCUSSION

Charles J. Pannill (communicated): I think Mr. Ginman has ably covered the situation in respect to radio in Alaska as operated by his company. He has however overlooked the various other systems of radio in Alaska that have been in operation for a number of years. The first point-to-point commercial operation of radio, I believe, was between Nome and St. Michaels, a comparatively short distance across water, but looked upon with pride in the early days of radio. These stations were erected by the Signal Corps and used in connection with their cable system. The Naval Radio Service maintains a chain of shore stations along the coast of Alaska from Sitka to St. Paul. Our stations in Alaska are open to commercial traffic both for radiograms to and from ships at sea, and domestic business between Alaskan points, and between United States and Alaskan points. We recently inaugurated a night letter service and a night service over our circuits. Traffic agreements have been entered into with the land line companies in the United States and the Marconi and cable systems in Alaska. There are several circuits between Alaskan points and United States points reached by the Postal and Western Union offices, via the cable transfer point at Seattle connecting with us at Sitka or Cordova, via our station at North Head, Wash., reached by the Western Union, via the Marconi service at Astoria to Ketchican and to Sitka, and via Vancouver Canadian service. I do not agree with Mr. Ginman that traffic via their Astoria circuit is handled more cheaply than via other circuits into Alaska. A careful study of the prevailing rates indicates that traffic may be handled by us thru North Head to Sitka and thence to the cable (paying a loop charge of 25-2 between Sitka radio station and Sitka cable office) to some points in the eastern part of Alaska at less than via their Astoria circuit. I followed the erection of the Ketchican station with a great deal of interest and I am glad that the work of Mr. Arthur A. Isbell, who erected the Marconi stations was so well performed.

AN IMPULSE EXCITATION TRANSMITTER*

By

Ellery W. Stone

(Assistant Radio Inspector, Department of Commerce.)

The "Preliminary Report of the Committee on Standardization of the Institute of Radio Engineers" for 1913 defines "impulse excitation" as follows:

"The term applied to a method of producing free alternating currents of relatively small damping by means of the actual or equivalent removal of a source of highly damped free alternating currents from the coupled secondary circuit. As a special case, the primary current may be very highly damped, but in all cases there must be in effect, a suppression of reaction between the circuits."

It is this "special case" with which we are concerned in the contemplation of the present systems of impulse excitation. However, the term "impulse excitation," as defined by the Standardization Committee, is quite broad, including as it does, all forms of quenched gap operations instead of limiting it to that particular form of quenched gap action with which it has of late been most generally associated. I refer to that form of transmitter in which the oscillations of the primary circuit are so abruptly damped out as to produce but single half cycles or impulses, causing a shock or impact excitation of the secondary circuit.

With the Telefunken and kindred quenched gap transmitters, the quenching action of the gap is confined to damping out the oscillations in the primary circuit when the current in the first *group* has become nil. In the strictly impulse excitation transmitter, the quenching properties of the gap are such as to damp out the current in the first *oscillation* or first *half cycle* before it passes thru zero, or at most, to prevent not more than one or two half cycles to follow, so that by far the greater portion of the energy is in the first half cycle. The difference between quenched gap excitation and impulse excitation, using the restricted meanings of these terms, is seen to be solely one of degree—of degree of damping of the primary current.

* Received for publication December 24, 1915.—Editor.

However, in practice, it is not customary to leave the attainment of high damping of the primary current to the gap alone.

The three constants in any alternating current circuit are resistance, R, inductance, L, and capacity, C. The proper value of R in the primary circuit is determined chiefly by the construction and action of the gap, that of L by the number of turns in the primary of the oscillation transformer, and of C by the capacity of the charging condenser.

From the formula $$d = \frac{R}{2fL},$$

we may obtain
$$\begin{aligned} d &= \frac{\pi R}{2\pi f L} \\ &= \frac{\pi R}{\omega L} \\ &= \pi R \omega C \end{aligned}$$

(where f is the frequency); or that the logarithmic decrement or damping of the free alternating current in a closed non-radiative circuit is a direct function of R and C and an inverse function of L. Hence, a decrease in the inductance and an increase of the resistance and the capacity of the primary will result in increased damping. A related conclusion may be drawn from Thomson's formula: namely, that when

$$R > 2\sqrt{\frac{L}{C}},$$

the current in the primary circuit is non-oscillatory. It may be seen from this formula as well, that the values of resistance and capacity must be high as compared to that of inductance to approach a condition of aperiodicity in the primary.

Mr. John Stone Stone, in a paper entitled "The Resistance of the Spark and its Effect on the Oscillations of Electrical Oscillators," in the December, 1914 issue of the PROCEEDINGS, takes exception to the logarithmic decrement theory as applied to those circuits in which the spark resistance constitutes the major resistance of the circuit, and hence to those formulas quoted above. However, Mr. Stone states on page 322 of his paper, "A great many oscillation circuits, and probably the majority of oscillation circuits, are intermediate between the two classes mentioned, and I hope soon to present a paper giving the theory of such circuits in some detail." (The "two classes" referred to are those two types of oscillation circuits to which the logarithmic decrement and Mr. Stone's linear decrement theories of current decay are respectively applicable.) Con-

sequently, until the advent of Mr. Stone's new paper, one is in a quandary as to which theory to quote, but as far as the case at hand is concerned, there is little conflict.

In place of the Thomson formula

$$R > 2\sqrt{\frac{L}{C}},$$

Mr. Stone substitutes

$$R > \frac{2}{\pi}\sqrt{\frac{L}{C}};$$

but the same conclusions are to be drawn from his formula as from Thomson's, i. e., that to approach a condition of aperiodicity in the primary circuit, the values of resistance and capacity must be large as compared to that of inductance.

There is still another point to be considered. The absorption of energy from the primary by the secondary results in increased damping of the current in the former circuit; so that, other things being equal, an increase in the coefficient of coupling between the two circuits will also increase the damping.

While L and C in the primary circuit remain constant, R does not, since R primarily is the resistance of the gap. For a quick discharge of the condenser, the resistance of the metallic circuit of the primary must be low, the surface of the gap large, the initial resistance of the gap high, and its recovery to its initial resistance rapid.

Before going into the details of a transmitter designed by the writer to incorporate these properties, a reference to previous work along these lines will be found of value.

From what information on impulse excitation the writer has been able to obtain, Mr. Roy E. Thompson, of the Kilbourne & Clark Mfg. Co., Seattle, seems to have been the pioneer in this work in the United States at least, having designed and constructed an impulse transmitter as far back as 1910. The writer must acknowledge his indebtedness to Mr. Thompson for most of the preliminary data gathered on the subject in the preparation of this paper.

A wealth of material covering this subject is to be found in the December, 1913 issue of the PROCEEDINGS in a paper entitled "The Multitone System" by Dr. Hans Rein. The values of inductance, capacity and resistance in a typical primary circuit are given, together with an excellent theoretical exposition of the principles of the Lorenz and other systems, with numerous photographs showing their practical application. Not the least

interesting is the appended discussion. Mr. Eastham's paper in the December, 1914 issue of the PROCEEDINGS will also be found to be of considerable bearing on the subject.

In the spring of 1913, the writer made some experiments on a rotary quenched gap of the Clapp Eastham type described in Mr. Eastham's paper. Recognizing the similarity between this gap and the Peukert gap, which latter employs a thin film of oil between the sparking surfaces, experiments were tried to see what effect would be realized by the introduction of oil into the Clapp Eastham gap. The only result noted was a decrease in antenna radiation, possibly due to excessive carbonization of the oil, and the experiments were temporarily discontinued.

In the spring of 1915, the subject was again taken up, utilizing a larger and an improved type of gap to see what effect the introduction of a liquid hydrocarbon would have on the quenching properties of the gap.

The experiments were conducted as follows. From the simple formulas quoted above, and from the work of Dr. Rein and others, the necessity of large capacity, high gap resistance and low inductance were recognized, so a primary circuit containing one turn of inductance and enough capacity to bring the circuit to about 670 meters was utilized. The necessity for a transformer giving a flat topped secondary wave to simulate a D. C. wave has long been appreciated in operating impulse transmitters on alternating current. Accordingly, a 2 kilowatt, 60 cycle transformer with a secondary voltage of 2,500, so designed as to secure a maximum of leakage and of iron saturation, was built and used in this work.

It was not possible with the facilities at hand to note the number of oscillations in the primary, so it was found necessary to resort to the following method to determine the degree of damping. Probably the most significant phenomenon attending the operation of impulse transmitters is the absence of necessity for resonant tuning between the primary and antenna circuits. As a matter of fact, with true impulse excitation, it is almost impossible to measure the wave length of the primary circuit even tho uncoupled, a resonance curve of the primary being practically without a peak. By the substitution of a plain gap, however, the wave length of the primary may be easily measured; and this procedure was followed in the present case, the primary being tuned to 670 meters, as previously noted. With true impulse excitation, it should be possible to detune the primary

and secondary circuits by any amount without a very great decrease of current in the secondary circuit and with the appearance of but a single frequency in that circuit.

The evolution of a gap to realize this state of affairs may be found of interest.

The first experiment tried was to tune the primary and antenna circuits to resonance, as in the usual type of quenched transmitter, to note the damping of the current in the antenna circuit, which would give some idea of the quenching properties of the gap. The coupling was extremely tight, the single turn of primary inductance being placed directly over one turn of the secondary winding. With a motor speed of 1,800 R. P. M., the logarithmic decrement of the oscillations in the antenna circuit, measured with a Kolster decremeter, was found to be 0.15, an exceedingly high value for a quenched gap, considering that the aerial employed was of the average ship type.

An increase of the motor speed to 3,400 R. P. M. reduced the logarithmic decrement to 0.06. From this, the quenching properties of the gap, without the introduction of gases, would seem to be a function of the speed of the revolving discharger. However, the impression should not be gathered that a speed below 3,400 would cause less damping of the primary oscillations or that a speed in excess of this would increase the damping. Good quenching, as determined by the logarithmic decrement of the secondary oscillations, was not obtainable until a speed of about 2,400 R. P. M. was reached. This gave a logarithmic decrement of 0.06 in the antenna circuit. From that speed up to 4,000 R. P. M., the limit of the motor, the logarithmic decrement remained constant at 0.06.

The next experiment was to detune the two circuits about 100 meters, with the result shown in the resonance curve, Figure 1. It was evident from the presence in the antenna circuit of the primary hump that impulse excitation had by no means been attained.

A light transformer oil was then introduced into the gap, flooding the gap completely, to see if the increased resistance would enhance quenching. But here, a mechanical difficulty asserted itself. Because of the presence of the oil (light as it was), the confined space in which the sectored plates revolved and the excessive churning up of the oil prevented the gap from coming up to speed with the size motor used, with the result that the sparking was irregular, the tone poor, and the condensers so overstrained as to cause puncture.

A gap was then constructed in which the sparking surfaces were mounted on an open frame instead of being enclosed in the usual casing. This open frame was then lowered into an earthen receptacle into which oil was poured. It was hoped by this new construction to overcome the friction caused by the violent

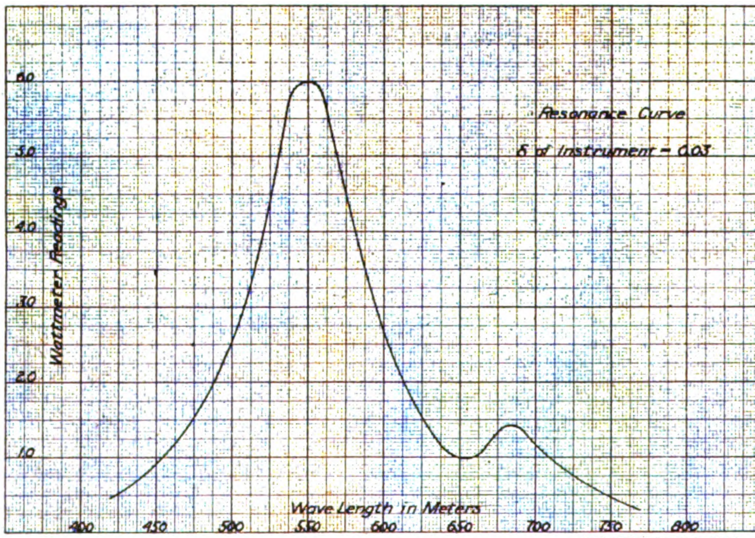


FIGURE 1

disturbance of the oil in so confined a space. The gap was more nearly brought up to speed, as had been expected, but the oil was freely thrown from the container, and air bubbles found their way into the sparking area with consequent explosions which caused the sparking to be so irregular as to make quantitative measurements with the decremeter impossible.

It was decided to return to the original type of enclosed gap, using a much larger motor to bring the gap up to the desired speed. But the largest motor at hand, 0.75 horse-power, failed to effect this; so the oil was removed by a pet-cock at the base of the gap and alcohol substituted in the hope that the lighter liquid would cause less friction. However, alcohol, when subjected to a potential difference of 2,500 volts across a distance of a few thousandths of an inch, exhibits slight insulating qualities, as was discovered; and leaked so badly as to prevent the passage of a spark.

The alcohol was accordingly allowed to run out of the gap, and measurements were again taken to confirm the results of the second experiment—to see if it were still impossible to detune the circuits without the generation of two frequencies in the antenna circuit.

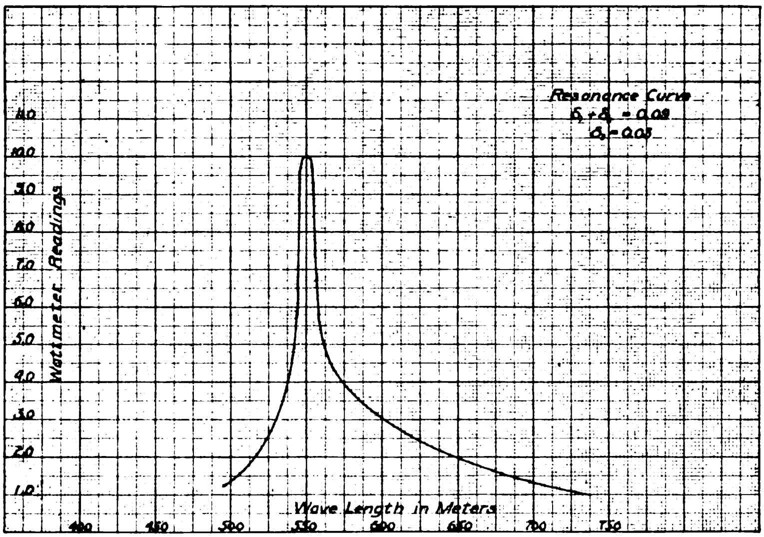


FIGURE 2

The same results as shown in Figure 1 were again obtained. To see if the gap were inadequately cooled, the spark was allowed to run five minutes, and when a resonance curve was taken at the expiration of that period, the curve shown in Figure 2 was obtained, showing a logarithmic decrement of 0.06.

The explanation was at once apparent. The pet-cock at the base of the gap was so mounted as to leave a small quantity of the alcohol in the base of the casing or housing. This, because of the heat of the spark, vaporized; thus enhancing the quenching properties of the gap. This action has been previously noted by Scheller, Poulsen, Eccles and Makower, and others.

Additional resonance curves were taken to check the first reading, the spark being allowed to operate continually. However, after ten minutes, sparking in the chamber became irregular, and was replaced by occasional and later by continuous sparking across the condenser safety gap, the electrodes of which

were separated a distance of 1.5 centimeter. It was of course obvious that the pressure of the alcohol vapor within the chamber had been so greatly increased by the continuous sparking, and consequent liberation of heat, as to reach a dielectric strength sufficiently high to prevent the passage of the spark.

To reduce the pressure within the gap, and thus to allow sparking to take place, the pet-cock was opened and a small quantity of the vapor allowed to escape. The high pressure within the spark chamber was clearly demonstrated by the almost explosive force of the escaping gas. Sparking again occurred but stopped after a few minutes because of the pressure again rising to too high a value. This condition was relieved by opening the cock a second time. The vapor was purposely ignited this time and burned quietly a distance of several centimeters from the opening, exhibiting by the great heat and colorless flame the presence of what appeared to be almost pure hydrogen.

The necessity for an automatic device to provide for a reduction of the gas pressure when it should reach too great a value was obvious. Accordingly, the pet-cock was removed, and an adjustable, poppet, release valve substituted. When the gap was again put into operation, this also served to handle the first explosion caused by the admission of air into the chamber when making alterations on it. This preliminary explosion is encountered in the operation of Poulsen arc transmitters, when air has been admitted into the arc chamber thru the replacement of a carbon.

The release valve was set so as to allow vapor to escape when the pressure reached a value beyond which a further increase in pressure would prevent the passage of the spark.

The quenching properties of this gap are thus seen to be wholly within the control of the operator or engineer, a unique feature, which, to the writer's knowledge, has not been used before.

The vaporization of alcohol or other hydrocarbons in a quenched gap is by no means a recent innovation. Poulsen, Scheller, Rein and others have vaporized the alcohol directly within the gap, and the Japanese have blown alcohol vapor, formed by passing a stream of air thru a sponge saturated in that liquid, into their quenched gaps. However, it is believed that the working of the hydrogen vapor within the gap at such an extreme pressure as to prevent sparking therein, except by automatic reduction of same, is a feature hitherto unused, and

which is solely responsible for the efficient performance of this gap for impulse excitation work.

The quenching properties of the usual quenched gap depend greatly on the extent to which the gap is cooled. With the gap herein described, the more heat generated within the gap, the greater is the gas pressure formed, so that the one action automatically compensates for the other. Cooling of the spark dischargers themselves is to be desired, and this is effected, as far as the stationary plate is concerned, by mounting radiating plates on the posts holding the plate to the casing.

Figure 3 illustrates the gap proper. The alcohol drip cup is shown at the top of the gap. This is equipped with a pressure equalizer, a tube running from the inside of the spark chamber to the top of the cup, to insure a steady flow of alcohol. With such an arrangement, it is necessary to keep the cap, by which alcohol is poured into the cup, air-tight. The safety release valve is shown at the bottom of the gap. As previously explained, the hand wheel on the valve is adjusted so as to keep the vapor pressure just below that value which prevents sparking. A rubber tube, not shown, serves to conduct the vapor from the mouth of the valve, acting as an exhaust pipe. The complete casing is of iron, leads being carried into the gap thru bakelite bushings. The construction of the stationary and movable plates is similar to that of the Clapp Eastham gap.

Figure 4 shows the complete 2 kilowatt transmitter. All wiring is done within the frame itself. Four variations of primary input are possible, being controlled by the switch at the right of the panel. At the rear of the panel are mounted the closed core transformer and the condenser. The latter is built up in one unit, using copper sheets with thin Belgian picture glass for dielectric, the whole impregnated in a non-hygroscopic compound. The low secondary voltage effectually prevents puncture, but the condenser is so mounted as to permit of the immediate substitution of a spare unit should any unforeseen accident to the condenser occur.

A closer view of the oscillation transformer is shown in Figure 5. Since tuning between the primary and secondary circuits is not necessary, the primary inductance is made non-adjustable. In the set shown, it consists of two turns. The greater coupling gained by using two turns instead of but one increased the antenna radiation, and did not seem materially to affect the damping of the primary current, the increased absorption by the secondary circuit probably compensating

for the less favorable effect of increased inductance in the primary.

Since all of the energy is to be delivered in one half cycle, or as nearly that as possible, it is necessary to have the resistance of the metallic circuit as low as practicable, and to have as

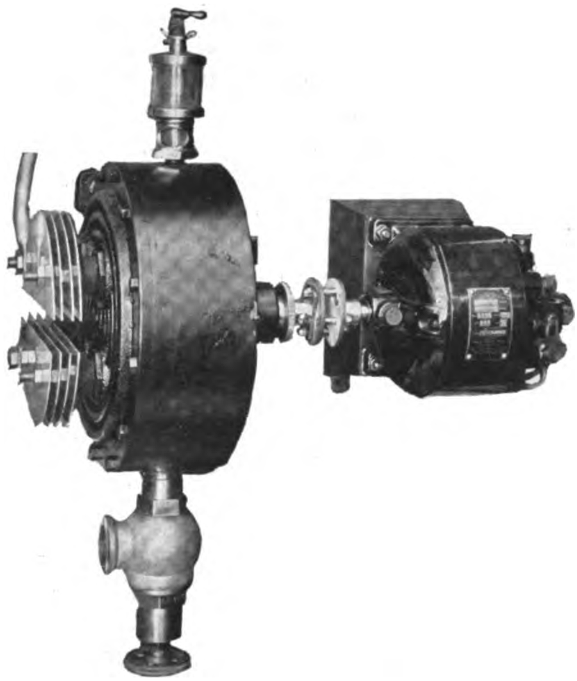


FIGURE 3

much of the primary inductance as possible effective in inducing energy in the secondary. One end of the primary inductance connects with the gap as shown, the other end to the condenser thru merely the thickness of the panel, 2.5 centimeters. The remaining lead from the gap to the condenser is about 8.0 centimeters in length.

The antenna loading inductance is mounted independently of the panel. This inductance is wound in the usual helical form and taps are taken from this to four plugs marked $\lambda = 300$, $\lambda = 400$, $\lambda = 500$, $\lambda = 600$, respectively. The position of these taps is of course determined by wave meter, being dependent on the fundamental wave length of the antenna. This inductance

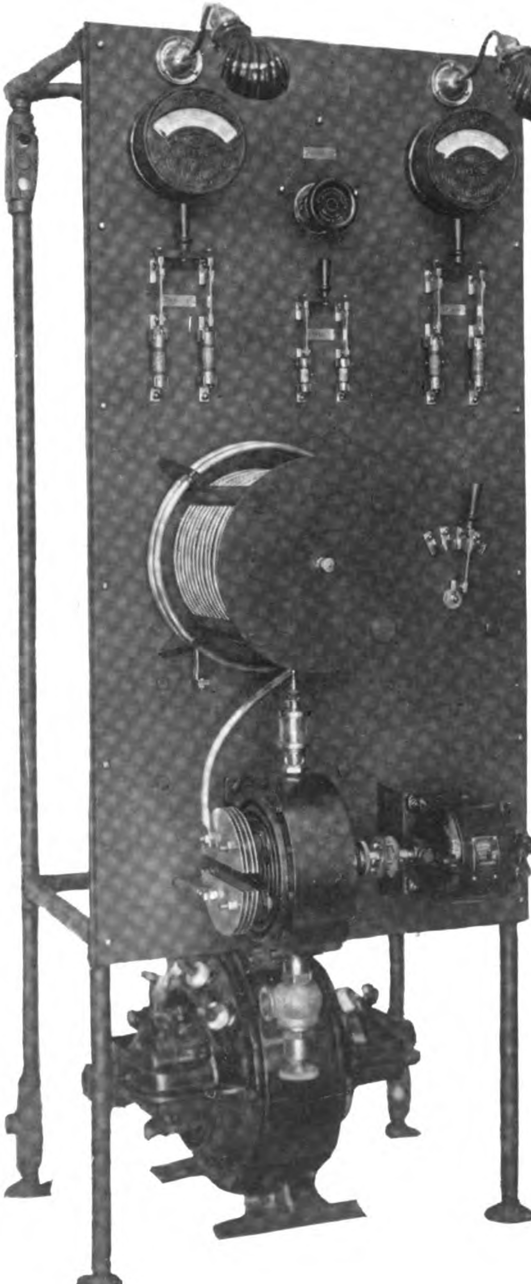


FIGURE 4

is mounted so as to permit of immediate change from one wave length to another. Since the primary wave length remains constant for any wave length it is desired to radiate from the antenna circuit, it is only necessary to insert the aerial-lead handle in any of the plugs mentioned.

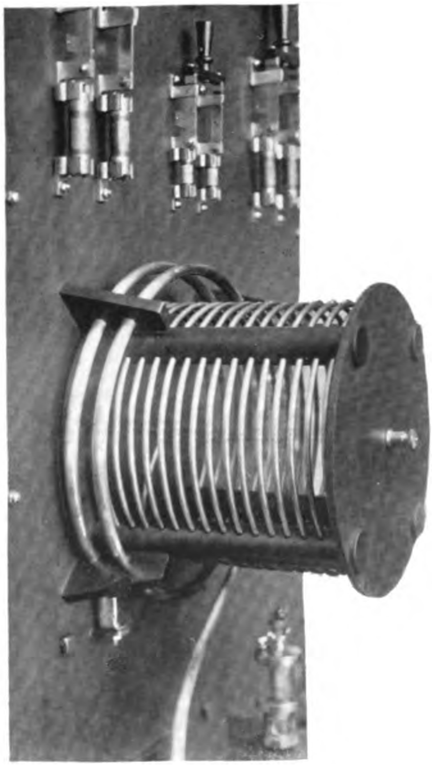


FIGURE 5

In operation, the tone of signals received from this transmitter is clear and piercing altho accompanied by a slight "feathery" tone, to use operator's nomenclature. This is probably due to the fact that discharges cannot always take place when the gap sectors are opposite each other, due to the non-synchronous revolution of the gap. However, this slight roughness is by no means displeasing; and even with local signals, the impulse group frequency of about 1,000 is not accompanied by the 60 cycle supply tone.

An antenna radiation curve of this transmitter is shown in Figure 6. In a true impulse excitation transmitter, other things being equal, one would expect, from Dr. Rein's paper, that the radiation would be constant, irrespective of the difference in wave length between the primary and antenna circuits. In other words, with the primary adjustment fixed, a curve of radiation current readings, plotted against different wave length settings of the antenna circuit, should be a flat linear curve, as against the sharp peak radiation curve of a resonant quenched transmitter. In Figure 6, the point at $\lambda = 300$ is not

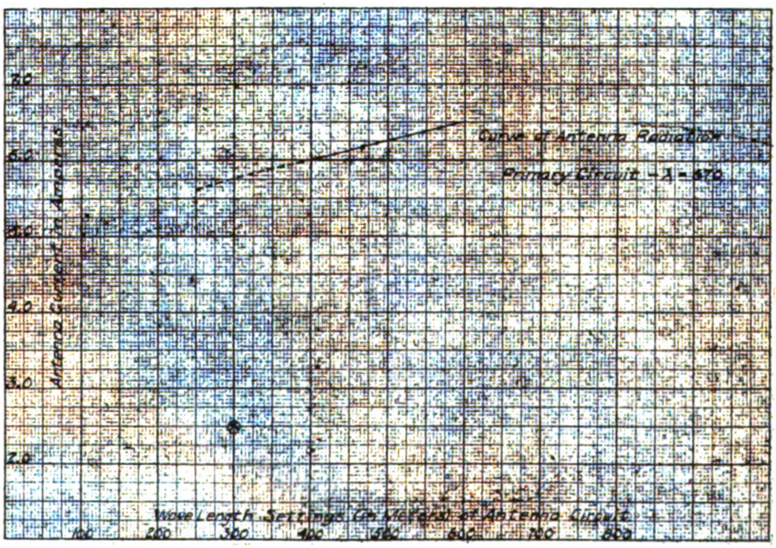


FIGURE 6

significant, since the fundamental wave length of the antenna necessitated the interposition of a series condenser in the antenna circuit for this wave length setting with the usual consequent decrease in radiation. The slight rise in the curve from 400 meters upward may be due to the diminishing antenna resistance at longer wave lengths just as much as to the fact that an approach to 670 meters in the antenna circuit places the primary and antenna circuits in resonance. This curve is similar to curves previously taken of the impulse excitation transmitters of the Kilbourne & Clark Mfg. Co.

Experiments were also conducted to observe the effect of

shunting a tone circuit across a gap, employing smooth discs in place of the usual sectored dischargers. The results were interesting. Without the tone circuit, the note obtained was a smooth, hissing one; signals being received far better on a Poulsen tikker than with the crystal or audion detector. The impulse frequency is above the limit of audibility, and that the spark is audible at all is due to the fact that the condenser is charged with alternating current instead of the direct current which should be used for ideal impulse excitation. This results in a somewhat irregular impulse frequency, due to the fact that the charging E. M. F. passes thru the zero point 120 times per second and also to the fact that the secondary wave of the transformer is probably not a perfectly rectangular flat-top one.

The quenching properties of the smooth gap seemed to be greater than those of the sectored gap, as evidenced by curve *b* of Figure 7. This curve is a resonance curve of the antenna

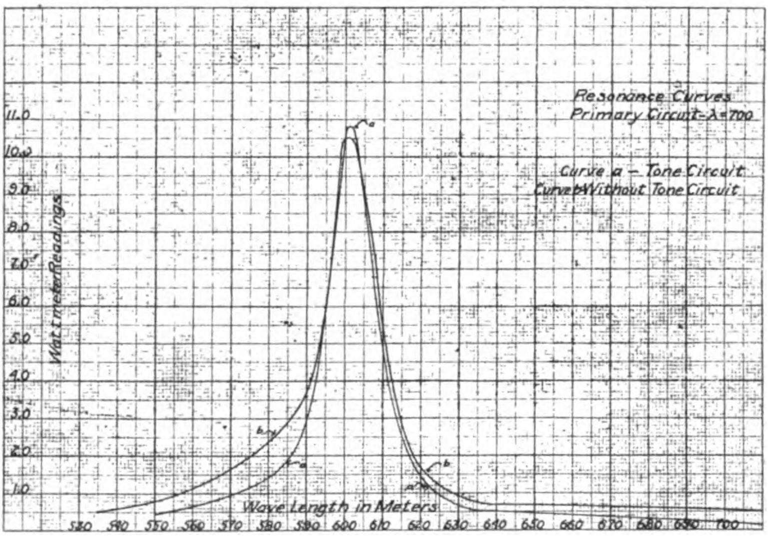


FIGURE 7

circuit, the logarithmic decrement being 0.052 as against the 0.06 decrement of the curve in Figure 2. The increased decrement is probably due to the larger gap or discharge surface.

A tone circuit, the capacity and inductance of which were determined by trial, was shunted across the gap, causing increased damping of the primary current as shown in curve *a*

of Figure 7, the decrement of which is 0.050. The absorption of energy by the tone circuit apparently assists in increasing damping in the primary circuit in the same fashion as the absorption of energy by the antenna circuit. While the antenna current was reduced about 1.5 per cent. by the use of the tone circuit, the height of curve *a* in Figure 7, when compared to that of curve *b*, shows that the energy at the oscillation frequency of the antenna circuit is slightly greater. (The coupling between the antenna circuit and the decremeter was constant in taking the data for both of these curves.)

Due to the alternating current, the note obtained with the tone circuit was not musical, but nevertheless was shrill, clear and piercing. At a receiving station, signals with the tone circuit were many times louder than without this circuit. Possibly, the note may be improved by the use of a higher frequency alternating current, say 500 cycles. Experiments will be undertaken later to observe this.

The tone circuit was then tried in conjunction with the sectored gap, but the resultant tone was poor. Certain speeds of the gap were found which tended to improve the tone greatly, but at no time was the note as clear as when the tone circuit was omitted. (These critical gap speeds were probably those which placed the impulse group frequency in resonance with the oscillation frequency, or a multiple thereof, of the tone circuit.)

On the whole, of all the experiments herein described, the best results were obtained using the smooth discs and the tone circuit. The addition of the tone circuit did not change the appearance of the transmitter as shown in Figure 4, the additional inductance and capacity being mounted on the rear of the panel.

SUMMARY: Ideal "impulse excitation," as opposed to the usual quenching gap phenomena, is described. The best conditions for impulse excitation are explained.

The development of a rotary sectored gap of small separation operating in a hydrocarbon atmosphere is considered. A 2,500 volt, 60 cycle transformer charges a large capacity which discharges thru the gap and a small inductance. Effective impulse excitation requires about 2,400 R. P. M. of the gap or more. Using alcohol vapor, an adjustable pressure, (safety) valve must be fitted to the gap to prevent excessive pressures which raise the gap voltage inordinately.

A complete 2 kilowatt transmitter of this type is described. The antenna circuit need not be in tune with the closed circuit; hence wave changing is accomplished by merely shifting the antenna lead along the antenna loading inductance. The radiation remains constant over a wide range of wave lengths without closed circuit tuning.

Smooth-disc gap experiments are also described.

DISCUSSION

Roy E. Thompson (communicated): Mr. Stone's excellent paper is of particular interest. He has shown very good judgment in attacking the problem along the line of the development of a spark gap which will effectively handle the tremendous currents in a circuit such as must necessarily be used in the production of pure impulse excitation.

Even tho it is assumed that the decrement of the impulse circuit is a logarithmic one (and therefore, that when

$$R > 2\sqrt{\frac{L}{C}}$$

no oscillations will occur in accordance with the Kelvin theory), or that no oscillations will occur so long as

$$R > \frac{2}{\pi}\sqrt{\frac{L}{C}}$$

(as pointed out by Mr. John Stone Stone); or tho we assume that the damping of the energy in the impulse circuit is intermediate between that of the logarithmically and linearly damped circuits, we must still keep the inductance in the circuit as small as possible, and the capacity as great as possible; for in order to secure a fair degree of efficiency the damping in the closed circuit must be due to the lack of inertia as represented by the inductance of the circuit and to the rapid transfer of energy from the impulse to an oscillating circuit.

It will easily be seen that if the damping of the impulse circuit is due to the resistance of the spark gap, great heat losses must necessarily result, which would not result if the damping was due to the other two causes pointed out above. Therefore, the ideal gap to be used in an impulse circuit is one that has extremely low resistance during the discharge, but which regains its initial resistance almost instantaneously after the passage of the discharge.

It is my opinion that Mr. Ellery W. Stone has more nearly approached the attainment of this ideal gap, than has any other worker along these lines.

In 1910 I designed and constructed an impulse transmitter, using a discharger in the form of a re-constructed Poulsen arc, the electrodes being of copper and aluminum with parallel circular faces of approximately 2 inches (5 cm.) in diameter and placed in a transverse magnetic field, the entire gap being contained in a chamber into which alcohol was introduced and

vaporized in a manner entirely similar to the present Poulsen arc method. However, instead of using an arc, I used a pure condenser discharge in the form of a spark. The capacity used was a one microfarad paper condenser and the inductance was probably between 500 and 1000 centimeters as afterwards calculated. With this device I was able to radiate widely different wave lengths without changing the characteristics of the impulse circuit.

In 1914, I modified the above design and interested the Kilbourne & Clark Mfg. Co. of Seattle, Wash., in its manufacture. Since that time more than forty of these transmitters have been put into commercial use on the Pacific Coast by this Company.

It might be of interest to call attention to the necessity of keeping the impulses in the closed circuit properly spaced in order to prevent the resultant wave trains in the antenna from over-lapping and interfering.

As the impulse frequency is a function of the charging potential, the capacity of the impulse condenser and the breakdown potential of the gaps used, it will be seen that even with a fixed capacity and a fixed gap; raising the charging potential may result in an impulse frequency which will cause wave train interference in the antenna.

The writer has worked out a formula by which the maximum permissible impulse frequency for use with an antenna with any decrement and wave length may be quickly determined:

Let δ be the decrement and λ the wave length of the antenna circuit.

Then
$$N < \frac{\delta\, 10^8}{\lambda}$$

where N is the frequency of the impulses in the closed circuit; or

$$T > \frac{\lambda}{\delta\, 10^8}$$

where T is the minimum time which must elapse between any two consecutive impulses.

The expression

$$\frac{\lambda}{\delta\, 10^8} = T$$

will be found approximately correct when T represents the time of duration of the effective energy in a train of oscillations in an antenna circuit.

I hope, at some time, to present a paper dealing with the development of the impulse transmitters manufactured by the Kilbourne & Clark Mfg. Co. and to describe certain phenomena encountered in the development of this apparatus, which I believe will prove of interest to radio engineers.

Mr. Stone's paper is undoubtedly a valuable contribution to this most interesting subject, and it is hoped that he will give us the benefit of his further contemplated work along these lines.

EXPERIMENTS AT THE U. S. NAVAL RADIO STATION DARIEN, CANAL ZONE*

By

Louis W. Austin

(Head, U. S. Naval Radio Telegraphic Laboratory; Past President of The Institute of Radio Engineers.)

The three towers for the Darien radio station were completed early in 1915. These towers are of the self-supporting type, each 600 feet (182 meters) high and approximately 900 feet (273 meters) apart, forming a triangle. The acceptance tests of the station gave another opportunity for carrying out long distance experiments in radio transmission which are, in a sense, a continuation of those earlier experiments carried on at Brant Rock and at Arlington which have already been described.[1]

The experimental work was begun in March, 1915. At this time the permanent antenna which consists of a triangular net of wires without spreaders, having a capacity of 0.01 μf. and an effective height of 480 feet (146 meters), was not in place, so the receiving during the first month was carried on with a 4-wire, flat-top antenna 400 feet (122 meters) long and 10 feet (3 meters) wide, stretched between two of the towers. The effective height of this antenna was calculated to be 480 feet (146 meters), and its capacity 0.003 μf. The ground system of the station consisted of a buried wire net covering the whole space inside the towers and extending to a considerable distance outside.

In the receiving experiments, a de Forest oscillating audion[2] with beat tone reception was used as a detector. This form of detector had been under investigation at the Naval Radio Laboratory for about a year before the Darien experiments were begun and had been found to give practically uniform sensitive-

*Presented before the Washington Section of The Institute of Radio Engineers, November 27, 1915.

[1] "Bulletin, Bureau of Standards," 7, p. 315. Reprint 159, 1911.
"Bulletin, Bureau of Standards," 11, p. 69. Reprint 226, 1914.

[2] "Proc. Inst. Radio Engineers," 3, pages 215 and 261, 1915.
"Journ. Amer. Soc. Naval Engineers," 27, page 358, 1915.

ness when properly adjusted,[1] except in the case of bulbs which, on account of imperfect exhaustion, behaved abnormally. Careful comparisons had been made of the relative sensitiveness of the oscillating audion and the electrolytic, the experiments showing that the normal oscillating audion gives from five hundred and one thousand audibility (depending on the telephone note) for unit audibility with the electrolytic.[2] It was also found that while the electrolytic and non-oscillating audion give telephone audibilities proportional to the square of the received current, the oscillating audion responds in proportion to the first power of the received current. Aside from the matter of telephone note, the sensitiveness seems to be the same for undamped and for damped oscillations, except when the spark trains are very short.

Figure 1 shows the circuits used for reception. It will be noted that the secondary receiving circuit is connected to the

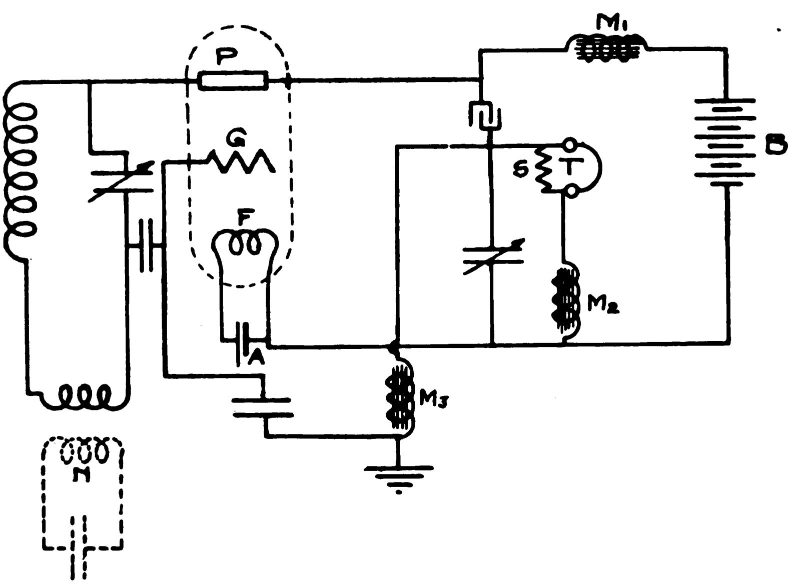


FIGURE 1

intermediate or grid electrode G in the audion, and to the plate electrode P, forming the ultraudion connection. The fila-

[1]The adjustment for greatest sensitiveness requires special skill on the part of the operator. Quantitative readings taken by untrained men will give considerably lower sensitiveness.

[2]"Bulletin, Bureau of Standards," 11, page 77. Reprint 226, 1914.

ment F is heated to incandescence by the storage battery A, while a steady flow of electrons is produced by the dry battery B. The telephones used in the experiments are placed in a shunt circuit in parallel with the audion, instead of in series with it as the more usual custom. The atmospheric disturbances are slightly less troublesome with this connection, and the sensitiveness to signals remains the same. The circuit described is designated the plain audion circuit, and for its best action the coupling between the antenna and secondary must be close, since the oscillating audion reaches its full sensitiveness only when the local oscillations are reduced in intensity by withdrawing energy into some neighboring circuit.

The sensitiveness may be increased some three or four times above that of the plain audion circuit by the use of a sensitizing circuit N for reducing the amplitude of the local oscillations. This consists simply of an inductance and capacity coupled to the secondary and tuned close to the resonance point. By the use of this circuit it is possible to work with a looser antenna coupling without loss of sensitiveness.

The strength of the received signals was measured by the shunted telephone method, the audibility of the signals being expressed in telephone current. The non-inductive resistance S (Figure 1) is placed across the telephone leads and the resistance reduced until the signal just remains audible. The unshunted telephone current is then

$$A = \frac{t+S}{S}$$

where t is the effective telephone resistance for the telephonic frequency used and S is the value of the shunt. The audibility A represents the ratio of the actual telephone current to the least audible telephone current of the same frequency. When the non-inductive shunt is placed across the telephones, it is necessary to have a choke coil M_2 or a second pair of telephones in series. On account of the effect of the observer's body, it is also necessary if the signals are strong, to earth one of the telephone leads thru a proper choke (2,000 ohm telephones). Which telephone lead should be earthed must be determined by trial.

Table I gives some of the results of the receiving experiments at Darien. As Arlington was the only station on which daily observations were made, the observations on its signals should be given much greater weight than the others in the table.

TABLE I

STATIONS RECEIVED AT DARIEN

	Distance Km.	I_s Amp.	λ Meters	h_1 Meters	R Ohms	W (Calc.) Watts	Audibility Calculated	Audibility Observed
Arlington	3,330	60	6,000	61[2]	23.2	$6.85 \cdot 10^{-8}$	7,500	5,000
Tuckerton	3,430	115	7,400	150	25.	$1.25 \cdot 10^{-6}$	32,000	10,000
Sayville	3,520	140	9,400	100	14.	$1.26 \cdot 10^{-6}$	32,100	7,500
San Diego	4,670	35	3,800	68	26.5	$1.63 \cdot 10^{-9}$	1,150	0–100
San Francisco (Federal)	4,820	40	6,500	120	23.5	$4.65 \cdot 10^{-9}$	2,050	0–1,000
Honolulu[1] (Federal)	8,500	60	10,000	120	13.5	$4.16 \cdot 10^{-10}$	580	150
Nauen	9,400	150	9,400	150	29.	$9.95 \cdot 10^{-10}$	900	200
Eilvese	9,160	140	7,400	150	25.	$5.67 \cdot 10^{-10}$	705	200

[1]Received on large antenna.
[2]h_1 corrected from short range observations. For other stations h_1 is uncorrected.

All of the signals of less than one thousand audibility were very much affected by the atmospheric disturbances, probably being inaudible many times on this account alone.[1] Column 1 gives the approximate distances of the various stations from Darien, Column 2 the strength of sending antenna current. This, however, was not known reliably in all cases. Column 3 gives the wave length, 4 gives the estimated effective height of sending antenna, 5 the total effective resistance of the receiving system for the given wave length, 6 gives the calculated received watts, 7 the calculated audibility, and column 8 the observed audibility.

TABLE II

Audion Audibility	Received Watts	Audion Audibility	Received Watts
5,000	$3.05.10^{-8}$	60	$4.41.10^{-12}$
4,000	$1.96.10^{-8}$	50	$3.05.10^{-12}$
3,000	$1.11.10^{-8}$	40	$1.96.10^{-12}$
2,000	$4.40.10^{-9}$	30	$1.11.10^{-12}$
1,500	$2.75.10^{-9}$	25	$7.66.10^{-13}$
1,000	$1.23.10^{-9}$	20	$4.40.10^{-13}$
800	$7.84.10^{-10}$	16	$3.14.10^{-13}$
600	$4.41.10^{-10}$	12	$1.765.10^{-13}$
500	$3.05.10^{-10}$	10	$1.23.10^{-13}$
400	$1.96.10^{-10}$	8	$7.84.10^{-14}$
300	$1.11.10^{-10}$	6	$4.41.10^{-14}$
250	$7.66.10^{-11}$	5	$3.05.10^{-14}$
200	$4.50.10^{-11}$	4	$1.96.10^{-14}$
160	$3.14.10^{-11}$	3	$1.11.10^{-14}$
120	$1.765.10^{-11}$	2.5	$7.66.10^{-15}$
100	$1.23.10^{-11}$	1.0	$1.23.10^{-15}$
80	$7.84.10^{-12}$		

Table II gives the received watts corresponding to the various audibilities using the oscillating audion without sensitizing circuit, as deduced from the experiments at the Naval Radio Laboratory which showed that for unit audibility with the electrolytic, the oscillating audion gave an audibility of 1,000. The watt sensitiveness of the electrolytic was taken to be 12.25×10^{-10} watts[2] using telephones of 2,000 ohms resistance and a current

[1]The normal disturbances at Darien except in the morning were so strong that with the telephones on the table a crackling rumble could be heard in all parts of the receiving room. To prevent the breaking down of the local oscillations due to these heavy atmospheric discharges, it was found necessary to earth the grid electrode of the audion thru a small capacity.

[2]"Bulletin, Bureau of Standards," 11, page 69, 1914. Reprint 226.

sensitibility of 5.10^{-10} amperes at a frequency of 1,000 per second. This table cannot lay claim to perfect accuracy as applied to the Darien receiving set, since it was derived from experiments with a different receiver, and it might be supposed that the sensitiveness of the oscillating audion might probably vary with the ratio of inductance and capacity. Experiments thus far made, however, do not indicate with certainty that there is any such variation. At any rate, it is safe to say that the values given in Table II are approximately correct.

All of the observations recorded in the table, except those on Sayville and Honolulu, were taken on the small receiving antenna during the month of March. A series of daily observations extending over a period of a week were taken on Honolulu early in May, using the large antenna. Tuckerton was measured about every second day during March, San Francisco, Nauen and Eilvese were measured only a few times so these observations have comparatively little value. San Diego with its short wave length coming all the way overland could not be expected to approach its calculated over-sea audibility. It will be noted that Arlington is the only station in which the observed audibility approaches the calculated, but in this case Arlington's effective height h_1, was determined experimentally from observations made at near-by stations and is only about one-half of the height to the geometric center of capacity. If the effective heights of Tuckerton, Sayville and Honolulu were reduced in the same ratio, the agreement of their observed and calculated values would be nearly as good. The great weakness of the day signals from Nauen and from Eilvese is astonishing, as in Washington they come in with their full calculated audibility.

Beginning with May, regular observations on the received signals from Darien were taken at the Naval Radio Laboratory at the Bureau of Standards. The signals were received on a flat-top antenna 450 feet (130 meters) long, having a capacity of 0.00155 μf and an effective height of 100 feet (30 meters). This effective height is practically the same as that of the old harp antenna described in former papers, and has the advantage over the harp of having a much lower ground resistance at the longer wave lengths.

TABLE III

DARIEN RECEIVED AT THE U. S. NAVAL RADIO LABORATORY, BUREAU OF STANDARDS

$h_1 = 146$ m. $h_2 = 30$ m. $I_s = 100$ amp. $\lambda = 6{,}000$ m.
$d = 3{,}330$ Km. $R = 75$ ohms. Calculated audibility $= 3{,}600$

1915	OBSERVED AUDIBILITY Total Average	Maximum	Minimum	Normal Average*	Number of Observations
May	8,700	20,000	1,000	3,100	11
June	33,000	100,000 (Estimated)	300	3,700	11
July	9,000	50,000	600	3,800	12
August	4,100	20,000	400	2,300	19
September	1,330	2,000	300	1,330	13
October	1,460	3,000	400	1,460	9
November	21,500	40,000	5,000	....	4
December	13,750	30,000	5,000	....	4

Table III gives the monthly averages of the results of these observations. A large number of measurements were made between May first and November first, and since that time have been taken weekly. During the summer, except in June, the general intensity was between 1,000 and 3,000 audibility, with occasional periods of greater intensity, going up to 30,000 or 40,000 audibility while on one or two occasions the intensity has been so great that the signals could be heard a hundred feet from the telephones without the use of any amplifying device. Since the first of November, the signals have been uniformly strong. The calculated value of audibility is given at the top of the table.

September and October are seen to be the months with the lowest averages. This is due not so much to exceptionally weak signals as to the fact that there were no periods of extraordinary intensity such as occurred in the other months.

One of the most interesting points in this table is the astonishingly high occasional values of the audibility, observed during June, July and August, a time of year which is generally supposed to be especially unfavorable for radio communication.

*The normal average excludes the occasional excessively high peaks of the audibility curve which are supposed to be produced by the same causes which produce the irregular and strong signals at night at shorter wave lengths. U. S. Naval Radio Laboratory.

Comparing the calculated values[1] of the Navy formula

$$I_R = 377 \frac{h_1 h_2 I_s}{\lambda d R} \varepsilon^{-\frac{0.0015 d}{\sqrt{\lambda}}}$$

and that of the Sommerfeld purely theoretical formula,

$$I_R = 377 \frac{h_1 h_2 I_s}{\lambda d R} \varepsilon^{-\frac{0.0019 d}{\sqrt[3]{\lambda}}}$$

we find that the Sommerfeld formula would give 15 times audibility for Eilvese received at Darien, and 20 times audibility for Honolulu received at Darien. These values are so far below those observed as to support the conclusion in the paper last cited that, in order to represent the usual observed values an additional term must be added to the theoretical formula, representing energy reaching the receiving station by reflection.

It seems possible that the Sommerfeld formula represents the very lowest values of received signals, and that these are ordinarily strengthened by energy from the upper atmosphere the intensity of which would probably depend on the wave length. On this supposition, the scattering term of the empirical formula would represent the sum of these two effects which in their combination might very possibly introduce the square root of the wave length instead of the cube root, as indicated by theory.

U. S. Naval Radio Laboratory,
Washington, February, 1916.

SUMMARY: The results of measurements of the strength of received signals at Darien from a number of stations are given. A specially modified ultraudion circuit is used. The Austin-Cohen formula is found to give much closer agreement with the observations than the Sommerfeld formula. Relations between received current and audibility are given for the audion and ultraudion.

[1] "Bulletin of the Bureau of Standards," 11, page 269. Reprint 226, 1915.

DISCUSSION

John L. Hogan, Jr.: This interesting paper of Dr. Austin's would have been of more value to me, and I think possibly to others of us, if a few specific points had been cleared up. I am sorry that Dr. Austin himself is not with us this evening, since he could without doubt explain the several relations of detector-organization sensitiveness which appear confusing.

The paper states that the oscillating audion, or auto-heterodyne, has been found to have from 500 to 1000 times the sensitiveness of the electrolytic detector, the exact ratio depending upon the telephone note. I do not understand whether this reference is to grouped-wave or to sustained wave reception. If the grouped waves were received, was the audion in the oscillating condition, and the group frequency tone destroyed, or was the tube in a critical condition and was amplification secured by regenerative action? If sustained waves were used for the comparison, was the electrolytic detector excited according to the heterodyne method, or was a chopper used?

At the end of the third paragraph of the paper it is stated that the oscillating audion or auto-heterodyne has the same sensitiveness, aside from the matter of telephone note, for slightly damped and for sustained waves. How are these measurements made, and what relation have they to the figures quoted above? Further, does this equality of sensitiveness hold when the sensitizing circuit N of Figure 1 is added? It is stated that the presence of this absorbing circuit increases the sensitiveness of the self-excited audion heterodyne by three or four times, giving apparently a total improvement in sensitiveness to a point some 4,000 times that of the electrolytic detector.

Since the ordinary amplification ratio of the single audion bulb is usually taken to be in the neighborhood of five, it would appear that Dr. Austin's work has been done under conditions in which the signals were continuously amplified by regeneration. This adjustment of circuits is notoriously unstable, and with it quantitative results showing consistent performance are very difficult to secure. The variation from day to day, or from one adjustment to an attempted repetition of it at some later time, is likely to be especially great when the regenerative audion is used to take measurements according to the shunted telephone.

With regard to table 1, it may be noted that the observed audibility ranges from one twentieth to one third the calculated

audibility. If the effective heights of the transmitting stations were halved, as suggested, somewhat better agreement would of course be secured. It appears to me, however, that one should consider the desirability of decreasing the assumed ratio of detector sensitiveness. If the sensitiveness of the ultraudion is taken to be only 500, instead of 1000 times that of the electrolytic, better agreement can be secured without the necessity of departing from the earlier conception of effective height. Until these measurements can be confirmed with so constant a device as the tikker, it would seem wise not to overthrow the relation between geometric and effective height which has been found to agree so well with quantitative results of many earlier observations.

This matter of checking ultraudion observations against tikker reception might also be borne in mind in attempting to pin down the causes for such tremendous variations in intensity as are indicated by table 3. Changes in received power so great as those implied by the observations of table 3 seem to indicate variations in net sensitiveness of the receiver, as well as changes of the medium between the two stations. Further, the effect of strong atmospherics, in reducing the apparent sensitiveness of receiving apparatus for telephone shunt observations, must not be underestimated.

The fact that in spite of a measured intensity of 5,000 audibility, it is very difficult for Darien to copy messages transmitted from Arlington, confirms the earlier indications that large numerical values of audibility to signal are useless in commercial radio telegraphy unless the intensity of response to strays is limited. In the absence of severe atmospheric disturbance, one can of course amplify feeble signals indefinitely, and in that way read messages which were entirely inaudible before successful telephone or radio frequency relays had been produced. When strays co-exist with the signals, however, amplification of the ordinary sort becomes futile. This indicates the need of a measurement of signal intensity which is based upon the ratio of signal strength to that of normal strays, for a given detector organization, rather than upon the mere audibility of signals in the absence of strays.

Leonard F. Fuller (communicated): Dr. Austin's work upon transmission formulas has required a vast amount of exacting and tedious measurement under difficult conditions. This was especially true at Darien where the atmospheric disturbances were very severe. Probably those who have attempted

such measurements can best appreciate the amount of detail, the trying difficulties and the chances of error.

The shunted telephone method is the only practical means developed at present for taking such data and since it involves the human ear, it is not surprising that results taken by different observers, or even the same observer at different times, vary widely. Furthermore, it involves telephone impedance which is determined by telephone resistance and reactance and is a function of the audio frequency.

In the reception of damped waves the group frequency is fairly constant and reasonably well known at the receiver, hence the correct telephone impedance value may be chosen reasonably well for the calculation of "observed audibility." In the reception of undamped waves, however, using an oscillating audion, with beat tone reception, as a detector, the audio frequency is altogether dependent upon the receiver adjustment and may be varied at the will of the receiving operator. In this case, therefore, choice of the proper value of telephone impedance is not an easy matter.

The determination of the correct resistance of the receiver is also a source of error and measured values of h_1 and h_2 are rarely available.

One should bear all these difficulties of observation and chances of error in mind when commenting upon such data as are given in Dr. Austin's paper, and should attempt to adjust the mind to consider differences of 100 per cent. between calculated and observed audibilities as we consider errors of 1 per cent. in many laboratory electrical measurements.

I believe Dr. Austin's formula gives a better approximation of actual results than any yet published. While the formula involving $\varepsilon^{-\frac{0.0015\,d}{\lambda^{1/2}}}$ discussed in the paper on "Continuous Waves in Long Distance Radio Telegraphy," "Proceedings A. I. E. E.," Volume 34, number 4 was derived from data taken with considerable care and while it checked Honolulu, San Francisco and Tuckerton, San Francisco data very nicely, it gives absurdly high values of calculated received watts when compared with the values observed in the receiving experiments mentioned by Dr. Austin.

The following experiments, involving the reception of Darien at Honolulu may be of interest, inasmuch as they cover signals in the reverse direction over the same path of 8,500 kilometers mentioned in Dr. Austin's paper.

On May 30, 1915 from 3 to 3:30 P. M., Washington time, (9:30–10 A. M., Honolulu time), Darien transmitted upon a wave length of 15,000 meters and observations of received signal strength were made at Honolulu using an oscillating audion receiver. The variables in the transmission formula were as follows:

$d = 8{,}500$ km.
$\lambda = 15{,}000$ m.
$h_1 = 146$ m.
$h_2 = 120$ m.
$I_s = 90$ amps.
$R_r = 25$ ohms approx.

The calculated audibility was 1,000.

The observed audibility was determined as follows:

Honolulu reported a shunt of 51 ohms on telephones having an impedance of 5,000 ohms per pair at 500 cycles with a telephone resistance of 2,400 ohms per pair. This gives a reactance of 4,385 ohms on 500 cycles or 8,770 ohms on 1,000 cycles, hence the impedance is 9,070 ohms at this frequency and the observed audibility 180.

Darien was audible at South San Francisco but unreadable.

Prior to and after this test Darien was received at Honolulu many times on wave lengths from 6,000–18,000 meters with similar results, but inasmuch as no previously planned tests were conducted, no further specific statement of observations is possible. It is reasonably probable, however, that during the test of May 30, 1915, conditions were approximately normal between Darien and Honolulu.

On March 27 and 28, 1916, 2:30 to 3:00 P. M., Washington time (9–9:30 A. M., Honolulu time), Darien transmitted in prearranged tests to Honolulu. The variables in the transmission formula during these tests were as follows:

$d = 8{,}500$ km.
$\lambda = 10{,}000$ m.
$h_1 = 146$ m.
$h_2 = 120$ m.
$I_s = 79$ amps. on March 27, 1916.
70 amps. on March 28, 1916.
$R_r = 26$ ohms. approx.

This gives a calculated audibility of 610 on March 27, 1916, and 537 on March 28, 1916.

The observed audibilities calculated in the same manner as the May, 1915, test were 110 for March 27, and 180 for March 28, 1916.

Since it was earlier in the year, the overland transmission over Mexico was considerably better than in May, 1915, so that whereas in May, 1915, Darien was barely audible at South San Francisco, in March, 1916, the signals were easily readable. Darien was also audible but unreadable on a small downtown office receiving antenna in San Francisco in this year's tests.

Continued tests during the months of May and June, 1915, wherein the Darien signals received at Honolulu were expressed in terms of commercial value rather than measured audibilities gave the following results with daylight over the entire path of transmission:—

Honolulu reported consistently that with a radiation of 75 amperes or below, the signals were weak but readable without interference; from 75 to 80 amperes fair, and from 85 to 100 amperes good readable signals. This referred to cipher and code on wave lengths of 10,000 meters and above.

Wave lengths of 15,000 and 18,000 meters gave an audibility ten times that observed on 8,000 meters. The 6,000 meter wave was a little weak but as a rule no great change in signal strength was noticeable from 6,000 to 10,000 meters, the great gain being from 10,000 to 18,000 meters.

At San Francisco the 15,000 and 18,000 meter waves gave better received signals than were obtained on waves of 10,000 meters and below, but on account of the increase in atmospheric disturbances they were no more readable than the shorter waves.

It is to be noted therefore that at Honolulu in March of this year, a 10,000 meter wave gave the same observed audibility as waves of 15,000–18,000 meters in May, 1915, and at San Francisco a considerably greater audibility.

Dr. Austin's Darien observations show calculated audibilities approximately four times the observed, suggesting as he mentions, the possibility of correcting h_1 and h_2 in the ratio found necessary at Arlington. However, the observed and calculated values check very well when receiving at the Bureau of Standards.

In the tests of Darien received at Honolulu, it is again to be observed that if calculated audibilities are altered by correcting h_1 and h_2 in the Arlington ratio the results approach more nearly the observed values.

The receiver resistance values for the receiving experiments at Honolulu are altogether approximations. I believe it would be of interest to many of the members of The Institute of Radio Engineers if Dr. Austin would describe in detail the method he used in determining this value in his work. A statement of his ideas on the error introduced by telephone impedance changing with audio frequency and the probable percentage error in his observations neglecting any errors in the h_1 and h_2 values would be most valuable.

Edwin H. Armstrong: Before discussing this paper, I would very much like to have a little more information about the operation of the apparatus that Dr. Austin used. I am fairly familiar with the regenerative audion and its use as a self-heterodyne, but nothing seems to have been published about the manner of operation of this so-called "ultraudion." This occasion is the first opportunity I have had for getting some first hand information about it, so I am going to ask Dr. de Forest if he will not be good enough to explain how it works. In the absence of an explanation by Dr. de Forest, I wish to advance the following explanation.

You might expect from the name that there is something super-mysterious about the action of this device, and from the manner in which the ultraudion circuit is drawn there is good ground for this belief. But when the circuit is re-drawn as in the accompanying sketch, it becomes at once evident that it is an ordinary regenerative circuit, dependent for its operation on a coupling between grid and wing circuits.

The wing circuit is coupled with the grid circuit thru the combined electrostatic and electromagnetic coupling of the condenser *C* and telephones *T* which are located in the common part of both circuits. Thru the medium of this coupling, some of the energy of the radio frequency current set up in the wing circuit by an incoming signal is transferred back into the grid circuit in the manner explained in my paper of March, 1915, in which the identically same form of coupling is shown.

That the arrangement regenerates can be shown experimentally (for the non-oscillating state), by measuring the current set up in the grid circuit by an incoming signal first with the audion disconnected from the rest of the circuit and second, with the audion connected and condenser *C* adjusted so that the system is fairly close to the point of oscillation. It will be found that the current will have increased very many times over

its value with the audion disconnected. Obviously the audion is supplying energy to the grid circuit and the only source from which this energy can come is the wing battery B. A current amplification of 50-fold can be obtained by adjustment of the coupling condenser C before the system begins to oscillate.

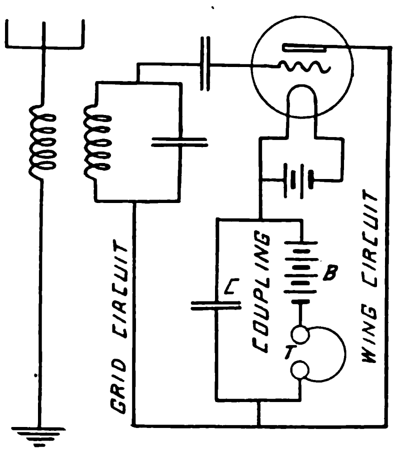


FIGURE 1

After local oscillations start the amplification can no longer be measured in this simple manner, but both theory and practical results show that the amplification due purely to the regenerative action apart from the added heterodyne amplification is markedly increased.

The sensitizing circuit of Dr. Austin is a very ingenious and interesting thing. In the non-oscillating regenerative circuit one can tune the grid circuit exactly to the incoming frequency so that (for continuous waves and loose coupling) the impedance of the circuit is equal to the effective resistance. When you make the system oscillate and receive by the beat principle then the circuit can have zero reactance for the local frequency only, and must oppose a definite reactance for the signaling frequency. The impedance of the circuit for the signaling frequency may thereby be greatly increased, particularly for the longer wave lengths, when the percentage mistuning necessary to produce a note of 1,000 cycles is considerable. What Dr. Austin does to the grid circuit by coupling another circuit to it is to give it two periods so that the reactance of the circuit is zero for two frequencies differing from each other by 1,000–

1,500 cycles. By adjusting the system to oscillate in one of these frequencies and having the other coincide with the signaling frequency the increase in signal strength is secured. It is possible to secure the same effect without the use of the sensitizing circuit by getting the two periods thru the medium of the antenna coupling, but as Dr. Austin points out this requires a relatively strong coupling. Despite the additional adjustments necessary, the use of the extra circuit is well worth while.

Lee De Forest (communicated): I would like to say that I also am in full agreement with the remarks made this evening concerning the uncertainty of audibility measurements. I cannot see that when "audibilities" of from 5,000 to 20,000 or 100,000 are obtained, we can be expected to handle them mathematically at all. With audibilities in amount up to one or five thousand this is possible, but above that I think we need a new unit. Where audibility comparisons are carried on, extending over a period of several weeks or months, and where different bulbs must be used and adjustments are changed, even if the circuits remain the same, the comparisons must be little better than guesses.

As to replying to any remarks of Mr. Armstrong's, I stated on a former occasion that I must refuse to be drawn into any discussion.

However, I wish to point out that it must be obvious to anyone examining, for example, circuit 1 of the de Forest-Logwood "ultraudion patent," No. 1,170,881, that the ultraudion circuit is not and cannot be a "regenerative circuit." There is only one oscillating circuit. This circuit is such that a sudden change of potential impressed on the plate produces in turn a change in the potential impressed on the grid of such a character as produce, in its turn, an opposite change of value of potential on the plate, etc. Thus the to-and-fro action is reciprocal and self-sustaining. It is "regenerative" in the sense that a reciprocating engine with piston and slide valve is "regenerative," or in the sense that an ordinary electric bell or buzzer, is "regenerative." If any member can obtain comfort from calling the ultraudion circuit "regenerative," he is entirely welcome so to do.

Louis W. Austin (by letter): I think that Mr. Hogan must have forgotten that in connection with the Arlington-Salem tests in 1912, experiments were carried out at the Bureau of Standards and at several other stations within ten miles of

Arlington in which the absolute received currents were measured, and in this way the field strength due to Arlington's radiation was determined. It was found that Arlington radiated like an ideal antenna or semi-doublet of less than half the height of the actual antenna, probably on account of the metal towers. (See "Bureau of Standards Bulletin," Reprint 226, page 74.) This is why the effective height of Arlington is taken as one-half the actual height. It seems probable that most land stations like Arlington and the Washington Navy Yard station have actual radiating heights less than the geometric heights. This may be brought about by imperfect ground conductivity under and near the antenna, or by the losses in the metal masts or towers now ordinarily used in radio installations.

The approximate equality of sensitiveness of the oscillating audion to damped and continuous oscillations was shown by a galvanometer arrangement which I described in the "Journal of the Washington Academy," 6, page 81, 1916. The loudness of signal in the telephones can of course also be compared even tho the notes are not the same, using a sending circuit which is first excited by a buzzer and then by an audion, the radio current in the circuit and the wave length remaining the same. In this case owing to the difference in note the audion signals seem to be about three to four times stronger than those from the buzzer.

It is perhaps not generally known that the remarkable sensitiveness of the oscillating audion depends very little on the presence of beats. Using broken up audion excitation with the receiving circuit tuned so closely to the incoming signal that no beats are heard, the signal is about one-third as loud as when the sending waves are not broken up but are received by the beat method with the best note for telephone sensitiveness.

The constancy of the audion when the circuits are adjusted in a perfectly uniform manner is remarkable, being quite equal, I believe, to the electrolytic. Different bulbs, except when evidently abnormal, also give sensibilities which agree within 20 or 30 per cent. The apparent variations are usually the result of imperfect adjustment. In this work the regular bulb was frequently tested by replacing it by a second bulb which could be instantly connected.

The first estimate of the absolute sensibility of the oscillating audion, assuming that it was 1,000 times as sensitive as the electrolytic at unit audibility, and that the electrolytic with the same telephones would respond to 1.225×10^{-9} watts in the antenna, gave 1.225×10^{-15} watts as its sensitiveness. Since

that time, further determinations have been made employing several different bulbs and different wave lengths. The method employed was the comparison of the deflection of a galvanometer connected to a silicon detector with the audibility observed with the oscillating audion. The same secondary circuit was used in both cases and the audion or detector thrown in by means of a two way switch, adjustments for tuning and best coupling being independently made in the two cases. The detector was calibrated by comparison with a thermo-element in the artificial antenna immediately before each experiment. The sending apparatus was a wave meter excited by a powerful audion. The average value of the energy for unit audibility on the audion was found by this method to be 1.5×10^{-15} watts in the receiving system. A paper on this and some connected lines of work is now in preparation.

As Prof. Zenneck suggests, it would of course be desirable to use a detector and galvanometer in all measurements of received signals, but in general for long distance work this is impossible.

If the detector is sensitive enough to produce deflections for weak signals, the atmospheric disturbances during a great portion of the time will make the readings even more unreliable than those taken with shunted telephones. I fully realize that the telephone method is far from satisfactory, altho it has been shown that telephone audibility, as taken by our methods, is proportional to the received antenna current in the case of the audion. This is shown in the following table of observations taken in the Naval Radio Laboratory with an artificial antenna.

Audibility	Radio Current	Audio Current
250	11	23
400	16	25
1,500	58	26
2,500	99	25
4,000	148	27
4,500	160	28

The experimental errors of the audibility measurements under station conditions amount frequently to 30 per cent. and the observation may sometimes be incorrect in the ratio of two to one, but except when the signals are nearly masked by heavy atmospherics, I do not believe that the errors are greater

than this. Bad as this is, it is certainly much better than no observation at all. The most disappointing fact in our work is the great irregularity in the signal strength which renders any comparison with the theory extremely unsatisfactory.

In beat reception, the telephone sensibility rises with the pitch of the note, but this is partly counteracted by the secondary circuit becoming more and more out of tune with the signal as the note rises. In addition, the audibility reading is lowered, due to the increase in telephone impedence. Thus, these effects to a considerable extent counterbalance each other over the range where the loudest signals are heard. If the operator readjusts for loudest signal after the audibility meter is set near the point of silence, the error due to these causes is not great, as is shown by our direct comparison of audibility and sending current using artificial circuits.

The resistance of the receiving system can be best determined, where continued oscillations are available, by exciting the antenna from a loosely coupled undamped circuit and then introducing enough resistance to reduce the antenna current to one-half. As the audibility meter is calibrated by comparison with a silicon detector and galvanometer, the amount of coupling resistance added in the calculations is that due to the silicon detector circuit. This, for best coupling, is always roughly seven-tenths of the antenna resistance.

THE MECHANISM OF RADIATION AND PROPAGATION IN RADIO COMMUNICATION*

By

FRITZ LOWENSTEIN

The energy to be transmitted by radio (high) frequency currents exists at the transmitter station alternately in the form of electric and magnetic energy. The electric energy is concentrated in that portion of the field which closely surrounds the transmitting antenna.

As many lines of force emanate from the antenna vertically upward as downward, as shown in Figure 1. This is so because

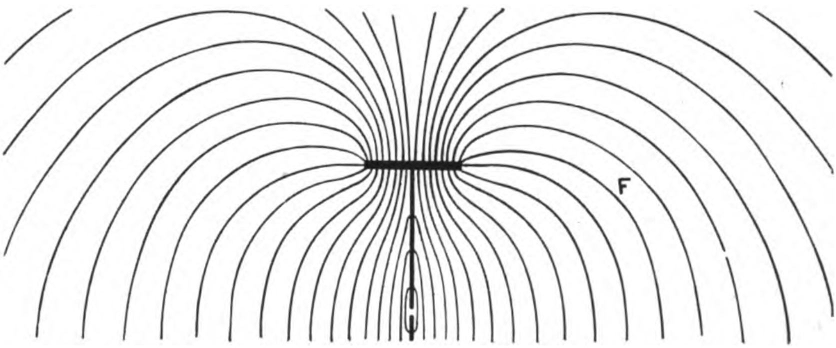


FIGURE 1

in an electric field whose two terminal surfaces are vastly different in size, the electric energy is concentrated near the smaller terminal surface (the antenna in this case), and the distribution of the electric lines near the smaller terminal surface is independent of the location of the larger terminal surface.

For a clear understanding of the distribution of the electric energy distribution in a field which ends in two terminal surfaces greatly different in size, I have shown in Figure 2 the field

* Presented before The Institute of Radio Engineers, New York, December 1, 1915.

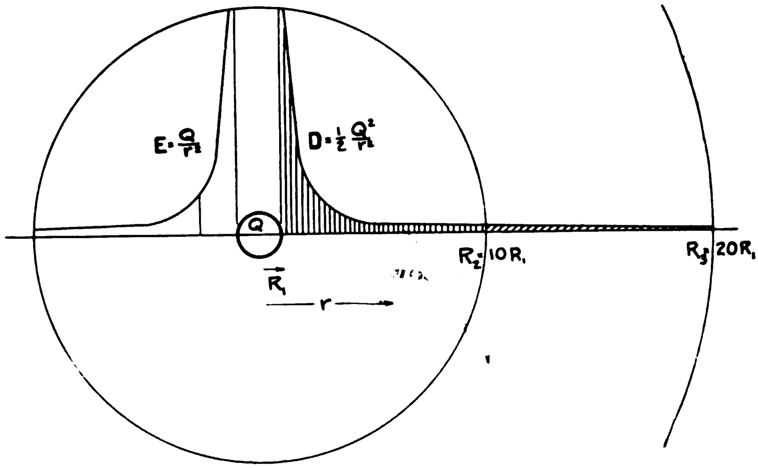


FIGURE 2

between two concentric spheres the radii of which are in the ratio of 1 to 10, and also for a ratio of 1 to 20.

The electric intensities are shown by the graph on the left side in Figure 2, wherein E represents the electric intensity in terms of r, the distance from the center. On the right side of Figure 2, D represents the lineal energy density:

$$D = \frac{1}{8\pi} E^2 \cdot 4\pi r^2 = \frac{1}{2} E^2 r^2 = \frac{1}{2} \frac{Q^2}{r^2}.$$

The vertically shaded surface indicates, therefore, the total energy in the field; whereas the outward continuation of this area, shaded slantingly, gives the energy which would be added to the field by increasing the radius of the outside terminal to a value $R_3 = 20\,R_1$. It is readily seen that the change of position of the larger terminal surface involves only a very slight change of the total energy, and therefore practically no change in the distribution of the field lines emanating from the smaller surface.

Such a comparison will appear still more striking if the smaller terminal surface be composed of conductors of a radius of a few millimeters and the distance between them and the ground be of the order of 100 meters.

The electric energy in the space closely surrounding the antenna will cause an effect in the receiving antenna in essentially the same way when radio frequencies are used as when the charge on the antenna is a stationary one. In Figure 3, Q may designate the quantity of electricity given to the antenna,

and h the height of the charge above ground. The force on a unit charge at the receiving point R will be equal to $\frac{Q}{r^2}$. The image of the charge Q produces an equal force F'. The resulting electric intensity is therefore

$$E = 2\frac{Qh}{r \cdot r^2}.$$

In a system of electrostatic transmission, the received intensity is therefore proportional to the electric moment Qh impressed at the transmitter station, and diminishes proportionately to the cube of the distance.

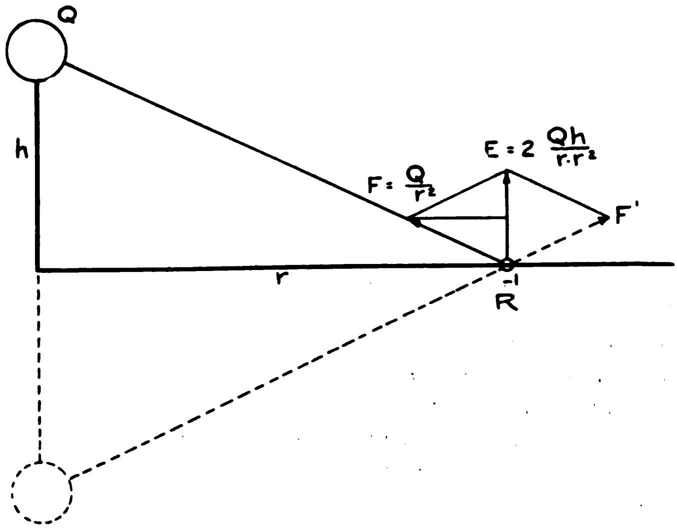


FIGURE 3

With a radio frequency current of value I flowing in the transmitting antenna, the received electric intensity is equal to

$$E = 4\pi\frac{Ih}{\lambda r} \cdot \frac{1}{3\,(10)^{10}}.$$

Substituting in this equation:

we have $$I = 2\pi nQ = 2\pi\frac{v}{\lambda}Q,$$

$$E = 2\frac{Qh}{r\left(\frac{\lambda}{2\pi}\right)^2},$$

where v = velocity of light $= 3(10)^{10}$ cm./sec.

Comparing this intensity obtained by radio frequency transmission with the value of the intensity obtained by the electrostatic method, we find that the intensity in both cases is proportional to the electric moment, inversely proportional to the transmitting distance r and inversely proportional to the square of the distance of the receiver from the charges producing the field.

In the case of the electrostatic transmission, this distance of the charge is identical with the transmitting distance. In the case of radio frequency transmission, it is the mean distance of the electrical charges near the receiving station from that station, and is expressed by $\frac{\lambda}{2\pi}$.

This analysis, I think, shows the beauty and advantage of substituting for the static method that of radio frequency currents, which latter method we owe to Mr. Nikola Tesla. My comparison above shows the great superiority of radio frequency currents for transmission in that the distance of electric action has been reduced from the total transmitting distance to a part of the wave length.

In view of the foregoing comparison between the various methods, it seems not out of place to review the history of the art of radio communication. Thomas A. Edison in 1885 applied for a patent, which was issued in 1891 as No. 465,971, wherein he proposed to use the static method. An electrostatic field produced at an elevated transmitting capacity is a source of electrostatic lines of force, which cut the receiving aerial, and a device is used for registering the potential difference which exists between the top of the receiving aerial and the ground, caused by the presence of lines of force. It must be borne in mind that the lines of force which strike the receiving aerial actually emanate from the transmitter capacity, and have a path starting from the transmitter capacity and ending on the ground near the receiving aerial.

In February, 1893, Nikola Tesla described before the Franklin Institute in Philadelphia, and again before the National Electric Light Association in St. Louis, in March, 1893, his system of "high" frequency radio transmission. In this paper he states that if radio frequency currents be caused to surge to and from an elevated capacity connected to ground by a vertical wire, it would not require a great amount of energy to produce a disturbance perceptible at great distances, or even all over the surface of the globe, particularly if the receiving circuit be

properly adjusted by means of inductance and capacity so as to be in resonance with the transmitted frequency.

In Edison's method, the electric action on the receiver came from the transmitter station directly; and therefore the distance of electric action was identical with the actual distance from the transmitter to the receiver.

Tesla's introduction of radio frequency transmission had the effect of bringing a considerable amount of the antenna charge into close proximity with the receiver, so that the electric intensity at the receiver was enhanced as the square of the ratio of the transmitter distance to the wave length of transmission.

The comparative analysis here given is the result of a conversation with Lieut.-Comm. A. J. Hepburn, U.S.N., to whom I hereby express my thanks.

The analysis of the complex wave form, found when transmitting over poorly conducting ground, as given by Sommerfeld, is of great value and interest. So are also the calculations of the effect caused by the spherical shape of the earth and of the effect of absorption, as given by the able treatment of Poincaré, Nicholson, Austin and Zenneck.

The fundamental question remains, however, whether we may designate the method now used as a transmission by Hertzian waves similar to those emitted by the Hertzian oscillator of the early Marconi apparatus, or as true conduction along the ground, as was proposed by Tesla in 1893. Sommerfeld's work only increased this doubt, inasmuch as the poorly conducting ground assumed to exist in his analysis led him to decompose the total wave action into a space wave and a surface wave. Thus the belief has been established in the minds of many radio engineers that transmission of the energy was carried on by two distinct and different phenomena.

To decide this question of principle, I assume the ground as plane and of good conductivity, an assumption absolutely permissible in the case of transmission over sea water.

The received electric intensity

$$E = 4\pi \frac{I h}{\lambda r} \cdot \frac{1}{3(10)^{10}}$$

may be expressed as

$$E = 8\pi^2 \frac{Q h}{r \lambda^2} \cdot \quad . \quad . \quad . \quad . \qquad (1)$$

If we designate by Q the electric charge in the antenna and by q the electric charge of each half wave length gliding along

the earth, then we find the mean density of charge on the zone Z (Figure 4) to be

$$\sigma_m = \frac{q}{2\pi r \frac{\lambda}{2}},$$

and the maximum density at the crest

$$\sigma = \frac{\pi}{2}\sigma_m = \frac{q}{2r\lambda}.$$

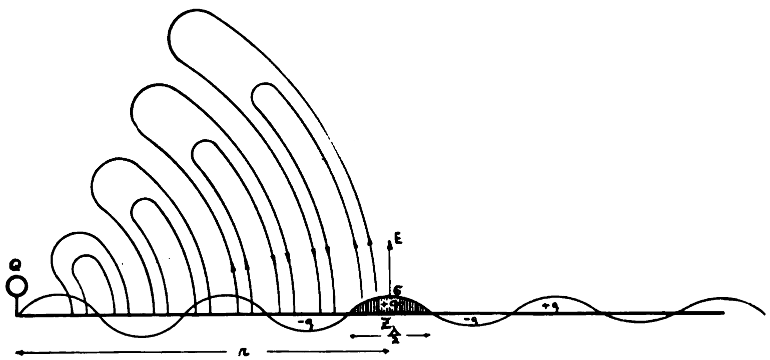


FIGURE 4

Therefore the electric intensity is

$$E = 4\pi\sigma = 2\pi\frac{q}{r\lambda} \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

From a comparison of equations (1) and (2), we find the radiated charge to be

$$q = 4\pi\frac{h}{\lambda}Q$$

and therefore independent of the distance.

We may therefore describe the phenomenon of transmission as follows. A maximum charge Q in an antenna oscillates to the ground and back, causing in every half oscillation the emission along the ground of a radiated charge q, where the ratio of that radiated charge to the full antenna charge,

$$\frac{q}{Q} = 4\pi\frac{h}{\lambda},$$

may be called the radiation factor, and appears to be independent of the distance.

The surging to and fro of the electric charge of the antenna entails, of course, a magnetic field, which represents part of the detached energy. During this process of detaching of energy, the magnetic energy is preponderant. Figure 5 shows the electric and magnetic intensities at various distances. At a distance of one-sixth of the wave length, the magnetic intensity M is 40 per cent. greater than the electric intensity E, at a distance

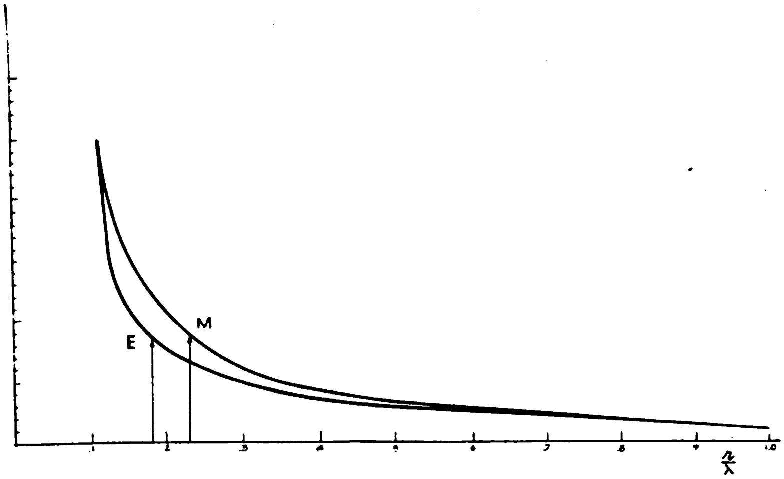


FIGURE 5

of one-half of the wave length the preponderance of magnetic over electric intensity is only 5 per cent.; and at a distance equal to the wave length there is practically no difference between the two intensities.

For convenience of computation of the values used in Figure 5 I have arranged the formulae for the electric and magnetic intensities as follows:

$$E = 2\frac{Qh}{r^3}\sqrt{1 - 4\pi^2\left(\frac{r}{\lambda}\right)^2 + 16\pi^4\left(\frac{r}{\lambda}\right)^4}$$

$$M = 2\frac{Qh}{r^3}\sqrt{4\pi^2\left(\frac{r}{\lambda}\right)^2 + 16\pi^4\left(\frac{r}{\lambda}\right)^4}$$

In a Hertzian oscillator, no electric charge or electrons move along the equatorial plane, whereas they do actually flow thru the ground in radio transmission and carry with them the energy which actuates the receiving devices.

Also, in view of the curvature of the earth, does it not seem more natural to speak of a conducted radio frequency current and to look upon the electric and magnetic field travelling with it as its accompanying result, than to designate radio transmission as identical with the radiation of a Hertzian oscillator *modified* by a conducting equatorial plane *and* by bending of radiation lines due to the earth's curvature *and* to the existence of the conducting upper strata?

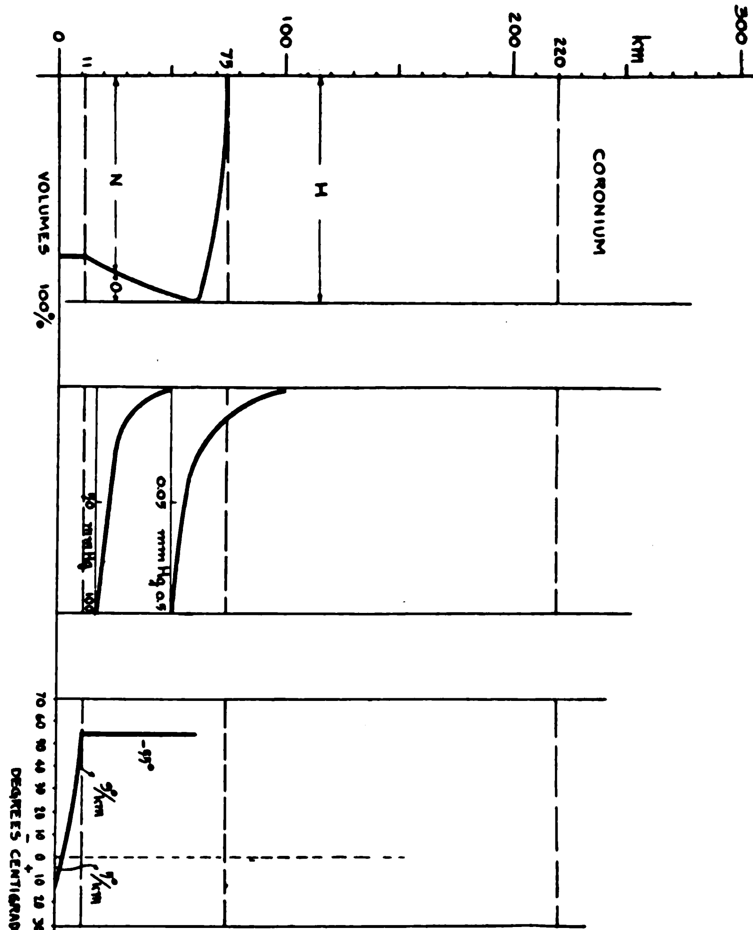
I have thought it of advantage to the development of radio art if the knowledge of our atmosphere, as herein given, be brought before every radio engineer who is in a position to make quantitative observations.

Figure 6 shows the chemical composition and pressure and the temperature of the atmosphere arranged for the varying heights.

Measurements of the intensity of light taken at sunset show three distinct discontinuities, when the last rays become tangent to the layers of air at the height of 11 kilometers, 75 kilometers, and 220 kilometers. In fact, test balloons with registering apparatus clearly show a very distinct change at the 11-kilometer height. The temperature which fell close to the earth at the rate of 5 degrees per kilometer, and at the height of 10 kilometers at the rate of 9 degrees per kilometer, suddenly becomes constant in regions from 11 to 75 kilometers, the mean temperature being minus 55 degrees Centigrade. A glance at the last graph of Figure 6 will make this discontinuity very apparent. The reason for the uniformity of temperature may lie in the fact that the pressures to which the atmosphere is subjected at these altitudes are suited to electric conduction, as may be seen from inspection of the graph in the center of Figure 6, wherein the pressures are given with their corresponding altitudes. This lower point of discontinuity (11 km.) has been further proven to exist by tests of chemical composition of the atmosphere.

I have indicated in the graph to the left of Figure 6 that the volumetric analysis of the atmosphere, as to its relative proportions of nitrogen and oxygen shows a constant proportion in this first layer of 11 kilometers, the reason being that air currents rise and fall therein, providing for thoro mixing, and not permitting the two gases to arrange themselves according to their densities.

This lower layer of 11 kilometers in thickness, the troposphere, is shown in the diagram by a vertical line, denoting a constancy of mixture up to 11 kilometers. Above that point, in the stra-



0
11
75
100
200
220
km
300
CORONIUM
N
H
VOLUMES 100%
0.05 mm Hg 0.5
DEGREES CENTIGRADE

FIGURE 6

tosphere, the two gases adjust themselves according to Dalton's Law, the heavier oxygen losing and the lighter nitrogen gaining in relative proportion. At 60 kilometers, the oxygen has practically disappeared, and the hydrogen becomes prominent. Because of the very great difference of density, however, the mixture of nitrogen and hydrogen extends over a comparatively thin layer of 15 kilometers only, and, as shown in the graph, there is a rather abrupt change from nitrogen to hydrogen at 75 kilometers.

The third sudden change in light intensity at sundown occurs when the last sun rays pass tangent to a layer at a height of 220 kilometers. Up to that moment, the sky has appeared blue by reason of the illumination of the hydrogen atoms which are able to send the shorter rays of the sun's spectrum to our eyes. The sudden disappearance of blue is due to the rather sudden disappearance of hydrogen from the atmosphere, a stratum of coronium beginning at that height.

It remains to consider the bearing of these various changes on the transmission of electric energy thru the space. The high pressures prevailing in the troposphere make it a perfect dielectric. The stratosphere is the layer of highest conductivity, bounded above by strata of such low pressures as to constitute practically dielectrics. *Gradual* changes of conductivity would entail considerable loss if they gave rise to marked reflections in electric wave propagation, and consequently a considerable amount of argument has been brought to bear against the possibility of efficient reflection by the upper strata. The *abrupt* changes shown to exist in the atmosphere certainly permit us with less hesitancy to ascribe strong variations of received signal intensities (according to the wave length employed), to efficient reflection.

SUMMARY: The intensity of the electric field at a distance from a statically charged antenna is calculated from elementary considerations. The same quantity is derived for the case in which the charge is oscillating at a radio frequency. It is shown that the total charges acting on the receiver in the two cases have a ratio equal to the square of the ratio of the transmitting distance to a certain part of the wave length; and hence the great advantage of the radio frequency transmission.

The theories of Edison, Tesla and Sommerfeld are historically considered. It is shown that there is no physical justification for the separation of the wave into surface and space waves. The electric and magnetic intensities at various distances from the antenna are calculated, and it is shown that they become practically equal at a wave length away. The author prefers to regard radio transmission as due to conducted radio frequency earth currents rather than modified Hertzian oscillator waves. The three distinct portions of the atmosphere: the troposphere, the stratosphere, and the coronium layer, are described, and their effect on radio transmission considered.

J. Zenneck: As regards the question whether the energy emitted by a transmitting antenna is propagated by conduction currents in the ground or by electromagnetic waves in the air, I agree with Mr. Lowenstein that this is largely a matter of expression. As a matter of fact, if a transmitting antenna is placed on the surface of the earth, at all distances from the antenna an electromagnetic field is present in the air as well as in the ground. It is therefore a matter of taste whether one describes the propagation of energy as taking place by means of earth currents or whether one describes it as being caused by electromagnetic waves in the air. Both of these descriptions are incomplete; the earth currents are always accompanied by electromagnetic waves in the air and the electromagnetic waves in the air always by currents in the ground.

Louis W. Austin (communicated): There are one or two points in Mr. Lowenstein's exceedingly interesting paper on the "Mechanism of Radiation" on which I would like to comment. In discussing the difference between the electrostatic field produced by the oscillator, and the electromagnetic, he substitutes the electrostatic expression for current in the electromagnetic equation for the electric intensity at the receiving station, and obtains a term which he calls the mean distance of the electrical charges from the receiving station. I am unable to give this any real physical interpretation, and unless this can be done, it hardly seems that the conception can be considered useful.

In the discussion of the transmission of the electrical waves along the surface of the earth, the paper might lead one to believe that the choice in theories lay between electromagnetic waves above the earth's surface or conduction along the surface; while as a matter of fact, under the given conditions neither can exist without the other, just as the current in a wire must always be accompanied by a field surrounding it. Hertzian waves will, I believe, always ground themselves on any conducting plane in their immediate neighborhood lying parallel to the direction of propagation. The moving charges on the grounded ends of the electrostatic lines of force will then produce currents such as are spoken of in the paper.

I am very glad that Mr. Lowenstein has called attention to the composition of the atmosphere in its different layers and the probable bearing of this on the variation in strength of the radio signals.

AMPLITUDE RELATIONS IN COUPLED CIRCUITS*

By

E. Leon Chaffee

The importance of a clear understanding of the action of coupled oscillatory circuits need not be emphasized. The theory of coupled oscillatory systems in general is not only applicable to electrical problems but is of interest in other branches of physics.

There has been a vast amount of material written on different aspects of the combined action of coupled circuits. Much of the work has been devoted to the relations between the coefficient of coupling, the natural wave lengths of the two circuits, and the resulting or "coupled" wave lengths. The effects of resistance in the circuits and the methods of measuring damping have been the subjects of many theoretical and experimental investigations. One might mention in this connection the work of Bjerknes[1], Oberbeck[2], Wien[3], Drude[4], and others.[5] It has seemed to the writer, however, that the *amplitudes* of the oscillations in coupled circuits has been less clearly set forth than have the other aspects enumerated above.

In the following paper the history of the changes in the amplitudes of the four component oscillations in coupled circuits as the constants of the circuits are changed are deduced and shown graphically. The results are derived directly from the familiar theory but the method of presentation may have in it some new features.

Figure 1 represents two circuits (1) and (2) made up of condensers C_1 and C_2, and inductances L_1 and L_2 as shown. The two inductances have a mutual inductance M or a coefficient of coupling $\tau = \frac{M}{\sqrt{L_1 L_2}}$. The resistances in the two circuits are

*Presented before the Boston Section of The Institute of Radio Engineers, November 24, 1915.

[1] V. Bjerknes; "Wied. Ann.," 55, p. 121.
[2] Oberbeck; "Wied. Ann.," 55, p. 623.
[3] M. Wien; "Ann. d. Phys.," 25, p. 625; "Wied. Ann.," 61, p. 151 "Ann. d. Phys.," 8, p. 685.
[4] P. Drude; "Ann. d. Phys.," 13, p. 512.
[5] L. Cohen; "Jahrb. der Drahtlosen Teleg.," 2, p. 448.

neglected in order to simplify the solution of the problem, but it can be shown that the relative results obtained are little affected by resistance unless the resistance is large.

If q_1 and q_2 represent the charges at time t on condensers C_1 and C_2, respectively, then the following equations express the performance of the two circuits:

$$\left.\begin{array}{ll}(1) & L_1 \dfrac{d^2 q_1}{d t^2} + M \dfrac{d^2 q_2}{d t^2} + \dfrac{q_1}{C_1} = 0 \\ (2) & L_2 \dfrac{d^2 q_2}{d t^2} + M \dfrac{d^2 q_1}{d t^2} + \dfrac{q_2}{C_2} = 0\end{array}\right\} \text{ and } \left\{\begin{array}{l} i_1 = -\dfrac{d q_1}{d t} \\ i_2 = -\dfrac{d q_2}{d t}\end{array}\right.$$

It may be noted for future reference that according to the convention adopted above the currents in the two circuits have the same sign if they produce fluxes in the same direction thru the coaxial coils L_1 and L_2.

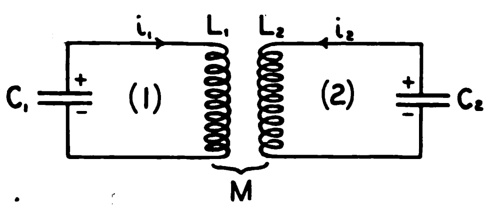


FIGURE 1

The above equations yield the two following equations, each involving a single dependent variable:

$$\begin{array}{ll}(3) & (L_1 L_2 - M^2) \dfrac{d^4 q_1}{d t^4} + \left(\dfrac{L_2}{C_1} + \dfrac{L_1}{C_2}\right) \dfrac{d^2 q_1}{d t^2} + \dfrac{q_1}{C_1 C_2} = 0 \\ (4) & (L_1 L_2 - M^2) \dfrac{d^4 q_2}{d t^4} + \left(\dfrac{L_2}{C_1} + \dfrac{L_1}{C_2}\right) \dfrac{d^2 q_2}{d t^2} + \dfrac{q_2}{C_1 C_2} = 0\end{array}$$

These equations may be abbreviated giving the two equivalent equations:

$$\begin{array}{ll}(5) & (1 - \tau^2) \dfrac{d^4 q_1}{d t^4} + (\omega_1^2 + \omega_2^2) \dfrac{d^2 q_1}{d t^2} + \omega_1^2 \omega_2^2 q_1 = 0 \\ (6) & (1 - \tau^2) \dfrac{d^4 q_2}{d t^4} + (\omega_1^2 + \omega_2^2) \dfrac{d^2 q_2}{d t^2} + \omega_1^2 \omega_2^2 q_2 = 0\end{array}$$

where $\omega_1^2 = \dfrac{1}{L_1 C_1}$, and $\omega_2^2 = \dfrac{1}{L_2 C_2}$.

The solutions of equations (5) and (6) are of the form

(7) $q_1 = A_1 \cos(\omega' t + \phi) + B_1 \cos(\omega'' t + \psi)$

(8) $q_2 = A_2 \cos(\omega' t + \phi) + B_2 \cos(\omega'' t + \psi)$, where

(9) $$\omega' = \sqrt{\frac{\omega_1^2+\omega_2^2+\sqrt{(\omega_1^2+\omega_2^2)^2-4\,\omega_1^2\,\omega_2^2\,(1-\tau^2)}}{2\,(1-\tau^2)}}$$

(10) $$\omega'' = \sqrt{\frac{\omega_1^2+\omega_2^2-\sqrt{(\omega_1^2+\omega_2^2)^2-4\,\omega_1^2\,\omega_2^2\,(1-\tau^2)}}{2\,(1-\tau^2)}}$$

Equations (7) and (8), when differentiated, give the expressions for the currents in the two circuits, and they are

(11) $$i_1 = A_1\,\omega' \sin(\omega' t+\phi) + B_1\,\omega'' \sin(\omega'' t+\psi)$$

(12) $$i_2 = A_2\,\omega' \sin(\omega' t+\phi) + B_2\,\omega'' \sin(\omega'' t+\psi)$$ or

(13) $$i_1 = I_1' \sin(\omega' t+\phi) + I_1'' \sin(\omega'' t+\psi)$$

(14) $$i_2 = I_2' \sin(\omega' t+\phi) + I_2'' \sin(\omega'' t+\psi)$$

Referring to expressions (9) and (10), it will be noted that ω'' is the smaller of the two angular velocities, and hence the length of the wave corresponding to ω'' is longer than the wave corresponding to the angular velocity ω'. In expressions (13) and (14) for the currents in the two circuits, there are four amplitudes: I_1' and I_2' are the amplitudes in circuits (1) and (2), respectively, of the *shorter* wave, and I_1'' and I_2'' are the amplitudes of the *longer* wave. The results which will first be derived are the four ratios of amplitude given below:

$$\frac{I_2'}{I_1'}\left[\frac{\text{Short in (2)}}{\text{Short in (1)}}\right] \qquad \frac{I_2''}{I_1''}\left[\frac{\text{Long in (2)}}{\text{Long in (1)}}\right]$$

$$\frac{I_1''}{I_1'}\left[\frac{\text{Long}}{\text{Short}}\text{ in (1)}\right] \qquad \frac{I_2''}{I_2'}\left[\frac{\text{Long}}{\text{Short}}\text{ in (2)}\right]$$

Derivation of Amplitude Ratios $\frac{I_2'}{I_1'}$ and $\frac{I_2''}{I_1''}$.

If expressions (7) and (8) be substituted in one of the original equations, say (1), there result the relations:

(15) $$A_1\left(\frac{1}{L_1C_1}-\omega'^2\right)=\frac{A_2\,M\,\omega'^2}{L_1}$$

(16) $$B_1\left(\frac{1}{L_1C_1}-\omega''^2\right)=\frac{B_2\,M\,\omega''^2}{L_1}$$ or

(17) $$\frac{A_2}{A_1}=-\frac{\omega'^2-\omega_1^2}{\tau\,\omega'^2}\sqrt{\frac{L_1}{L_2}}$$

(18) $$\frac{B_2}{B_1}=\frac{\omega_1^2-\omega''^2}{\tau\,\omega''^2}\sqrt{\frac{L_1}{L_2}}$$ and

(19) $$\frac{I_2'}{I_1'}=-\frac{\omega'^2-\omega_1^2}{\tau\,\omega'^2}\sqrt{\frac{L_1}{L_2}}$$ *short waves*

(20) $$\frac{I_2''}{I_1''}=\frac{\omega_1^2-\omega''^2}{\tau\,\omega''^2}\sqrt{\frac{L_1}{L_2}}$$ *long waves*

Since ω'^2 is always greater than ω_1^2, and ω''^2 always less than ω_1^2, it is evident that $\frac{I_2'}{I_1'}$ is always negative, while $\frac{I_2''}{I_1''}$ is always positive. This shows that the short waves in two coupled circuits are, neglecting resistance, always opposite in phase whereas the long waves are always in the same phase. In other words, the magnetic fluxes due to the short wave components of the currents in the two coils L_1 and L_2 of the oscillation transformer are opposite and give a small resultant flux, while the magnetic fluxes due to the long wave components are in the same direction giving a resultant flux equal to the sum of the two component fluxes.

For some purposes it is more convenient to express the ratios of amplitudes in terms of wave lengths instead of angular velocities as in (19) and (20). If λ' and λ'' are the wave lengths corresponding to ω' and ω'', respectively, then

(21) $\lambda' = \frac{2\pi V}{\omega'}$ and $\lambda'' = \frac{2\pi V}{\omega''}$, where V is the velocity of light. Similarly, if λ_1 and λ_2 are the wave lengths of the natural free vibrations of circuits (1) and (2), respectively, then

$$\lambda_1 = \frac{2\pi V}{\omega_1} \text{ and } \lambda_2 = \frac{2\pi V}{\omega_2} \tag{22}$$

Using relations (21) and (22), expressions (9) and (10) become

$$\lambda' = \sqrt{\frac{\lambda_1^2+\lambda_2^2-\sqrt{(\lambda_1^2-\lambda_2^2)^2+4\tau^2\lambda_1^2\lambda_2^2}}{2}} \tag{23}$$

$$\lambda'' = \sqrt{\frac{\lambda_1^2+\lambda_2^2+\sqrt{(\lambda_1^2-\lambda_2^2)^2+4\tau^2\lambda_1^2\lambda_2^2}}{2}} \quad \text{or} \tag{24}$$

$$\frac{\lambda'}{\lambda_1} = \sqrt{\frac{1+\left(\frac{\lambda_2}{\lambda_1}\right)^2-\sqrt{\left[1-\left(\frac{\lambda_2}{\lambda_1}\right)^2\right]^2+4\tau^2\left(\frac{\lambda_2}{\lambda_1}\right)^2}}{2}} \tag{25}$$

$$\frac{\lambda''}{\lambda_1} = \sqrt{\frac{1+\left(\frac{\lambda_2}{\lambda_1}\right)^2+\sqrt{\left[1-\left(\frac{\lambda_2}{\lambda_1}\right)^2\right]^2+4\tau^2\left(\frac{\lambda_2}{\lambda_1}\right)^2}}{2}} \tag{26}$$

Similarly the expressions (19) and (20) are transformed by use of relations (21) and (22) into

$$\frac{I_2'}{I_1'} = -\frac{1-\left(\frac{\lambda'}{\lambda_1}\right)^2}{\tau}\sqrt{\frac{L_1}{L_2}} \tag{27}$$

$$\frac{I_2''}{I_1''} = \frac{\left(\frac{\lambda''}{\lambda_1}\right)^2-1}{\tau}\sqrt{\frac{L_1}{L_2}} \tag{28}$$

Since the values of $\frac{\lambda'}{\lambda_1}$ and $\frac{\lambda''}{\lambda_1}$ are used in the calculation of (27) and (28), the equations (25) and (26) have been plotted for various values of τ and the results appear in Figure 2.

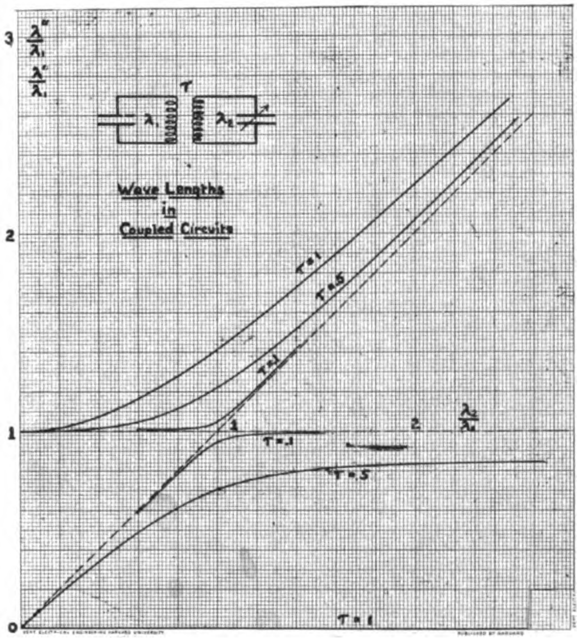


FIGURE 2

The abscissas are the values of the ratio $\frac{\lambda_2}{\lambda_1}$, and the ordinates are the corresponding values of $\frac{\lambda'}{\lambda_1}$ and $\frac{\lambda''}{\lambda_1}$. The coördinates of these curves are *ratios* of wave lengths and, therefore, the plots are applicable to any two circuits. It is simplest to think of λ_1 as remaining constant and that λ_2 is varied by changing C_2.

The relations (27) and (28) are plotted in Figure 3. Here again the coördinates are independent of the particular circuits and give the relation between the amplitudes of the same wave in the two circuits as the capacity in one circuit is changed. The abscissas are as before the values of the ratio $\frac{\lambda_2}{\lambda_1}$, and the

ordinates the values of the ratios $\frac{I_2'}{I_1'} \div \sqrt{\frac{L_1}{L_2}}$ and $\frac{I_2''}{I_1''} \div \sqrt{\frac{L_1}{L_2}}$. For any particular case it is only necessary to multiply the ordinates by $\sqrt{\frac{L_1}{L_2}}$.

Experimental Determinations of Ratios $\frac{I_2'}{I_1'}$ and $\frac{I_2''}{I_1''}$.

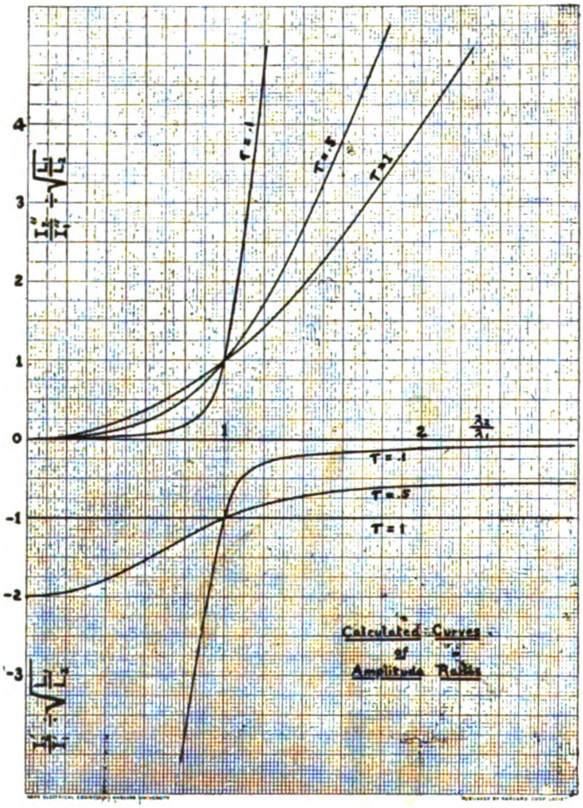


FIGURE 3

The ratios of amplitudes, as shown in Figure 3, of corresponding waves in the two oscillatory circuits can be easily obtained experimentally. Two very small coils of but a few turns each are set accurately with their planes at right angles. The coils of the particular apparatus used were each wound with six

turns in two layers of number 16 B. and S. wire,* forming a coil about an inch and an half (3.8 centimeters) in diameter. One of these coils were placed in each of the oscillatory circuits. A third larger coil (6 inches (15 centimeters) in diameter) of many turns, similar in shape to the coil of a tangent galvanometer, is mounted so that it can be rotated about its vertical diameter, this diameter being coincident with the common diameter of

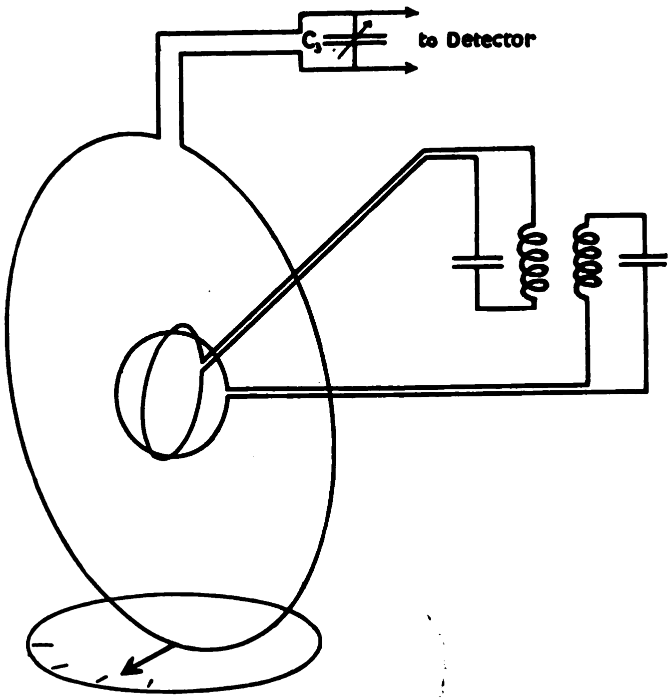


FIGURE 4

the two smaller coils. The latter are placed centrally inside the larger coil. The angle thru which the large coil is rotated can be accurately read on a scale. The arrangement is diagrammatically shown in Figure 4. The large coil is connected with a small variable condenser C_3 and to an audion detector.

Let a_1 be the direction and the maximum value of the flux due to a current in the small coil in circuit (1), and similarly, a_2 be the corresponding vector for circuit (2). R, the resultant of a_1 and a_2, is then the resultant flux if the fluxes have the same

* Diameter of number 16 B. and S. wire = 0.0508 inch = 0.129 centimeter.

period. If the plane of the movable coil is in the direction a_1 there will be no response of the detector connected to this coil due to currents in circuit (1). Similarly for the position parallel to a_2. When both coils are active, there will be zero response of the detector in the third circuit when the direction of the plane of the large coil is parallel to the resultant R, and if θ is the angle from position (1) then

$$tan\ \theta = \frac{a_2}{a_1} = \frac{I_2}{I_1}.$$

If circuits (1) and (2) are coupled with mutual inductance M, there will be two waves in each circuit. In order, therefore, to separate the two waves, the third circuit is resonated by means of C_3 to the particular wave, the amplitude of which is being measured. Either circuit (1) or (2) may be excited and in fact it will be found convenient to excite one circuit for some observations and the other circuit for others.

The curves of Figure 5 were obtained in the manner outlined above.

Derivation of Amplitude Ratios $\frac{I_1''}{I_1'}$, and $\frac{I_2''}{I_2'}$.

The ratio of the amplitudes of the two waves in each circuit depends upon which one of the two circuits is excited and upon the mode of excitation. There are two modes of excitation, namely: the condenser in the excited circuit may be charged and allowed to discharge, or the electromagnetic field associated with the inductance of this circuit may be suddenly released so that its energy goes to establishing oscillations in the circuit. In the first case the excited or primary circuit possesses potential energy at the start, while in the second case the circuit possesses kinetic energy when time equals zero. The methods used in radio telegraphy of exciting the primary circuit of the transmitting station is an example of the potential energy method of excitation, while the familiar method of the production of oscillations in a circuit by means of a buzzer illustrates the inductance excitation.

(a) **Condenser Excitation.** The relation between A_1 and B_1 and between A_2 and B_2 of expressions (11) and (12) can easily be obtained when oscillations are excited by the discharge of the primary condenser by imposing the initial conditions for this case. These initial conditions are

$$\text{when } t = 0 \begin{cases} q_1 = Q_o \\ q_2 = 0 \\ i_1 = 0 \\ i_2 = 0 \end{cases}$$

Supplying these values in (7) and (8) it follows directly that

$$Q_o = A_1 \cos\phi + B_1 \cos\psi \qquad 0 = A_1\,\omega' \sin\phi + B_1\,\omega'' \sin\psi$$

$$0 = A_2 \cos\phi + B_2 \cos\psi \qquad 0 = A_2\,\omega' \sin\phi + B_2\,\omega'' \sin\psi$$

It is apparent that $\phi = 0$, and $\psi = 0$.

The desired ratios of current amplitudes are, therefore,

$$\frac{I_1''}{I_1'} = \frac{\omega'^2 - \omega_1^2}{\omega_1^2 - \omega''^2} \cdot \frac{\omega''^3}{\omega'^3} = \frac{1 - \left(\frac{\lambda'}{\lambda}\right)^2}{\left(\frac{\lambda''}{\lambda}\right)^2 - 1} \cdot \frac{\left(\frac{\lambda'}{\lambda_1}\right)}{\left(\frac{\lambda''}{\lambda_1}\right)} \tag{29}$$

$$\frac{I_2''}{I_2'} = -\frac{\omega''}{\omega'} = -\frac{\left(\frac{\lambda'}{\lambda_1}\right)}{\left(\frac{\lambda''}{\lambda_1}\right)} \tag{30}$$

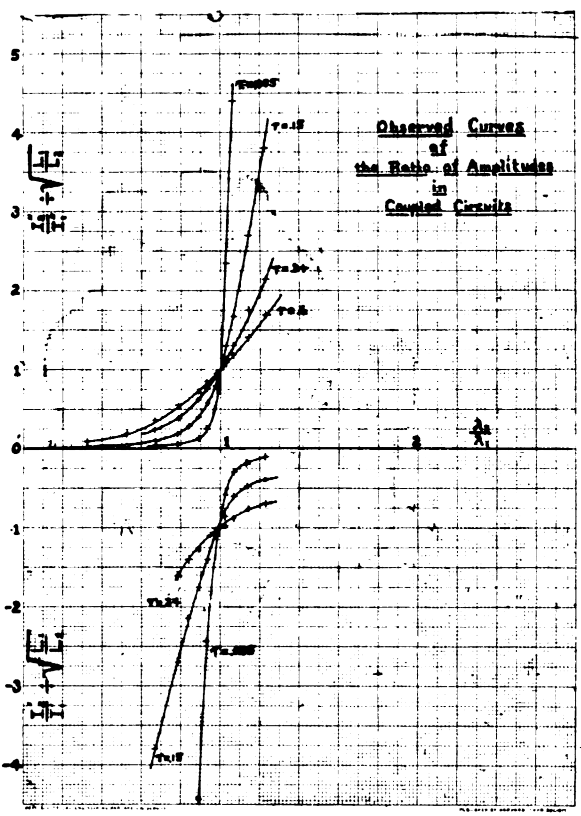


FIGURE 5

period. If the plane of the movable coil is in the direction a_1 there will be no response of the detector connected to this coil due to currents in circuit (1). Similarly for the position parallel to a_2. When both coils are active, there will be zero response of the detector in the third circuit when the direction of the plane of the large coil is parallel to the resultant R, and if θ is the angle from position (1) then

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$$Q_o = A_1 \cos\phi + B_1 \cos\psi \qquad\qquad 0 = A_1\,\omega' \sin\phi + B_1\,\omega'' \sin\psi$$

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It is apparent that $\phi = 0$, and $\psi = 0$.

The desired ratios of current amplitudes are, therefore,

$$(29) \qquad \frac{I_1''}{I_1'} = \frac{\omega'^2 - \omega_1^2}{\omega_1^2 - \omega''^2} \cdot \frac{\omega''^3}{\omega'^3} = \frac{1 - \left(\frac{\lambda'}{\lambda_1}\right)^2}{\left(\frac{\lambda''}{\lambda_1}\right)^2 - 1} \cdot \frac{\left(\frac{\lambda'}{\lambda_1}\right)}{\left(\frac{\lambda''}{\lambda_1}\right)}$$

$$(30) \qquad \frac{I_2''}{I_2'} = -\frac{\omega''}{\omega'} = -\frac{\left(\frac{\lambda'}{\lambda_1}\right)}{\left(\frac{\lambda''}{\lambda_1}\right)}$$

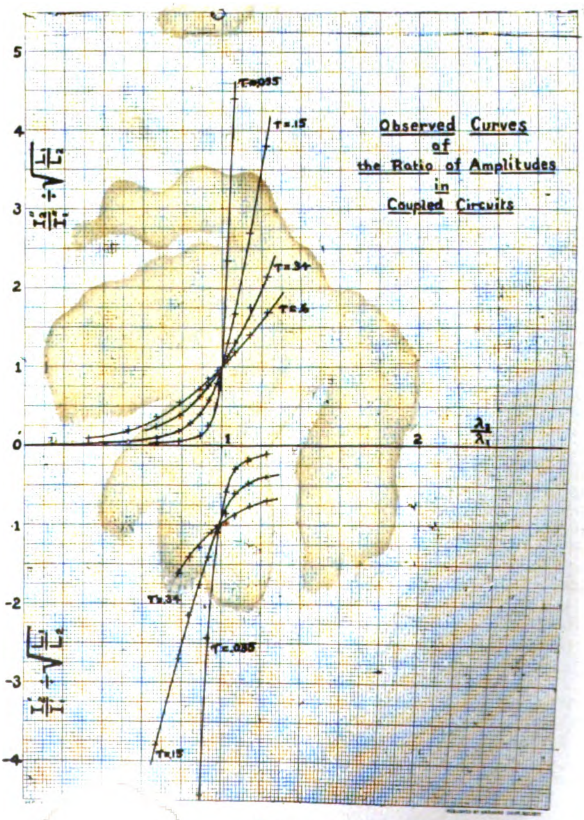

FIGURE 5

These conditions supplied in equations (7), (8), (11) and (12) give

$$\left.\begin{aligned} 0 &= A_1 \cos\phi + B_1 \cos\psi \\ 0 &= A_2 \cos\phi + B_2 \cos\psi \\ I_o &= A_1\,\omega' \sin\phi + B_1\,\omega'' \sin\psi \\ 0 &= A_2\,\omega' \sin\phi + B_2\,\omega'' \sin\psi \end{aligned}\right\}$$

In order to satisfy these equations $\phi = 90°$ and $\psi = 90°$.

The last two relations in conjunction with equations (17) and (18) give for the amplitude ratios

$$(41)\qquad \frac{I_1''}{I_1'} = \frac{\omega'^2 - \omega_1^2}{\omega_1^2 - \omega''^2} \cdot \frac{\omega''^2}{\omega'^2} = \frac{\lambda_1^2 - \lambda'^2}{\lambda''^2 - \lambda_1^2}$$

$$(42)\qquad \frac{I_2''}{I_2'} = -1$$

These expressions are plotted in Figure 7.

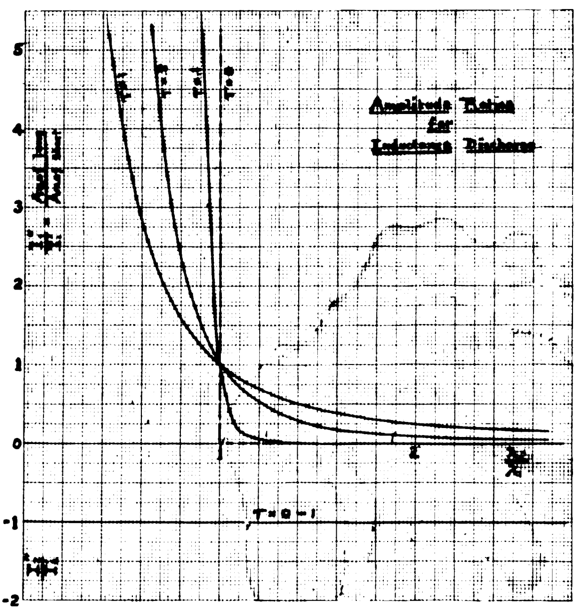

FIGURE 7

The expressions for the primary and secondary currents for this case are

$$(43)\quad i_1 = I_o \left\{ \frac{\left(\frac{\lambda''}{\lambda_1}\right)^2 - 1}{\left(\frac{\lambda''}{\lambda_1}\right)^2 - \left(\frac{\lambda'}{\lambda_1}\right)^2} \cos\frac{2\pi V}{\lambda'} t + \frac{1 - \left(\frac{\lambda'}{\lambda_1}\right)^2}{\left(\frac{\lambda''}{\lambda_1}\right)^2 - \left(\frac{\lambda'}{\lambda_1}\right)^2} \cos\frac{2\pi V}{\lambda''} t \right\}$$

$$(44)\qquad i_2 = -I_o\sqrt{\frac{L_1}{L_2}}\left\{\frac{\left[\left(\frac{\lambda''}{\lambda_1}\right)^2-1\right]\left[1-\left(\frac{\lambda'}{\lambda_1}\right)^2\right]}{\tau\left[\left(\frac{\lambda''}{\lambda_1}\right)^2-\left(\frac{\lambda'}{\lambda_1}\right)^2\right]}\cos\frac{2\pi V}{\lambda'}t - \frac{\left[\left(\frac{\lambda''}{\lambda_1}\right)^2-1\right]\left[1-\left(\frac{\lambda'}{\lambda_1}\right)^2\right]}{\tau\left[\left(\frac{\lambda''}{\lambda_1}\right)^2-\left(\frac{\lambda'}{\lambda_1}\right)^2\right]}\sin\frac{2\pi V}{\lambda''}t\right\}\quad \text{or}$$

$$(45)\qquad i_1 = I_o\left\{P_1'\cos\frac{2\pi V}{\lambda'}t + P_1''\cos\frac{2\pi V}{\lambda''}t\right\}$$

$$(46)\qquad i_2 = I_o\sqrt{\frac{L_1}{L_2}}\left\{P_2'\cos\frac{2\pi V}{\lambda'}t + P_2''\cos\frac{2\pi V}{\lambda''}t\right\}$$

$$(47)\qquad I_1' = I_o P_1' \qquad\qquad I_1'' = I_o P_1''$$

$$(48)\qquad I_2' = I_o\sqrt{\frac{L_1}{L_2}}\cdot P_2' \qquad\qquad I_2'' = I_o\sqrt{\frac{L_1}{L_2}}\cdot P_2''$$

The values of P_1', P_1'', P_2' and P_2'' are shown as broken-line curves in Figures 8, 9, 10 and 11.

Conclusion and Remarks

An examination of the results given lead to certain interesting conclusions.

Mention has already been made of the fact that the fluxes of the long waves in the two coils are together in phase while those of the short waves are opposite in phase. Looking again at expressions (27) and (28) it will be seen that at resonance $\frac{I_2'}{I_1'}\sqrt{\frac{L_2}{L_1}} = -1$ and $\frac{I_2''}{I_1''}\sqrt{\frac{L_2}{L_1}} = 1$, or *at resonance the magnetic fluxes of corresponding waves are equal in magnitude.*

If the coils L_1 and L_2 are placed coaxially, then the flux change due to the long wave is greatest along the axes of the coils, while the change of flux due to the short waves is greatest in a plane between the coils and is zero if the coupling is 1. This explains the reason why it is possible with a wave meter to find one of the two coupled waves when the wave meter coil is in one position while for the other wave another position of the wave meter coil will usually give louder response.

The amplitudes of the oscillations in both circuits alternately increase and decrease as the two waves are alike and opposite in phase, giving rise to the phenomenon of beats. The energy surges between the primary and secondary circuits so that

KEY TO FIGURES 8, 9, 10 AND 11

	SHORT WAVE	LONG WAVE
Condenser Excitation	$I_1' = \frac{2\pi V Q_o}{\lambda_1} \cdot K_1'$	$I_1'' = \frac{2\pi V Q_o}{\lambda_1} \cdot K_1''$
	$I_2' = \frac{2\pi V Q_o}{\lambda_1} \cdot \sqrt{\frac{L_1}{L_2}} \cdot K_2'$	$I_2'' = \frac{2\pi V Q_o}{\lambda_1} \cdot \sqrt{\frac{L_1}{L_2}} K_2''$

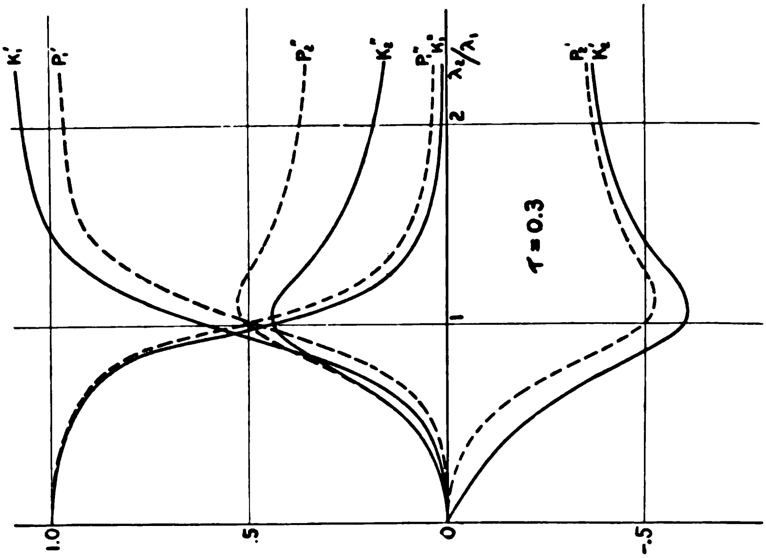

FIGURE 9

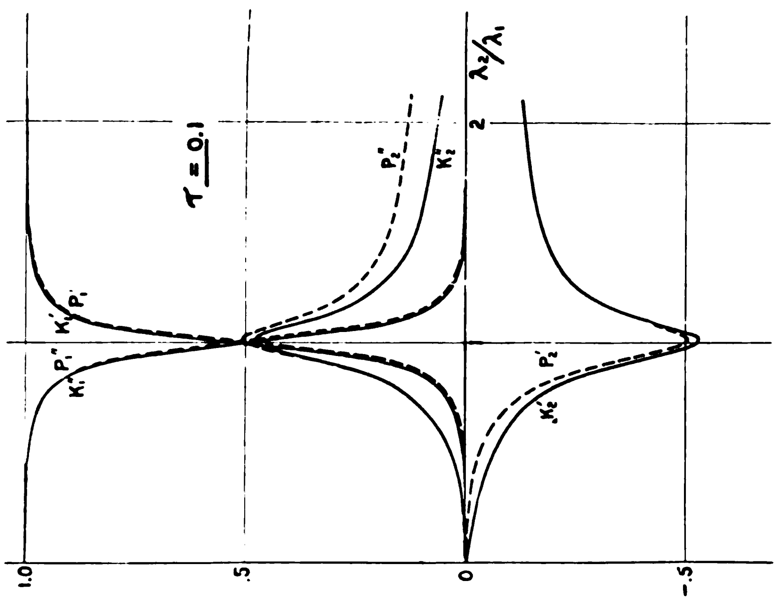

FIGURE 8

KEY TO FIGURES 8, 9, 10 AND 11 (*Continued*)

	SHORT WAVE	LONG WAVE
Inductance Excitation	$I_1' = I_o\,P_1'$	$I_1'' = I_o\,P_1''$
	$I_2' = I_o\sqrt{\dfrac{L_1}{L_2}}\cdot P_2'$	$I_2'' = I_o\sqrt{\dfrac{L_1}{L_2}}\cdot P_2''$

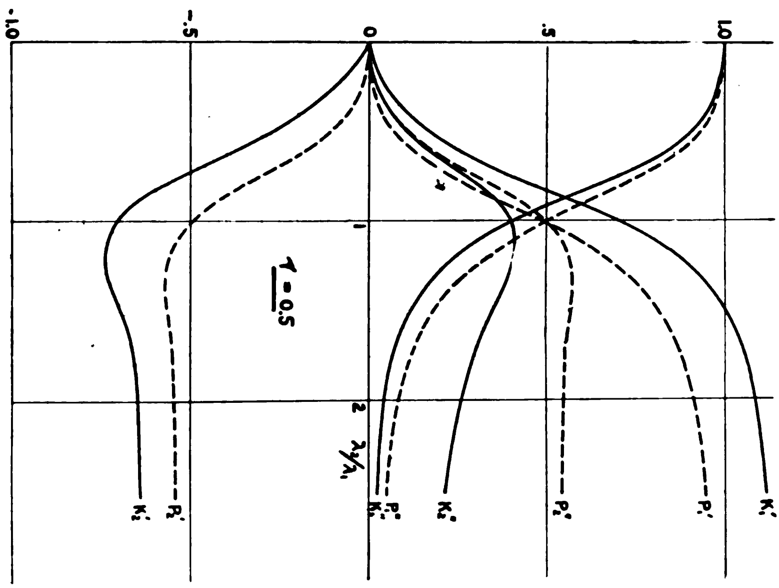

FIGURE 10

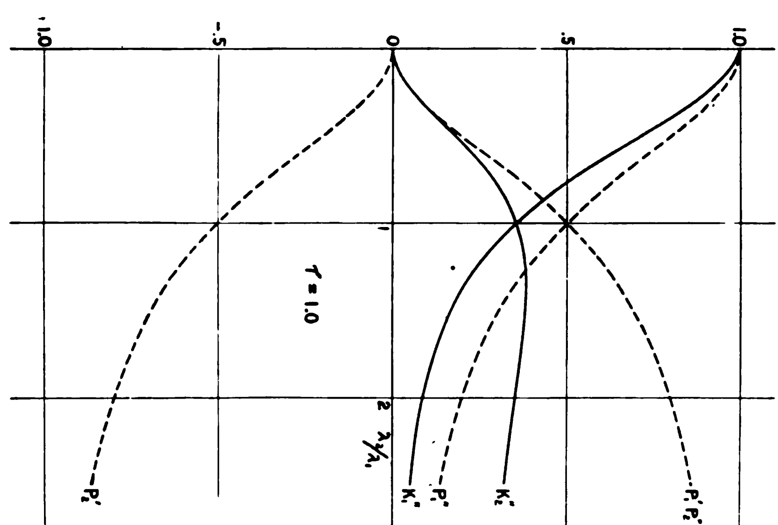

FIGURE 11

KEY TO FIGURES 8, 9, 10 AND 11

	SHORT WAVE	LONG WAVE
Condenser Excitation	$I_1' = \frac{2\pi V Q_o}{\lambda_1} \cdot K_1'$	$I_1'' = \frac{2\pi V Q_o}{\lambda_1} \cdot K_1''$
	$I_2' = \frac{2\pi V Q_o}{\lambda_1} \cdot \sqrt{\frac{L_1}{L_2}} \cdot K_2'$	$I_2'' = \frac{2\pi V Q_o}{\lambda_1} \cdot \sqrt{\frac{L_1}{L_2}} K_2''$

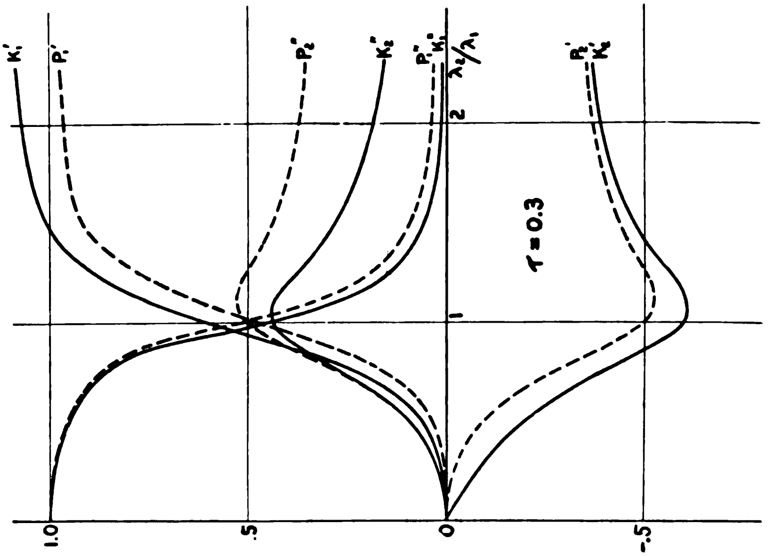

FIGURE 9

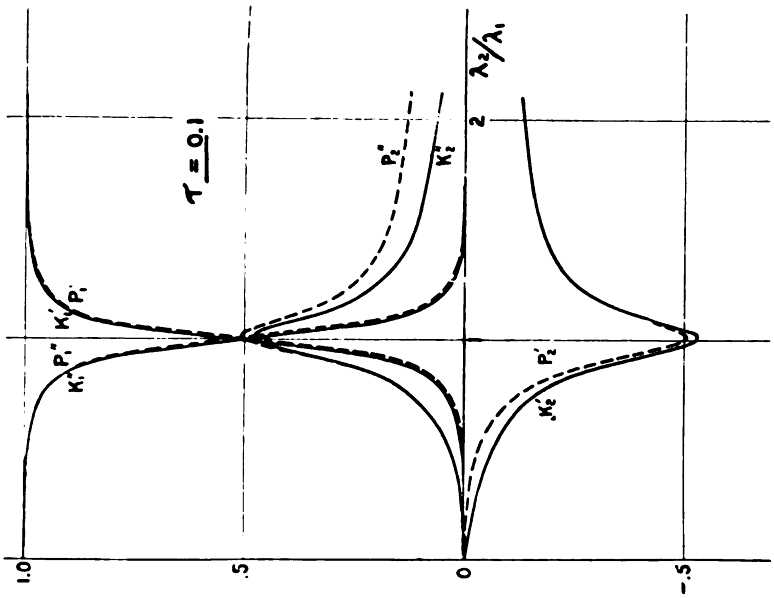

FIGURE 8

KEY TO FIGURES 8, 9, 10 AND 11 (*Continued*)

	SHORT WAVE	LONG WAVE
Inductance Excitation	$I_1' = I_o \, P_1'$	$I_1'' = I_o \, P_1''$
	$I_2' = I_o \sqrt{\frac{L_1}{L_2}} \cdot P_2'$	$I_2'' = I_o \sqrt{\frac{L_1}{L_2}} \cdot P_2''$

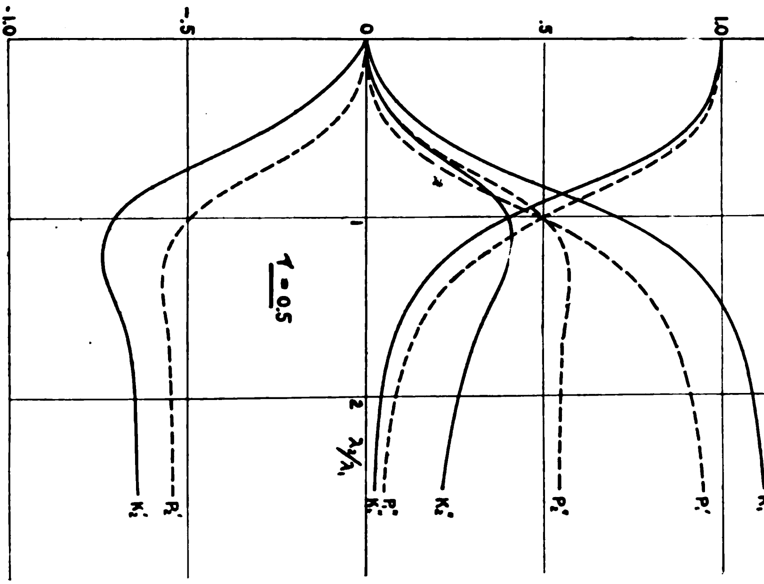

FIGURE 10

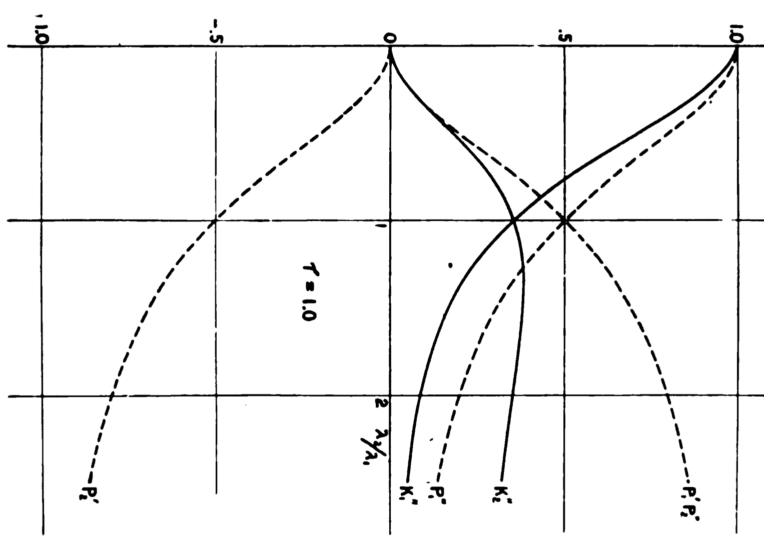

FIGURE 11

when the amplitude of the one circuit is greatest and equal to the sum of the component amplitudes, the amplitude of the current in the other circuit is a minimum and equal to the difference of the component amplitudes. The closer the coupling and the more $\frac{\lambda_2}{\lambda_1}$ differs from 1, the more rapid are these beats. It will be seen on examination of Figures 8, 9, 10 and 11, that when the primary is excited by a discharge of the magnetic field of the inductance, the secondary component amplitudes are *equal.* The amplitude of the secondary current, therefore, for this case periodically becomes zero, whereas, if the primary is excited by a condenser discharge the secondary amplitude never becomes zero because the short wave component always has a greater amplitude than that of the long wave. Similarly, the beating of the primary current, in the case of inductance excitation, causes the amplitude periodically to vary between 1 and zero, whereas for the condenser excitation the primary current amplitude changes between a maximum value *greater than* 1 and a minimum value *greater than zero.*

Furthermore, it is to be noted that the maximum amplitudes of the component oscillations in the secondary circuit and, therefore, the maximum amplitude of the current is obtained when $\frac{\lambda_2}{\lambda_1}$ is somewhat *greater* than 1, the value depending upon the coefficient of coupling. In other words *the maximum effect in the secondary is not obtained when the natural periods of the two circuits are the same.* If the expression for the amplitude of the secondary wave, in the case of inductance excitation, be differentiated with respect to $\frac{\lambda_2}{\lambda_1}$ to find what value of the latter ratio will give a maximum amplitude, the result is

$$(49) \qquad \left[\frac{\lambda_2}{\lambda_1}\right]_{max.} = \frac{1}{\sqrt{1-2\tau^2}} \Bigg|_{\tau=0}^{\tau=\frac{1}{\sqrt{2}}}$$

This expression is applicable between $\tau=0$ and $\tau=\frac{1}{\sqrt{2}}$. For values of τ above $\frac{1}{\sqrt{2}}=0.707$, the secondary amplitude has no maximum except at $\frac{\lambda_2}{\lambda_1}=\infty$. Equation (49) is plotted in Figure 12. The points on the curve show experimental verification of the above relation. A similar relation could be deduced

for the condenser excitation but it is evident from the curves of Figures 8, 9, 10 and 11 that the value of $\left[\frac{\lambda_2}{\lambda_1}\right]_{max.}$ for this case would be always less than the corresponding value for the inductance excitation.

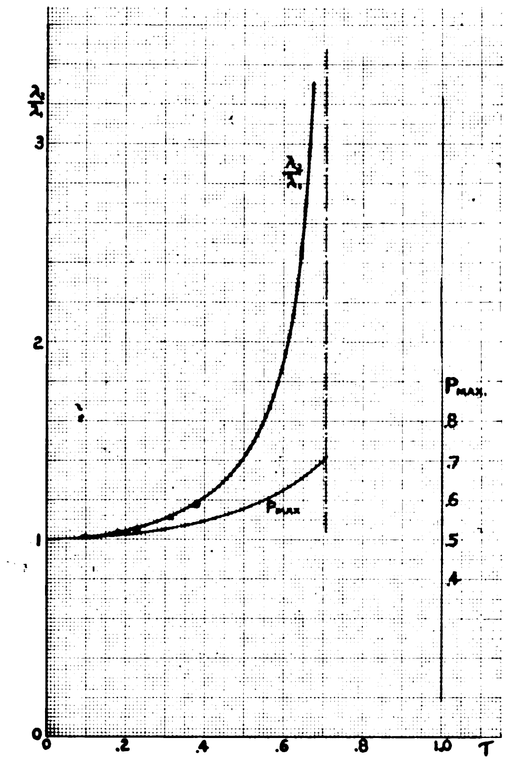

FIGURE 12

The above consideration is very important in the use of a wave meter. If the coupling between the wave meter and the circuit, the wave length of oscillations of which is being measured, is appreciable, the setting of the wave meter will be in error. A coupling of 0.1 will cause over 1 per cent error in the determination.

If (49) be substituted in the expression for the amplitude in

the case of inductance excitation, the maximum value of the amplitude will be given and is

$$I_2''_{max.} = I_2'_{max.} = I_o\sqrt{\frac{L_1}{L_2}} \cdot \left. \frac{1}{2\sqrt{1-\tau^2}} \right|_{\tau=0}^{\tau=0.707} = I_o\sqrt{\frac{L_1}{L_2}} \cdot P_{max.}$$

The value of $P_{max.}$ is plotted in Figure 12.

If Figures 8, 9, 10 and 11 be once more examined it will be seen that the secondary component amplitudes rise to a maximum much more sharply for values of $\frac{\lambda_2}{\lambda_1}$ less than 1 than for values greater than 1, where $\frac{\lambda_2}{\lambda_1}$ is plotted on a uniform scale. If the secondary circuit (for instance a wave meter) be tuned by varying its capacity, the change of $\frac{\lambda_2}{\lambda_1}$ is not proportional to the angle thru which the variable capacity is rotated but increases less rapidly as the capacity becomes greater. This would even more accentuate the difference in steepness, with reference to variations in C_2, of the amplitude curve on the two sides of the maximum. It is, therefore, evident why, as the capacity of a wave meter or similar circuit coupled with an oscillating circuit is increased thru the resonant value, the current in the wave meter rises very rapidly as resonance is approached and then decreases very gradually as the capacity passes its resonant value.

Cruft Laboratory, Harvard University.

SUMMARY: An historical survey of previous work on coupled circuits is made.

There are formed, in general, in coupled circuits two waves in both primary and secondary circuits. The amplitude ratios of the longer two of these waves and of the shorter two of these waves are calculated. Neglecting resistance, the shorter waves are opposite in phase, the longer waves in phase.

A simple experimental method for rapidly verifying the theoretical relations is described.

The ratios of the amplitude of the longer to the shorter primary wave and of the longer to the shorter secondary wave are theoretically determined for both condenser excitation and inductance excitation.

It is shown that at resonance the magnetic fluxes of corresponding long or corresponding short waves are equal numerically. It also appears that maximum secondary effect is not obtained for equal natural periods of the circuits.

The extent to which wave meter close coupling affects accuracy of wave length determination is considered. An explanation of dissymmetry of resonance curves about the resonance point is also given.

All relations deduced are graphically illustrated.

DISCUSSION

H. G. Cordes (communicated): It may be of interest to supplement Dr. Chaffee's paper by discussing the amplitude relations in a direct coupled circuit. A different notation will be used, but the results can be readily compared with those given.

Figure 1 represents a direct coupled circuit in which there is no mutual inductance between L_1 and L_o.

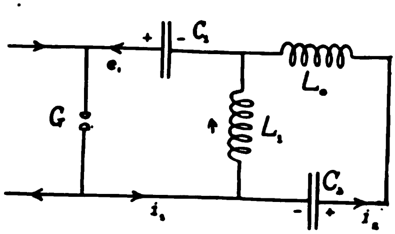

FIGURE 1

In the figure L_1, C_1, G represents the primary and L_o, L_1, C_2 represents the secondary oscillating current circuit. The initial ($t=0$) condition is represented by current I flowing into condenser C_1, an initial charge, e_a ($=e_1$) and e_p ($=e_2$), on condensers C_1 and C_2 respectively, and an initial current, i_p ($=i_2$) in the secondary circuit. The condensers when charged as indicated will be considered positively charged, and current in the direction of the arrows will be considered positive.

Let $L_1 = \rho L_o$

" $C_1 = a C_2$

" $v = \dfrac{a\rho+\rho+1}{2a\rho}$

" $u = \dfrac{\sqrt{(a\rho+\rho+1)^2-4a\rho}}{2a\rho}$

" $r = \sqrt{v+u}$

" $s = \sqrt{v-u}$

" $\omega = \dfrac{1}{\sqrt{L_o C_2}}$

" $\omega_o = \dfrac{1}{\sqrt{(L_1+L_o)C_2}}$

Referring to equation (14) of Mr. Chaffee's paper

Let $I_2' = \sqrt{A^2+B^2}$, $I_2'' = \sqrt{C^2+D^2}$, $\omega' = \omega r$, $\omega'' = \omega s$, $\phi = tan^{-1}\frac{B}{A}$, $\psi = tan^{-1}\frac{D}{C}$.

These conditions are satisfied when

$$A = \frac{C_2\,\omega_o\sqrt{1+\rho}}{2u\cdot 10^6}\left[\frac{a\,(1-s^2)+1}{a\,r}\cdot e_p + r\,e_a\right]$$

$$B = \frac{a\,(1-s^2)\,i_p - I}{2\,u\,a}$$

$$C = \frac{C_2\,\omega_o\sqrt{1+\rho}}{2u\cdot 10^6}\left[\frac{a\,(r^2-1)-1}{a\,s}\cdot e_p - s\,e_a\right]$$

$$D = \frac{a\,(r^2-1)\,i_p + I}{2\,u\,a}$$

If C_2 is expressed in microfarads, e_a and e_p in volts, and i_p and I in amperes, i_2 will be expressed in amperes.

To obtain the equation for i_1 from i_2, substitute the following in equation (13)

$$I_1' = a\,(r^2-1)\,I_2' \quad \text{and} \quad I_1'' = a\,(s^2-1)\,I_2''$$

Certain relations of s, r, a and ρ may be noted.

When $\quad a\,\rho = 1,$

then $\quad s\,r = 1.$

$$\frac{a\,(r^2-1)-1}{a\,s} = \frac{a\,(1-s^2)+1}{a\,r}$$

$$a\,(1-s^2)+1 = a\,(r^2-1)$$

$$s\,(r^2-1) = r\,(1-s^2)$$

In all cases $r\,s = \dfrac{1}{\sqrt{a\,\rho}}$.

and $\omega = \omega_o\sqrt{\rho+1}$.

For resonance $\quad r = 1$

$$\rho\,a = \rho+1$$

$$u = \sqrt{\frac{1}{a}}$$

$$r = \sqrt{1+u}$$

$$s = \sqrt{1-u}$$

$$A = \frac{C_2 \omega_o r \sqrt{\rho+1}}{10^6 \cdot 2} \left[e_p + \sqrt{a}\, e_a \right]$$

$$B = \frac{1}{2} (i_p - u I)$$

$$C = \frac{C_2 \omega_o s \sqrt{\rho+1}}{10^6 \cdot 2} (e_p - \sqrt{a}\, e_a)$$

$$D = \frac{1}{2} (i_p + u I)$$

Let (27a), (28a), etc., be the equations of the ratios of amplitudes corresponding to equations (27), (28), etc., of Mr. Chaffee's paper.

$$(27a) \qquad \frac{I_2'}{I_1'} = \frac{1}{a\,(r^2-1)}$$

$$(28a) \qquad \frac{I_2''}{I_1''} = \frac{1}{a\,(s^2-1)}$$

which is negative since s is less than unity.

$$(29a) \qquad \frac{I_1''}{I_1'} = \frac{s\,(1-s^2)}{r\,(r^2-1)}$$

$$(30a) \qquad \frac{I_2''}{I_2'} = -\frac{s}{r}$$

For inductance excitation consider all initial conditions zero except I, which is flowing when the spark gap, G, is short-circuited.

$$(41a) \qquad \frac{I_1''}{I_1'} = \frac{1-s^2}{r^2-1}$$

$$(42a) \qquad \frac{I_2''}{I_2'} = -1$$

For resonance $\quad a\,(r^2-1) = a\,(1-s^2) = \sqrt{a}$

" " $\quad r^2 - 1 = 1 - s^2 = u$

In comparing the phase relations in inductively coupled circuits with those of conductively coupled circuits it is evident that when there is mutual inductance between L_1 and L_o, the effects due to the mutual inductance are in phase with the effect due to the conductive coupling.

From the conclusion reached in the case of inductively coupled circuits it would appear that a circuit coupled to L_1 would give a loud response to the long waves and a feeble response to the short waves, since the current in this coil is the resultant of the primary and secondary currents.

The curves shown were plotted for condenser excitation where $a=5$, $\rho=0.2$, and $\omega_o=2\pi\times10^5$. Curve "R" is plotted for $\omega_o t=0°$ to $400°$, and is for reference only. Curve "P" is the primary current, while curve "S" is the secondary current

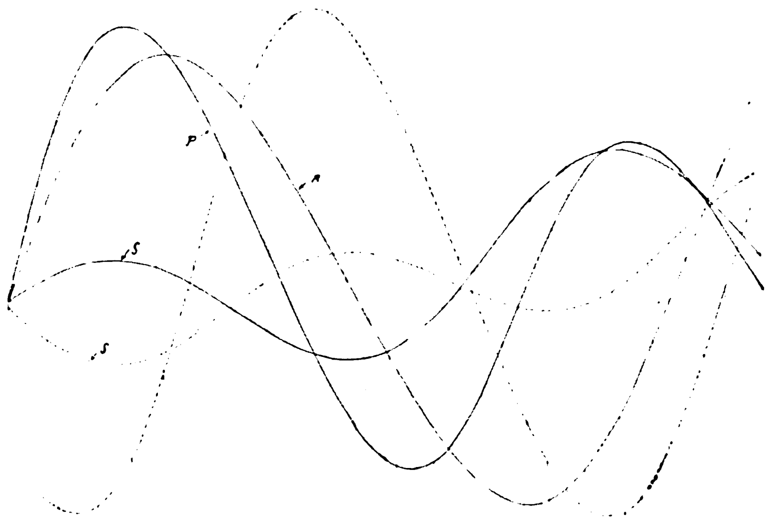

FIGURE 2

for the interval $\omega_o t=0°$ to $400°$. Curves P' and S' are the primary and secondary currents respectively during the interval $\omega_o t=2{,}000°$ to $2{,}400°$.

Inspection of these curves shows how the resultant frequency is continually changing. Curves P' and S' show the phase relations of the resultant current at a point of re-transfer of energy.

SUSTAINED WAVE RECEIVING DATA *

By

LEONARD F. FULLER

(CHIEF ELECTRICAL ENGINEER, FEDERAL TELEGRAPH COMPANY.)

On September 1, 1915, the steamer "Ventura" of the Oceanic Steamship Company left San Francisco for Sidney, Australia. Installed on the ship was a 5-kilowatt Federal-Poulsen arc set. At San Francisco a 30-kilowatt set was used. The antenna current was 50 to 60 amperes, and the wave length 8,000 meters. The following reception was accomplished on the "Ventura":

A distance of 3,830 miles (6,150 km.) from San Francisco, the signals could be copied on the typewriter, in September, 1915, by daylight.

At a distance of 4,200 miles (6,750 km.), the messages could be copied by pencil in daylight thru heavy strays.

At a distance of 5,140 miles (8,260 km.), the messages could be copied by pencil in daylight thru light strays.

In the early evening in September, 1915, the ship being on a course between Hawaii and Samoa, the signals from Tuckerton, N. J. were copied on the typewriter in the early evening. The "Ventura" was then 3,840 miles (6,180 km.) from San Francisco.

Evening signals in September, 1915, from Tuckerton were copied by pencil on the "Ventura" when 530 miles (850 km.) southwest of Samoa, 5,320 miles (8,550 km.) from San Francisco, and approximately 8,000 miles (13,000 km.) from Tuckerton.

This reception from Tuckerton was often duplicated. Tuckerton used a 60-kilowatt arc set and an antenna current of 100 to 120 amperes. The signals from the Tuckerton alternator were also received when 3,840 miles (6,180 km.) from San Francisco.

In May, 1915, the steamship "Sierra," 1,700 miles (2,600 km.) west of San Francisco, copied messages from Nauen, Germany, by pencil, the total distance being approximately 8,600 miles (14,000 km.).

In December, 1914, the South San Francisco station copied

* Presented before The Institute of Radio Engineers, Washington Section, December 29, 1915.

by pencil in daylight the Nauen, Germany, signals at a distance of approximately 7,000 miles (11,000 km.).

Eilvese, Germany, has also been heard 1,700 miles (2,700 km.) west of San Francisco on board ship and at night at the Honolulu station of the Federal Telegraph Company.

All the above data relative to the reception of messages from South San Francisco and Tuckerton have been frequently duplicated, and are not to be classified as "freak" work. The signals from Nauen were not duplicated frequently, and are to be classified as more or less "freakish" or erratic.

SUMMARY: The shipboard reception by daylight of sustained wave signals from a 30 kilowatt arc at distances of the order of 4,000 miles (7,000 km.) and from a 60 kilowatt arc at distances of approximately 7,000 miles (11,000 km.) in the evening are instanced.

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PROCEEDINGS

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ALFRED N. GOLDSMITH, Ph.D.

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On page 268 of the June, 1916, "PROCEEDINGS," in heading of third column of the numerical table, for

"Audio Current"

read

"Audibility/Current."

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RECENT STANDARD RADIO SETS*

By

HARRY SHOEMAKER

(RESEARCH ENGINEER, MARCONI WIRELESS TELEGRAPH COMPANY OF AMERICA)

The standard sets to be described are made in two sizes; namely, the 2-kilowatt, 500-cycle set, known as type "P-4," and the ½-kilowatt, 500-cycle set, known as type "P-5." As the ½-kilowatt size is practically the same as the 2-kilowatt size, except in detail, I will confine my description at present to the 2-kilowatt set and later on point out the slight differences. These set are made at Aldene, New Jersey, by the Marconi Wireless Telegraph Company of America.

In designing the set, our first consideration was given to meeting all government requirements and next to our own commercial requirements. As these sets were to be used on shipboard, it was necessary to construct them so as to occupy a minimum space—especially floor space. After careful consideration, the panel type of equipment was decided on and the sets constructed accordingly.

As these sets were required to operate on a current furnished by the ship's dynamo, it was necessary to use a motor generator which would successfully operate on a direct current having varying voltages over wide limits, and deliver a 500-cycle, single phase current. As this motor generator would necessarily be of considerable size and weight, we found it necessary to make provision for mounting it on the floor of the operating room.

All the other appliances, with the exception of the transformer and starting resistance, we were able to mount either directly on the front of the panel or on the back of the panel. In this way we were able to get a set which would occupy a minimum floor space.

All switches and handles necessary for the control and manipulation of the set are mounted on the front of the panel so as to be readily accessible to the operator. All the radio frequency and high potential circuits and appliances are mounted

* Presented before The Institute of Radio Engineers, New York, March 1, 1916.

on the back of the panel and supported by insulators. The panels are made of "bakelite dilecto" which has been found to be the most suitable material for this purpose on account of its great strength and good insulating properties. The insulators or rods which support the radio frequency circuits and appliances are also made of this material. All wood-work and inflammable material has been entirely eliminated, where possible.

As the circuits used in these sets are so well known, I will not attempt to describe them in any great detail but will endeavor to confine such description to the novel points. The radio frequency circuits consist of a closed oscillating circuit, and an open oscillating or radiating circuit which are coupled inductively.

The closed oscillating circuit consists of an inductance, a condenser, and a spark gap which are connected in series. In these sets, two types of spark gaps are provided either of which can be used as desired. A double throw double pole switch is provided so that either spark gap can be inserted in the circuit. One type of spark gap used is known as the quenched gap. The other type is known as the rotary synchronous gap. In practice, the quenched gap is used almost entirely and the rotary gap merely serves as a spare.

The condenser used consists of six Leyden jars connected in multiple, each having a capacity of 0.002 μf. and a total capacity of 0.012 μf. As it is desired to transmit three different wave lengths; namely, 300, 450, and 600 meters, provision is made for connecting in the proper inductance to produce these wave lengths. The inductance of the circuit consists of a copper strip wound in a spiral and mounted on a bakelite dilecto plate. Taps at the proper points are provided which, in conjunction with a switch known as the primary wave changing switch, enables the wave length or natural period of this circuit to be readily changed. As the capacity used is of too great a value to reach the 300-meter wave length and still maintain sufficient inductance in the primary coil for coupling purposes, the wave-changing switch is so constructed that the capacity used on the 300-meter wave length is reduced to 0.006 μf. This requires only three jars and, consequently, the power used at this wave length is reduced to one-half or one kilowatt.

The secondary circuit consists of the aerial, the aerial inductance which is variable, the loading inductance, the wave-changing switch, the secondary, and the heating element of the

radiation meter. These parts are all connected in series. A series condenser known as the short wave condenser is provided for the purpose of shortening the aerial circuit when its natural period is too high to operate on the 300-meter wave length.

The coupling or inductive relation between these two circuits can be varied by varying the distance between the primary and secondary—the primary being movable with respect to the secondary.

On the front of the panel, commencing at the top, are mounted the following instruments and appliances:

A radiation meter, a wattmeter, a motor field rheostat, a generator field rheostat, a handle for varying the aerial inductance, a handle for operating the wave changing switch, a switch for operating the set on low power, a handle for varying the coupling between the closed oscillating circuit and open oscillating circuit, and two bushings thru which the leads to the quenched gap are brought. These are all mounted on the upper section of the panel. The quenched spark gap is mounted on a separate panel which is mounted on the frame-work by means of hinges so that the panel can be opened from either side. This permits the replacement or inspection of the condenser jars which are mounted directly behind this panel. On the lower panel are mounted the automatic starter, together with the D. C. line switch, generator field switch, and A. C. line switch. An over-load release relay is also mounted on this panel which protects the D. C. circuit from over-loads. A row of studs mounted at the lower edge of this panel permits all connections from the panel to the motor generator and external circuits to be made. On the back of the panel, commencing at the top, are the following appliances:

A variable inductance, known as the tuning inductance, a loading coil, the secondary wave changing switch, the primary wave changing switch, a heating element for the radiation meter, the secondary coil, the primary coil, the condenser jars, the change-over switch and an air duct.

The motor generator, the transformer and starting resistance units are mounted on the floor of the operating room directly back of the panel.

The closed oscillating circuit is adjusted for the three wave lengths at the factory and the open or radiating circuit is adjusted when the apparatus is installed and connected to the aerial. The wave changing switches of both circuits are controlled by a single handle so that both circuits are changed

simultaneously. This not only changes the wave length of each circuit but also varies the coupling between the two circuits by the proper amount so that it is not necessary to re-tune the two circuits when changing the wave length. This is accomplished by varying the amount of inductance in the secondary while maintaining the total inductance of the radiating circuit constant. This latter method is the one used when operating with the quenched spark gap.

The radiation meter used in this set is of a comparatively new design and is constructed as follows. A number of thin strips constructed of a platinum silver alloy are connected between two terminals which are inserted in series with the aerial circuit. These strips are heated by the aerial current. They are of such thickness and so disposed that they have practically a constant resistance for any frequency. The number of these strips used depends on the amount of current which the instrument has to measure. A thermo junction, consisting of two small wires of different materials, is soldered to the center of one of these strips. The other two ends of the thermo junction are connected to two terminals which have practically the temperature of the surrounding air. These terminals are in turn connected to a very sensitive indicating instrument of the D'Arsonval type. When the strips are heated by the antenna current, the thermo junction is heated and current flows thru the indicating instrument. The deflection of the indicating instrument will be proportional to the square of the current flowing thru the heating strip. The thermo junction and heating strips are made in a separate unit which can be mounted in any convenient place. Connection between the thermo junction and the indicating instrument is made by means of lead covered wire with the covering properly grounded. In the later type of instrument the indicating instrument has the heating strip and thermo junction mounted inside its case.

The condenser of the primary circuit has its terminals connected to the secondary terminals of the transformer. The primary of this transformer is connected to the terminals of the generator thru the necessary controlling appliances.

This transformer is of the closed core, non-leakage type. The coils are placed inside an iron case and the terminals are brought out thru the top of this case. All the windings are immersed in an oil which is solid at ordinary temperatures. A safety gap is provided to protect the secondary against excessive potentials. This transformer is designed to operate with

a total capacity of 0.012 giving one discharge per half cycle, or 1,000 discharges per second. When operating on the 300-meter wave length and with a capacity of 0.006 μf., it is necessary to use a reactance in series with the generator and transformer primary. This reactance is of such value that two discharges per cycle are obtained. When operated on other wave lengths, this reactance is automatically short-circuited by the wave changing switch.

Figure 1 is a front view of the set complete, Figure 2 is a side view and Figure 3 is a back view. The different parts are numbered and are as follows:

FIGURE 1

1 is the radiation meter which measures the antenna current. This instrument is of the D'Arsonval type and is operated by means of a thermo junction which is heated by the aerial current.

2 is a wattmeter which measures the A. C. watts at the terminals of the transformer primary.

3 is the generator field rheostat which permits variation of the generator voltage.

4 is the motor field rheostat which permits speed variation of the motor generator.

5 is a scale which indicates the number of turns in the variable tuning inductance.

6 is the handle for varying this inductance.

7 is a handle which permits the transmitted wave lengths to be changed and at the same time indicates this wave length.

8 is the low power switch which cuts in sufficient resistance to operate the set on a sufficiently low power to reduce interference to a minimum. This reduces the power of the transmitted wave to approximately 10 watts.

9 is a scale which indicates the proper coupling for the different wave lengths when using the rotary gap. This coupling is not varied when operated on the quenched gap as the switch controlled by handle 7 takes care of the coupling adjustments as well as the wave length adjustments.

10 is a handle which permits the varying of the coupling between the closed oscillating circuit and the open oscillating circuit.

11 and 12 are flexible leads which connect the quenched gap 13 in the closed oscillating circuit.

14 and 15 are hinges which permit the panel carrying the quenched gap to be swung open from either side.

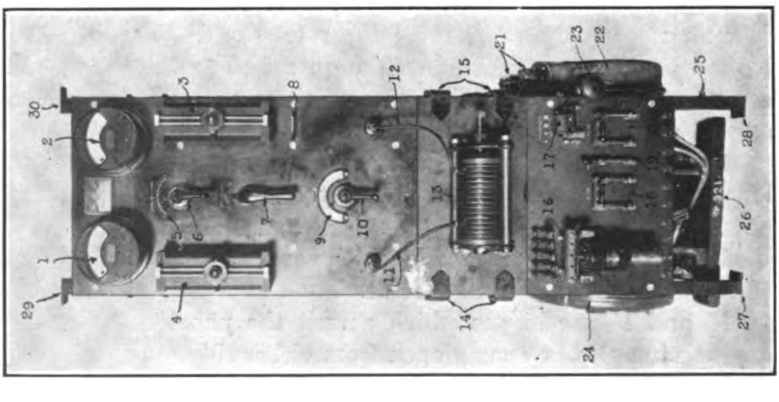


FIGURE 1

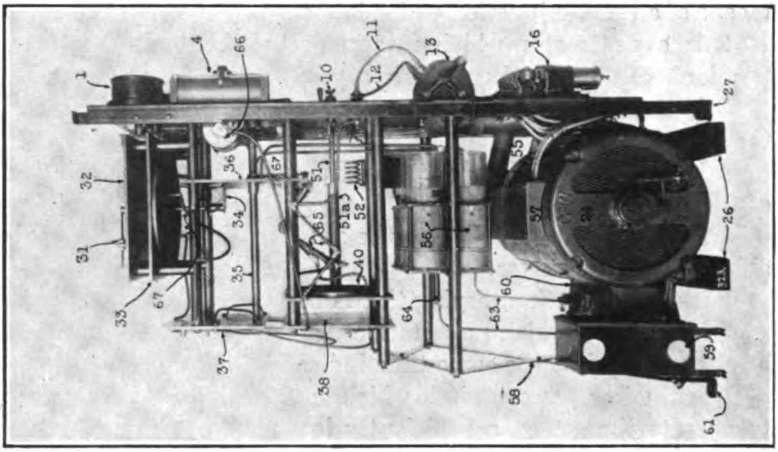


FIGURE 2

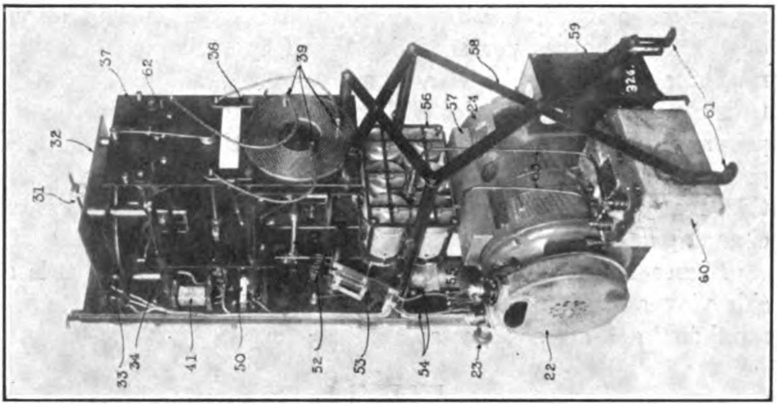


FIGURE 3

16 is the automatic starter movement which permits the motor generator to be started and stopped by means of a single pole switch situated on the operator's table. This movement also serves to open and close the D. C. circuit, cut out the starting resistance in proper order, and connect in the generator field after all starting resistance has been cut out. This last mentioned function prevents the operator from transmitting until the motor generator is up to full speed.

17 is an overload relay which is so constructed that when the set is overloaded the starting movement is released which, in turn, opens the motor circuit. This circuit remains open until the control switch is opened and again closed, in which case the motor will again start unless there is a permanent short circuit on the line.

18 is the D. C. line switch which permits opening or closing the D. C. line.

19 is a generator field switch which permits opening or closing the D. C. generator field.

20 is the A. C. line switch which completely connects or disconnects the A. C. generator from the transformer primary.

21 are the rotary spark gap terminals.

22 is the rotary spark gap casing.

23 is a handle which permits the rotation of the spark gap casing and terminals for phase adjustment.

24 is the motor generator.

25 is the ground terminal which is directly connected to the panel frame.

26 is one of the skids on which the motor generator is mounted. These skids are secured to the floor of the operating room by lag screws.

27 and 28 are the lower ends of the panel frame which permits the panel to be secured to the floor by means of lag screws.

29 and 30 are extension angles which slide in the panel frame and which permit the panel frame to be secured to the ceiling of the operating room. The height of these angles can be adjusted to suit the height of the operating room.

FIGURES 2 AND 3

31 is a movable contact which is controlled by the handle 6 and permits the variation of inductance in the aerial or opened oscillation circuit.

32 is the antenna inductance which is of spiral form and mounted on a plate of bakelite dilecto.

33 is a spiral inductance mounted on a plate of bakelite dilecto and is called the loading coil. This inductance is in series with inductance 32 and is so located that connection can be made to any part of it by means of flexible connections and clips.

34 is a bevel gear which gears the handle 6 to the movable contact 31 so as to permit its rotation.

35 is an insulating rod which transmits the motion of the handle 7 to the arm 62 of the wave length switch 37. This also transmits the motion of the handle 7 to the arm 67 of the primary wave length changing switch.

36 is the primary wave length changing switch.

38 is the secondary coil or inductance and consists of a spiral mounted on a bakelite dilecto plate. Connection can be made to any portion of this inductance by means of the clips and flexible leads 39.

40 is the primary coil or inductance which consists of a spiral mounted on a bakelite dilecto plate. This plate is movable with respect to the secondary coil by means of the screw 51 and handle 10. This movement permits the variation of coupling between the primary and secondary. Connection is made between this movable coil and the switch 36 by means of the links 65.

51–a is a bakelite dilecto tube rigidly connected to the plate carrying the primary 40, the other end having a collar adapted to engage the screw 51.

41 is an impedance coil which is automatically connected in series with the primary of the transformer when the handle 7 is set for the 300 meter wave length.

50 is a series resistance which is thrown in series with the generator field when switch 8 is open for the purpose of operating on low power.

52 is a compensating inductance which is connected in series in the closed oscillating circuit when the quenched gap is being used. This inductance compensates for the inductance in the leads 54 when the rotary spark gap is being used.

53 is a change-over switch which permits cutting out the quenched spark gap and connecting in the rotary spark gap in the closed oscillating circuit.

55 is the air duct which carries the air from the combined rotary gap and blower to the quenched gap.

56 is the condenser of the closed oscillating circuit which consists of 6 jars connected in multiple, each having a capacity

of 0.002 μf. making a total capacity of 0.012 μf. These jars are mounted in a cast iron rack which, in turn, is supported by 2 bakelite dilecto rods.

57 is a cast iron cover which covers the terminals of the motor generator and the protective condensers used for protecting the low potential circuit from the radio frequency potentials.

58 is a support which supports the rods carrying the radio frequency appliances.

59 is the starting resistance unit which is mounted on the floor.

60 is the transformer which transforms the generator voltage from 140 volts to approximately 12,000 volts.

63 are the secondary leads which connect the secondary of the transformer 60 to the terminals of the condenser 56.

61 are adjustable feet for securing support 58 to the floor.

Figure 4 is a diagram of connections of complete set and shows the relative positions of the different parts of the apparatus. In this drawing all the different parts are so named that the reader will readily understand it.

The motor generator of this set was especially designed to operate under our service conditions which, in most cases, require a motor which will run at practically constant speed with a voltage variation of from 95 volts to 120 volts. This was accomplished by making the motor of such size that it could be operated with the field below saturation when operating on the highest voltage. The motor is also differentially compounded to insure constant speed with varying load. In practice this motor showed a maximum speed variation of 5 per cent. with a variation of voltage from 95 to 120. The variation of speed from no load to full load was within 5 per cent. The speed of the motor can also be controlled over a limited range by means of a field rheostat. This field rheostat enables the operator to get the proper speed for best tone. The generator is of the wound armature type having an open circuit voltage of approximately 350 and a load voltage of 140. The synchronous impedance of this machine was found to be approximately 17 ohms and inductance 15 ohms. The voltage of this generator is controlled within working range by a field rheostat. This field rheostat enables the operator to adjust voltage for best tone. The windings of both motor and generator are protected from the radio frequency induction by means of condensers. Each terminal of a winding is grounded thru a condenser.

An automatic starter of the remote control type is provided, which enables the motor to be started and stopped either by means of the antenna switch or a separate control switch. A dynamic brake is also provided which quickly brings the motor to a stop when the control switch is open. Wtih this type of starter the motor can be brought up to full speed in approximately 10 seconds and completely brought to a stop in approximately 15 seconds. This starter is so designed as to operate with a voltage varying from 95 to 120. An overload relay is provided which opens the motor circuit when the current becomes excessive. This relay automatically holds the circuit open until the control switch is opened. When the control switch is again closed, the overload relay automatically closes the motor circuit so that the motor will immediately start. In case of a short circuit, this overload relay will immediately open the circuit again.

A switch is provided which, when opened, inserts a high resistance in the generator field. This reduces the voltage of the generator so that the set can be operated with one gap of the quenched gap in the circuit. Under these conditions the set radiates about 10 watts. This arrangement was provided for the purpose of operating over short distances with a minimum of interference. In practice it has been found that the sets will work easily 50 miles on this low power adjustment and, in some cases, it has been reported that the sets have worked over 100 miles. For the purpose of signalling, a small hand key, known as the type "C" key, is used, and serves to open and close the A. C. circuit. A switch known as the type "S. H." antenna switch is provided which permits the operator to connect in the receiving circuits or transmitting circuits at will. When in the transmitting position, it starts the motor generator, closes the generator field, closes the A. C. line, and connects the antenna to the transmitter. It also, when in this position, protects the receiving circuits. When in the receiving position, the antenna is connected to the receiver and the transmitter circuits are opened so that it is impossible to transmit.

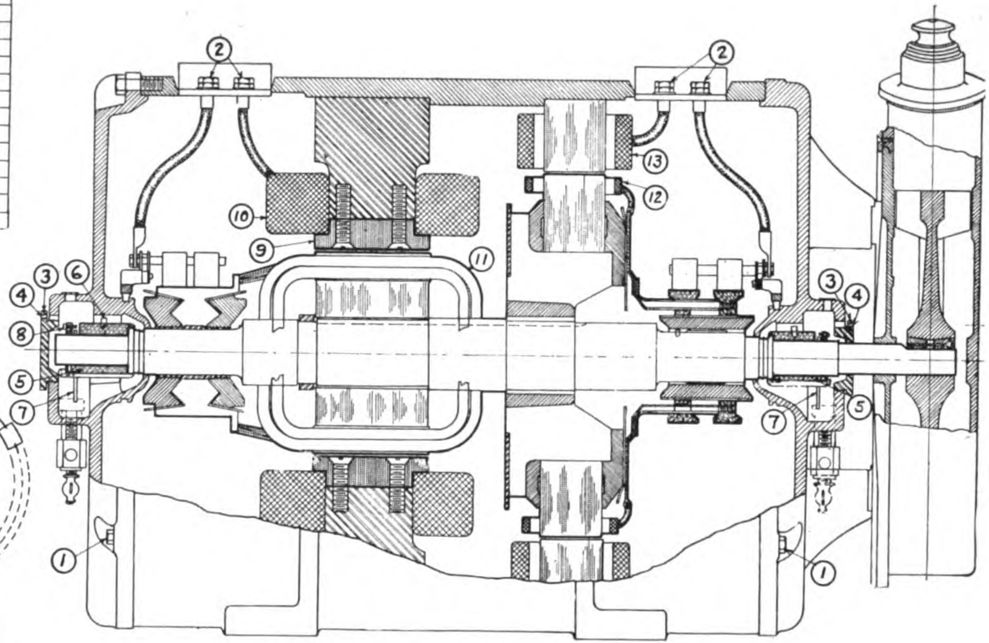
The receiver used with these sets is of the coupled circuit type and is known as the type "106" receiver. There are two circuits in this receiver, the one circuit known as the open or antenna circuit, and the other as the closed tuned circuit. These two circuits are coupled inductively and means are provided for varying the inductive relation of the two circuits. The primary circuit consists of the antenna, a loading coil, a primary

coil, a series condenser, and a ground connection, all being connected in series. By means of a switch any amount of the loading coil in the primary can be thrown into circuit at will. This enables the operator to adjust readily this circuit to the received signals. In case the wave length of the received signal is shorter than the natural period of the antenna, the series condenser is thrown in the circuit which brings this circuit to the desired period. The secondary circuit consists of a secondary coil or inductance which is movable with respect to the primary. This coil is in series with a variable condenser. A switch is provided which permits any amount of the secondary to be thrown in the circuit. The variable condenser permits the variation of the capacity of this circuit so that it may be adjusted for resonance with the primary circuit. By having both the inductance and capacity of this circuit variable, the ratio of capacity to inductance can be varied while keeping the period of the circuit constant. This enables the operator to obtain the best adjustment for operating the detector. The detector used in this receiver is of the crystal type. A battery and potentiometer are provided so that a crystal can be operated either with or without battery as desired. The detector circuit consists of the potentiometer, a stopping condenser and the telephones. The potentiometer and stopping condenser are connected in series with each other and in shunt to the variable condenser. The telephones are connected in shunt with the stopping condenser. A test circuit is provided which enables the operator to excite the antenna circuit at will so that he can adjust the detector for maximum sensitiveness. These circuits are all shown in Figure 4. A careful consideration of this drawing will enable the reader to understand fully the complete circuits of both transmitter and receiver.

Figure 5 shows a section of the motor generator and the combined rotary spark gap and blower. The different parts of this motor generator are indicated by the table and reference numbers.

Figure 6 shows the details of the combined rotary spark gap and blower. This type of spark gap is so well known that it does not need any detailed description. This figure also shows the method of grounding the shaft of the motor generator to prevent currents flowing thru the bearings.

Ref	Detail
1	Cap Screw for removing end shield.
2	Terminal Nuts for removing lead.
3	Thrust nut which holds bearing in place.
4	Set screw for loosening thrust nut
5	Bearing
6	Pin which holds bearing from turning.
7	Oil ring
8	Screw which holds oil ring in place
9	Pole shoe which holds motor field coil in place
10	Motor field Coil
11	Armature winding.
12	Generator armature coil
13	Generator field coil



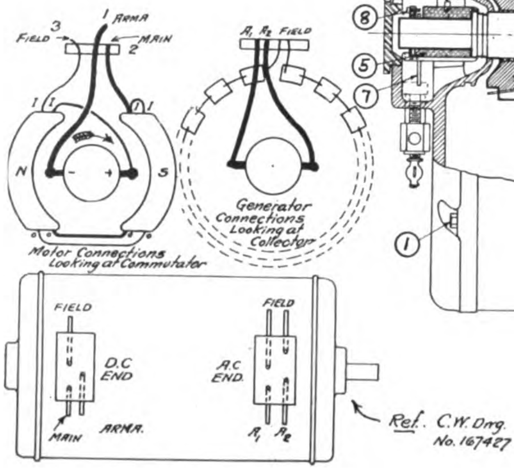


Figure 5

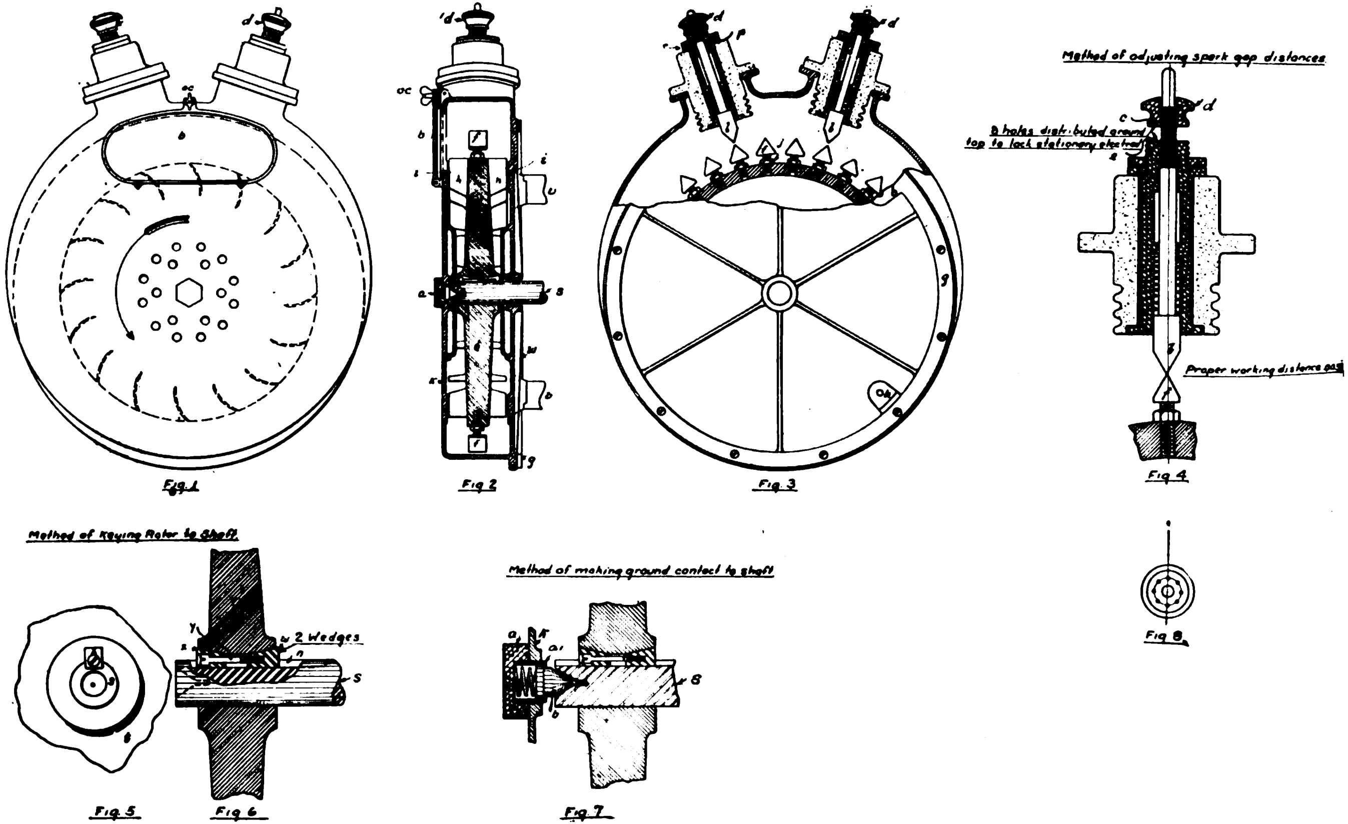


Fig 1
Fig 2
Fig 3
Method of adjusting spark gap distances
8 holes distributed around top to lock stationary electrode
Proper working distance
Fig 4
Fig 8
Method of Keying Rotor to Shaft
2 Wedges
Fig 5
Fig 6
Method of making ground contact to shaft
Fig 7

FIGURE 6

Figure 7 shows the signalling key used with these sets.

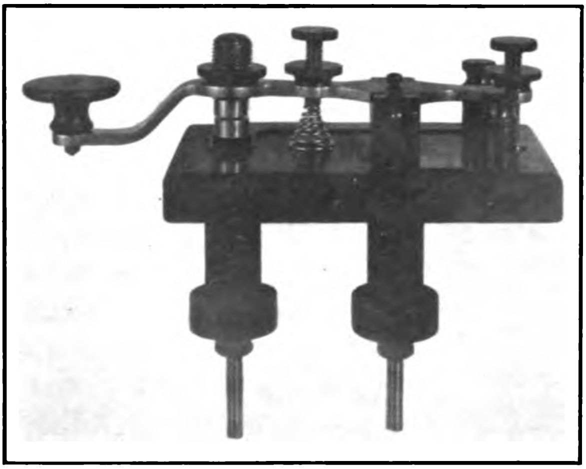


FIGURE 7

Figure 8 shows the complete receiver.

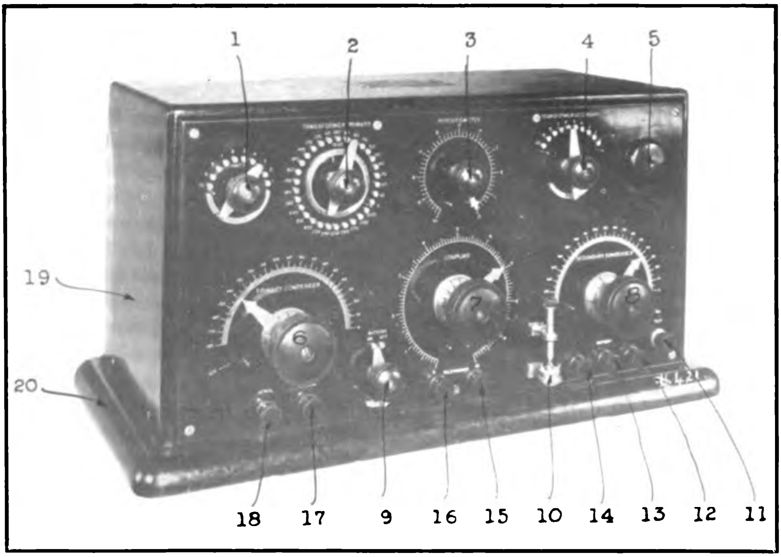


FIGURE 8

1 is the unit switch which permits 1 turn variation in the primary.

2 is the primary switch which permits 10 turns variation in the primary and at the same time cuts out the unused portions of the primary.

3 is a potentiometer.

4 is the secondary of the transformer. This switch enables the operator to cut in the necessary amount of inductance properly to tune the secondary to the primary circuit.

5 is the test buzzer.

6 is the primary condenser. When in the "out" position, it is short-circuited.

7 is the coupling mechanism for varying the inductive relation between the aerial and closed circuit.

8 is the secondary condenser.

9 is the battery switch.

10 is the detector.

11 is a switch for operating the buzzer or test circuit.

12, 13 and 14 are battery posts for the receiver and buzzer.

15 and 16 are the telephone posts.

17 and 18 are the ground and aerial posts.

19 is the case and 20 the base of the receiver.

Figure 9 is the quenched gap showing part in section.

k and *c* are the end plates.

dd etc. are bolts holding end and plates together.

b is the clamp nut.

a is a lock nut.

ee are tubes of bakelite dilecto which cover the clamp nuts, and serve to hold the plates in position.

nn, etc., are the gaskets or separators which keep the plates the proper distance apart and at the same time excludes the air from the sparking surface.

hh, etc., are the spark plates which are made of copper.

j is the sparking face of the spark gap which is made of pure copper and carefully soldered to the larger plates.

oo, etc., are the coupling flanges.

ff are plates of insulating material which insulate the end plates from the frame.

p and *q* are the terminals which serve to connect the spark gap in the closed oscillating circuit.

Figure 10 is a front view and Figure 11 is a rear view of the ½-kilowatt set.

1 is the motor field rheostat.
2 is the generator field rheostat.
3 is the wattmeter.
4 is a handle controlling the aerial inductance.
5 is a low power switch.
6 is the wave length changing switch.
7 is the radiation meter.

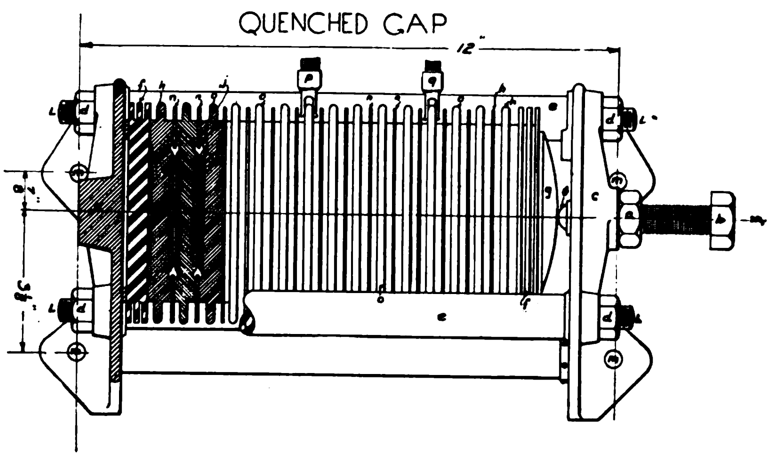


FIGURE 9

8 is a lightning switch which permits the grounding of the antenna when desired.

9 is the handle for varying the coupling.

10 and 11 are the flexible leads for connecting the quenched spark gap in the closed oscillating circuit.

12 is the quenched spark gap.

13 is the automatic motor starter.

14 is the D. C. line switch.

15 is the generator field switch.

16 is the A. C. generator line switch.

17 is a handle for adjusting the rotary spark gap for proper phase relation.

18 is the casing of the rotary spark gap.

19 are the terminals of the rotary spark gap.

20 is the motor generator.

21 is the resistance unit of the motor starter.

22 is the frame work supporting the panel.

23 and 24 are extension angles for securing the set to the ceiling of the operating room.

25 is the variable inductance.

26 is the loading coil.

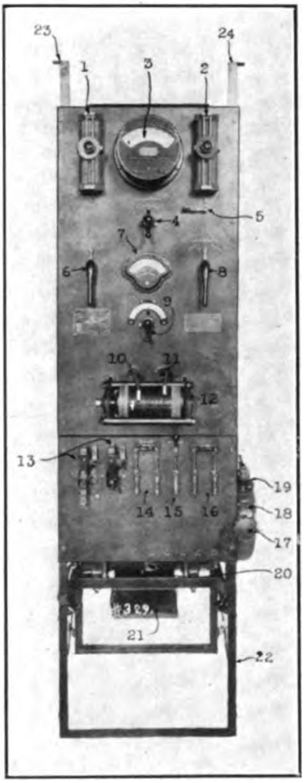


FIGURE 10

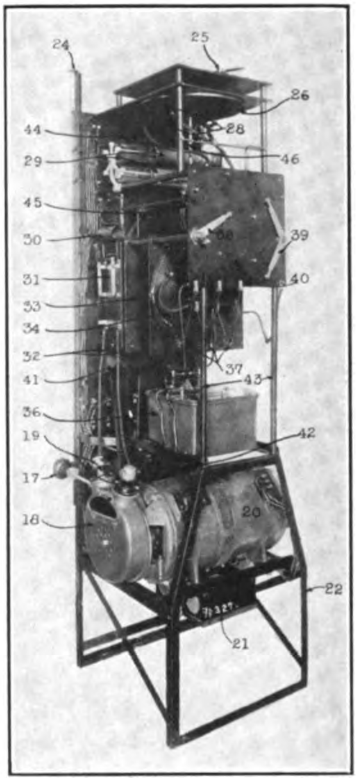


FIGURE 11

28 are adjustable contacts for making connections with the loading coil.

29 is the primary condenser.

30 is the compensating inductance.

31 is the switch for changing over from quenched gap to rotary gap.

32 are the quenched gap leads.

33 is the primary inductance.

34 is the secondary which is movable relative to the primary.

36 is the transformer.

37 are the links for connecting the movable secondary with the switch 40.

38 is the lightning switch arm.

39 is the wave length changing switch arm.

41 is an air duct carrying the air from the combined rotary gap and blower to the quenched spark gap.

42 are the protective condensers mounted on the back of the panel.

43 are rods of bakelite dilecto supporting the switch 40.

44 are the terminals of the wattmeter.

45 is a gear which enables the coil 25 to be varied by the handle 4.

It will be seen from Figures 10 and 11 that the ½-kilowatt set is practically the same as the 2-kilowatt set.

Figure 12 is a complete diagram of connections of this set, which needs no detailed description.

TUNING

The tuning of these sets is quite simple when the method is properly understood. As the closed oscillating circuit is adjusted for the three wave lengths used, the set can be completely tuned without the use of a wave meter. It will, however, require the use of a wave meter to determine the logarithmic decrement of the oscillations.

In tuning the set, it is necessary to bear in mind the fact that a certain definite amount of inductance must be included in the open radiating or aerial circuit to bring its period up to the required wave length and that the coupling between this circuit and the closed oscillating circuit is obtained, not by varying the distance between the secondary and primary inductances, but by means of varying the amount of inductance in the secondary circuit in such a manner as to obtain the proper mutual inductance between the two circuits. When the inductance in the secondary circuit is varied, the amount of inductance in the loading coils must be varied also to keep the total inductance constant. The primary inductance is made movable with respect to the secondary so that the coupling can be varied within limits. The scale which indicates the coupling has a mark which indicates the normal setting for use with the quenched gap. The coupling can either be increased or decreased from this normal setting. This is necessary for the tuning-up operation.

In tuning a set, the wave changing switch is set to the desired

wave length, say, 600 meters. An approximate setting is made by means of the adjustable contact on the secondary inductance. Another contact is made on the loading coil in such a manner that the aerial circuit includes a certain number of turns in the loading coil. The set is then operated and the variable inductance in the aerial circuit is varied while the indications on the radiation meter are noted. If the amount of inductance is too small in the antenna circuit, then an increase of inductance will give an increased amount of current. If the inductance in the antenna circuit is too large, then an increase of inductance will give a decreasing current in the antenna circuit. Inductance is then adjusted until a maximum current is obtained; and at the same time, the coupling is varied within limits, so that by means of varying the inductance in the antenna circuit, and varying the coupling, a maximum current is obtained in that circuit. Now the position of the coupling indicator is noted. If the coupling is less than the normal amount it indicates that there is too much inductance in the secondary. If it is greater than the normal amount it indicates that there is too small an amount of inductance in the secondary. The amount of inductance in the secondary is then either increased or decreased as desired and the amount in the loading coil is varied accordingly so as to keep the total amount equal. The exact amount is generally obtained by two or three trials. The 450 and 300-meter wave lengths are adjusted in the same manner.

In tuning up the sets it is generally advisable to use low power; that is, adjust the set so that it operates with two or three plates in the quenched gap. It is also advisable to adjust the power so that a half note is obtained during the tuning operation. This prevents the gap from being subjected to excessive heating caused by the two circuits being out of tune a considerable portion of the time. After these sets have been tuned up all that is necessary to change from one wave length to the other is to throw the wave length switch to the desired position. It has been found that sets which have been tuned at the dock generally are out of tune when the ship moves away from the dock. This is probably due to the increased capacity of the antenna due to the close proximity of the dock's structure. The operator can easily compensate for this by varying the variable inductance in the radiating circuit until the maximum current is indicated in the radiation meter.

When using the rotary gap it is necessary to use a much looser coupling than with the quenched gap and as this rotary

gap is not intended for continuous use, it is necessary to vary the coupling for each different wave length. When the set has been tuned for operation with the quenched gap, it is then operated on the rotary gap and the position for the proper coupling with the different wave lengths is marked on the scale of the coupling indicator. This enables the operator to adjust for proper coupling without the use of the radiation meter.

In most cases, storage batteries are provided for operating the motor generator in case of emergency. The set can then be operated continuously from 4 to 8 hours on these batteries.

SUMMARY: The most recent standard 2-kilowatt and ½-kilowatt (transformer input) sets of the Marconi Company are described fully. Details are given of the design of the switchboards, gaps, condensers, radiation thermo ammeter, motor generator, transformer, automatic starter, overload relay, low power switch, wave changer, and receiver. The adjustment of these sets is carefully considered.

DISCUSSION

J. Zenneck: There is only one objection that I can raise in connection with this paper, and that is to the name "radiation meter." This instrument measures effective current in the antenna, and not true radiation. Indeed, were a true radiation meter to be invented, it would be a most useful and valuable instrument.

However, it must be admitted that "radiation meter" is not the only misnomer which has been applied to instruments of this type. In Germany, an instrument of this sort calibrated to read currents squared has been called a "watt meter." Of course, this name is quite as incorrect as "radiation meter," since the instrument indicates watts absorbed in the instrument itself and not at all in the outside circuit.

I suppose that the use of an auxiliary source of potential or battery across the detector by the Marconi Company is to enable working at a special point of the detector characteristic. The superposition on the alternating current in the telephone of a direct current certainly changes the sensitiveness of the telephone, but probably decreasingly. This, however, is a more or less secondary effect compared with the total increase in sensitiveness of the detector, resulting from working at the proper point of the detector characteristic.

Louis R. Krumm: A discussion of the new panel sets being installed by the Marconi Company is a very practical question with me. Mr. Shoemaker states in his paper that the first intention was to comply with the Government requirements, and I am pleased to be able to say that they do this and more. The 2-kilowatt set can communicate considerably more than the 100 miles (160 km.) required by the Radio Laws, and the decrement obtained is considerably better than that required.

However, there is a practical side to this set to which I would like to invite attention. The development of the panel set appears to have been carried somewhat to the extreme in this case. A primary object of the panel arrangement is to bring all the equipment into the most compact form, so that all the switches and other apparatus are readily available to the operator, as well as shortening or lengthening the wave length of all the circuits. In the case of these 2-kilowatt sets, the physical dimensions (that is, the width and depth), are determined mostly

by the motor-generator, which is installed at the bottom. This has resulted in a panel set of a size which does not always meet the space requirements of the radio room in which it must be installed.

Radio engineers may appreciate the advantages of a panel set, but ship companies using them are not so well informed, and in many cases provided radio rooms of such size that the 2-kilowatt panel equipment cannot be properly arranged. Its lack of flexibility has produced some curious results. One case I have in mind is where the panel had to be installed in such a manner that it was very difficult to get at the switches, and it was necessary for the operator to go outside and look thru the window of his cabin if he desired to read his instruments. Such conditions, of course, entirely nullify the advantages of a panel equipment, and it occurs to me that there would be several advantages if the motor generator had been separated from the panel, permitting a more compact and flexible arrangement of the remaining equipment. Even the ability to move the motor generator 5 to 10 feet (3 meters) would have greatly improved the conditions under which many of these sets were installed, and the difficulties of running the high-tension circuits to the synchronous rotary gap are not insurmountable. In a word, unless the radio company also provides the quarters, a 2-kilowatt panel set is hardly practicable for a ship set under present conditions.

Altho the three wave lengths provided, namely, 300, 450 and 600 meters, are easily obtained by means of one switch, I doubt very much if the operators are taking proper advantage of these wave lengths.

The Traffic Department of the Marconi Company can no doubt inform you much better than I how much their operators are using the wave lengths other than 600 meters. This would have a direct bearing on the practicability of adopting a common calling wave length and then passing to another one as a transmitting wave.

Another objection might be made to 2-kilowatt sets on ships. I believe that there is more power provided than is necessary for a large proportion of the vessels on which they are installed. Many of the ships in coastwise trade never get more than 50 or 75 miles (120 km.) from land stations, nor into the region of heavy strays. This results in the operator being the determining factor in the amount of interference which will be created by the use of unnecessary power, and experience indicates that operators are inclined to use full power at all times.

Ralph H. Langley: This paper is very interesting in that it discusses fully the mechanical details of the design of quenched spark apparatus of the latest Marconi type. There are two or three questions I should like to ask Mr. Shoemaker. The first is concerned with the continuously variable inductance supplied in the antenna circuit. This set is arranged for three wave lengths. Is a separate inductance, independently variable, supplied for each wave length, or is there but one inductance supplied? It is evident that with but one inductance to make final adjustment of each wave length, it will usually happen that this inductance, or variometer, as we call it, must be adjusted each time the wave length is changed. If separate inductances are supplied, each remains in proper adjustment, and is not effected by changes in the inductance for another wave length.

In all quenched spark transmitters, it is found that there are two or more points of coupling at which proper tuning can be obtained. In other words, there are two or more points of coupling that give current maxima in the antenna circuit. I should like to ask Mr. Shoemaker whether in this transmitter, where the inductance of the coil is varied, and not its position in space, the same points of near-resonance are found, and if so, at which one of these points is it found most advantageous to work. While the method of coupling variation is by inductance rather than motion of the coil, it is apparently found necessary to provide in addition a method of moving the coil. I should like to ask Mr. Shoemaker whether it is possible to move the coil sufficiently to reach all of the two or more points of coupling mentioned above, with any given adjustment of the remaining variables in the circuits remaining constant.

Harry Shoemaker: As regards the inductance available for exact tuning and slight correction when changing from one wave length to another, this was not exactly described in the paper. It consists of approximately ten turns, and the normal setting is at about five turns. There is therefore room for increase or decrease if necessary. Any effect which influences the antenna tuning after the set has been adjusted can be compensated for by this means. Of course, in the original tuning of the set, this inductance is set at its normal mid-way position.

It is interesting to note that if a ship be tuned while in dock and then goes out to sea, the change in antenna current may be an ampere or an ampere and a half. While this would not interfere with the working of the set to any marked extent, it might

be desirable to bring the radiation back to full value, and this can readily be done by the use of the compensating inductance mentioned.

As regards the term "radiation meter," I must disclaim responsibility for this term which originated elsewhere in this country. Professor Zenneck's criticism of the term is fully justified.

SOME SMALL DIRECT CURRENT SETS*

By

Bowden Washington

(Radio Engineer, Cutting & Washington, Cambridge, Massachusetts)

Altho many radio engineers are undoubtedly familiar with the action of the Chaffee gap as used in the sets which are to be described, a short description of its behavior and characteristics may not be out of place.

This gap, originated by Dr. E. Leon Chaffee of Harvard University, consists of a copper anode and an aluminum cathode, in an atmosphere of moist hydrogen.† Figure 1 is a photo-

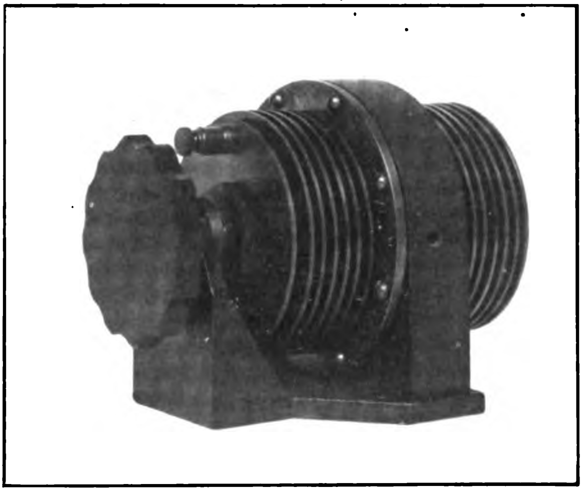


Figure 1

graph of our standard gap; Figure 2 a section. We have found it extremely difficult to build a gap which is adjustable for gap length, and is also hydrogen-tight. All manner of threads and

* Received by the Editor, February 5, 1916.

† "A New Method of Impact Excitation of Undamped Oscillations, and Their Analysis by Means of Braun Tube Oscillographs," by E. Leon Chaffee, "Proceedings of the American Academy of Arts and Sciences," November, 1911.

stuffing boxes have been tried, without great success. The total adjustment required is very small, probably never being over 1/16 inch (1.5 mm.), so we have adopted the thin phosphor-bronze diafram shown in the section at "A." This pushes in and out like the bottom of an oil-can, and we have found it very satisfactory. The periphery of this diafram is clamped against

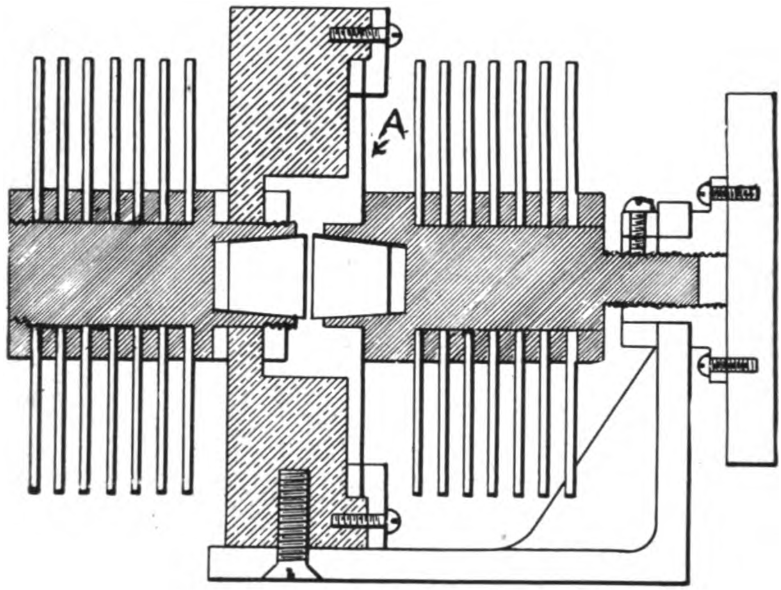


FIGURE 2

a soft rubber packing by a brass ring, held to the bakelite gap chamber by eight "10–32" machine screws. The base is a small composition casting. Cooling fins and the adjusting handle are shown.

The connections are as in Figure 3. When the direct current feed circuit is closed, the primary condenser, C_1, is charged until a potential is reached which is sufficient to break down the gap, G. The condenser discharges thru the gap in a single loop, or half-cycle. The gap then goes out, and leaves the secondary circuit free to oscillate at its own period. Meanwhile, the condenser, C_1, is being recharged from the generator. When it has reached a potential almost sufficient to break down the gap again, the e. m. f. induced in the primary circuit by the oscillating antenna is sufficient to "trigger off" the gap in the proper phase-

relation to the antenna; which process continues indefinitely. The number of antenna oscillations which occur between gap discharges, called by Dr. Chaffee, the "Inverse Charge Frequency," and indicated hereafter by the abbreviation I. C. F., can be varied from two to almost any number, depending on the amplitude of the feed current, the size of the primary con-

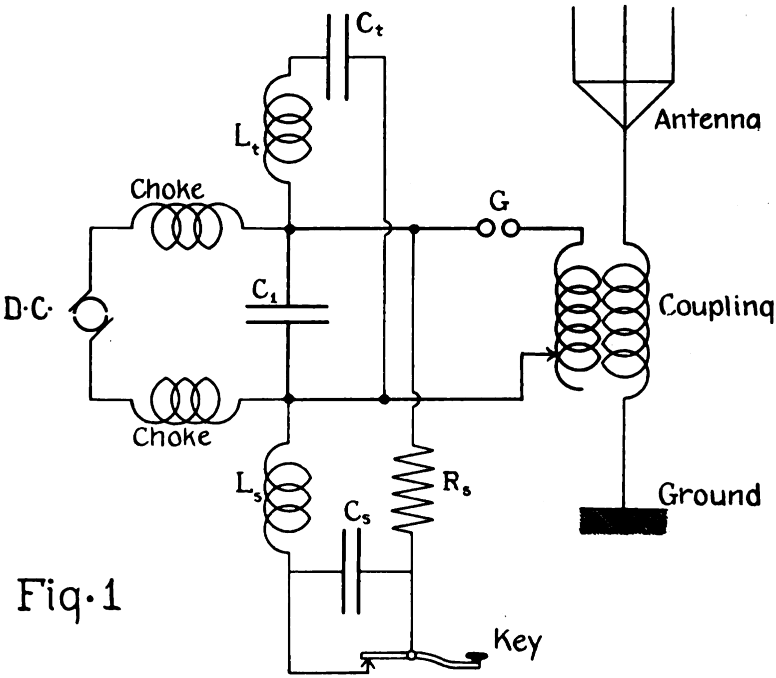


FIGURE 3

denser, the length of the gap, etc. It will be readily seen that with an antenna of normal resistance, and an I. C. F. of 2 to 6, a sustained, and in fact practically undamped, oscillation will be maintained in the secondary. To illustrate this more clearly, the following Braun tube oscillograms may be helpful.

Figure 4 illustrates the current-voltage characteristic of the gap, the current vertical, and the voltage horizontal.

Figure 5 shows the primary half-loop, with an I. C. F. of 2, and was taken by deflecting the beam vertically with the primary current, and horizontally with the secondary current.

Figures 6 and 7 show the secondary wave-train, with an antenna resistance of 40 ohms and an I. C. F. of 9 and 6 respectively.

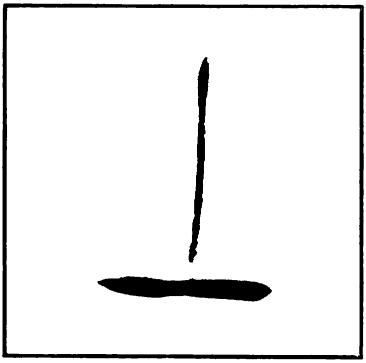


FIGURE 4

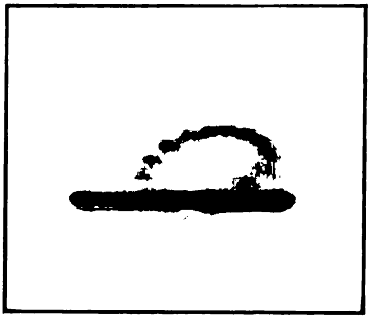


FIGURE 5

Figures 8, 9 and 10 show the secondary wave-train with an I. C. F. of 4, 3 and 2, and an antenna resistance of 5 ohms. These were taken by deflecting the beam vertically with the secondary current and horizontally with the potential of the primary condenser. It will be apparent that when the gap discharges, the spot returns, and one antenna oscillation occurs during this return.

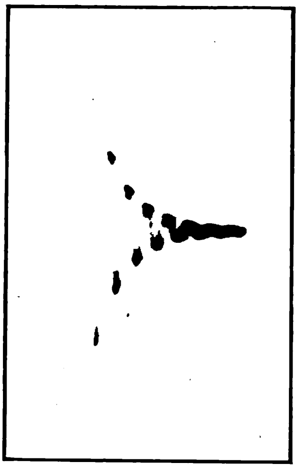


FIGURE 6

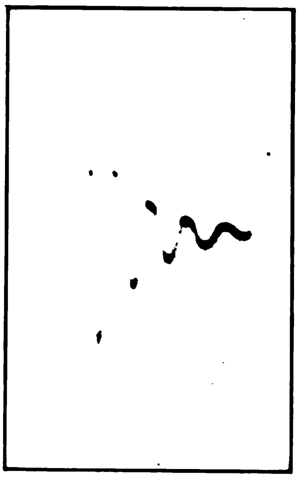


FIGURE 7

Figure 11 is the potential current curve of the secondary, with an I. C. F. of 4, and a resistance of 40 ohms.

Figure 12 shows the production of beats, and was made with two secondary circuits.

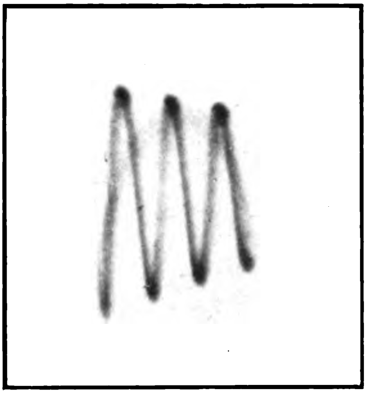


Figure 8

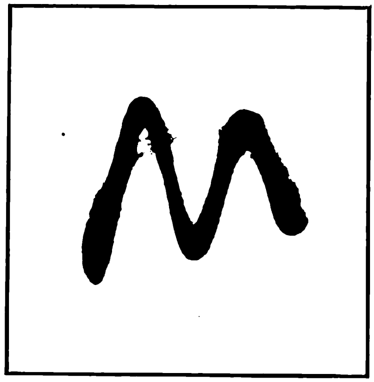


Figure 9

The power and efficiency curves for changing feed current, Figure 13, taken from Dr. Chaffee's paper referred to above, may be of interest, and require no explanation. One gap is capable of efficiently handling about 200 watts input. We find that when two or three gaps are used, with the same primary condenser as with one, it is advisable to double or triple, as the case may be, both current and voltage, thereby keeping the I. C. F. the same, so that it may be roughly stated that the

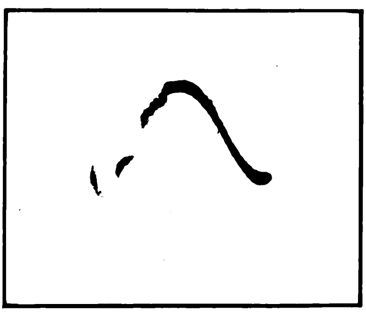


Figure 10

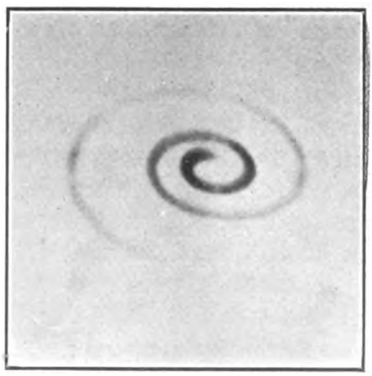


Figure 11

power increases with the square of the number of gaps. However, when powers of 1/2 kilowatt or larger are required, we use a later development of the Chaffee gap, which consists of a rapidly rotating aluminum disc from which the spark passes to a stationary copper electrode, running in an atmosphere of hydrocarbon vapor; and used in conjunction with the Cabot high-

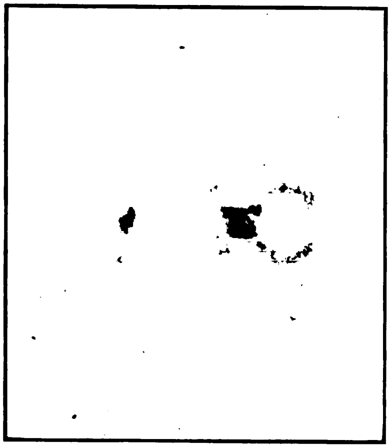


FIGURE 12

voltage, direct current machine. We have shown with the Braun tube that the action of these gaps is identical, as is to be expected; but the efficiency of the rotary type is remarkably high, from 60 per cent. to 70 per cent.

There is no resonance between the primary and radiating circuits; and to obtain the best energy transfer it has been found advisable to tune the primary to a wave-length 1.5 to 2 times that of the secondary. However, the smaller the I. C. F., the more desirable it becomes to tune the two circuits to a ratio of 1.7 to 1. It will be seen that the efficiency from generator to antenna of these sets is quite low, judged by modern standards; however, rather remarkable distances have been covered with small powers, due probably to the radiation of a single wave, and to its maintained nature.

Some tests were made not long ago with a single-gap aeroplane set, which I will describe in detail later. The input was about 150 watts, and the antenna used consisted of two vertical or nearly vertical wires 130 feet (40 m.) long, spaced 4 feet

(1.2 m.) apart, and having a natural period of 205 meters. The wave-length was 480 meters, and the radiation meter showed 1.5 amperes. This test was not pre-arranged in any way. The stations with which we obtained communication were simply called in the ordinary manner. Communication was established with Highland Light, a distance of about 51 miles (80 km.),

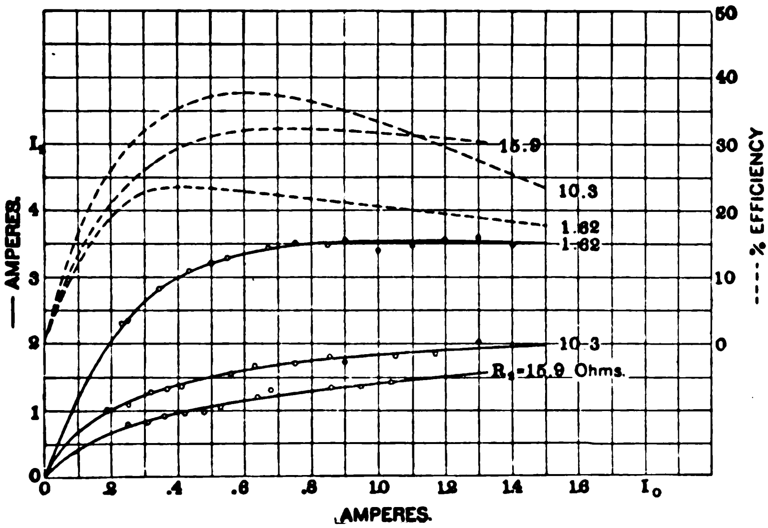


FIGURE 13

and our signals were reported strong. We then worked with the S. S. *Virginian*, which reported 78 miles (125 km.), signals good. Then we got the U. S. S. *Gresham*, presumably between 40 and 50 miles. We also worked several other stations at about these distances, with equal success.

ONE-QUARTER KILOWATT PANEL SET

Figures 14 and 15 are the front and side views, respectively, of a ¼- kilowatt panel set, as used in the Cruft Laboratory at Harvard University. The panel is ⅜-inch (9.5 mm.) bakelite, and is 14 inches (35 cm.) wide, and 24 inches (60 cm.) high. It is bolted to the 1 inch (2.5 cm.) by ⅛-inch (3 mm.) angle-iron frame with black-oxidized cap screws. In the upper left-hand corner will be seen the direct current ammeter, which is in the feed circuit, and has a full scale deflection of 1.5 amperes. In the right-hand corner is a hot-wire radiation meter. Below

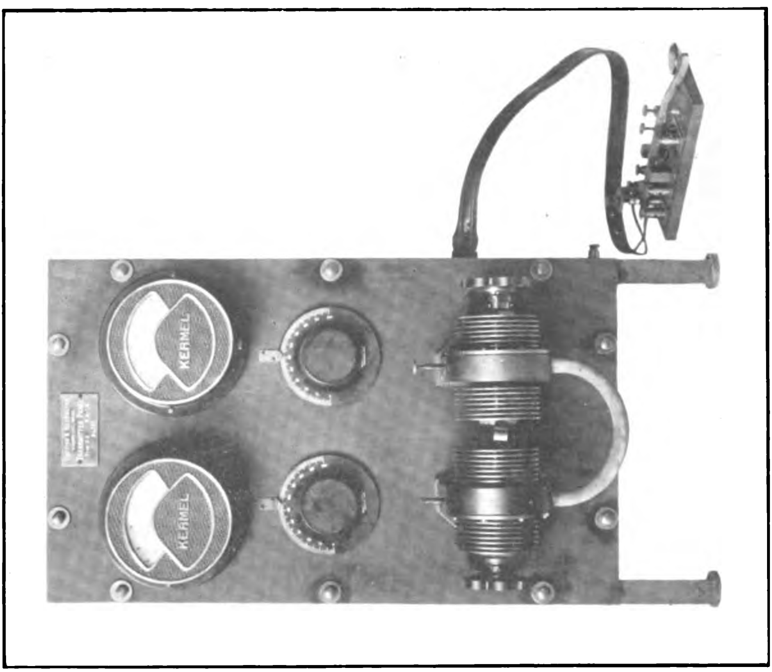


FIGURE 14

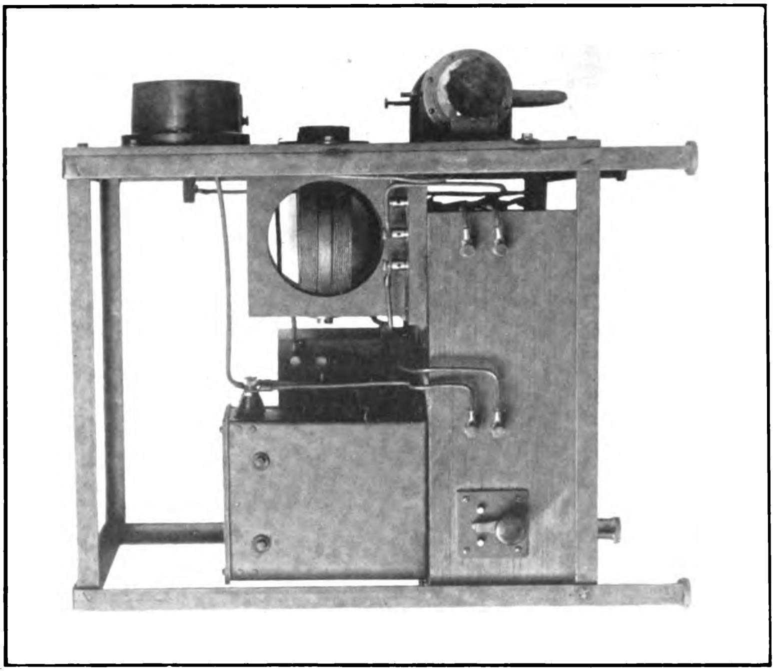


FIGURE 15

are the handles for the two variometers, which are situated at the back of the board. These are wound with separately insulated, stranded wire. The large coil of each is form-wound, and held on the inside of the wooden frame, so that there need be only a sufficient air gap between the two coils to keep them from rubbing, thus giving very close coupling, and therefore a low minimum inductance. One of these is common to both circuits, the other in the antenna. Below the variometer handles are seen the two gaps, which are connected in series.

The large box filling the whole lower part of the frame contains the "tone circuit," which consists of a circuit, shunted across the primary condenser, containing such capacity and inductance as to be of an audible frequency ($C_T L_T$, Figure 3). Its action is similar to that of the tone circuit in the Rein "Multitone" sets.* The inductance is tapped in three places, and provided with a switch so the three tones can be obtained. This box also contains a "starting circuit" ($C_s L_s R_s$, Figure 3). These gaps, of course, require a good deal higher potential in starting than the average drop across the gap, which latter is all the generator need supply. If one uses a generator with a high initial voltage and a very steep falling characteristic, the operation of the generator is apt to be unstable, for the load "kills" the field, and the voltage drops below that necessary to keep the gap in operation, builds up and starts again. A moderately falling characteristic, however, seems to be desirable, as apparently the gap is in the most stable state when the generator and gap characteristics cross at nearly right angles at the working point. This starting circuit consists of an inductance of the order of 0.1 henry, and a capacity of, say, 0.06 μf. (microfarad), and a resistance approximately equal to the average resistance of the gaps. The key is shunted across this condenser, and open circuits when depressed. The current from the generator flows thru the key, inductance and resistance. When the key is opened, it breaks the current quickly, owing to the condenser, and thus the energy stored up in the choke coils and the "starting inductance" produces a high voltage (which the Braun Tube showed in one case to be about 1,500 volts, while the average gap drop is about 150 volts per gap), sufficient to start the gaps, which continue to operate in a normal manner. The resistance in this circuit is, by the way, sufficient to keep it from causing a tone effect.

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*See PROCEEDINGS OF THE INSTITUTE OF RADIO ENGINEERS, Volume 1, 1913.

The small box at the rear of the set contains the choke coils: a closed magnetic (iron) circuit wound with two coils, one of which is connected in each side of the line, and having a total inductance of about 40 henries. Placed beside this is the primary condenser, consisting of aluminum plates in oil, contained in an oil-tight steel tank, and having a capacity of about 0.006 μ f.

The connectors accompanying this set, one of which is shown leading to the key at the right, consist of two stranded silk-covered wires, sewn in a thin leather strap and terminating, when convenient, in small so-called stage connectors. There are two of these from the key to the panel, one from the receiving set to the panel and one from the generator or line to the panel.

A detailed photograph of the key is shown in Figure 16. The pair of contacts attached to the key lever at the back, and insulated from it, are led to the antenna binding post thru a piece of Belden braid. The upper adjustable contact is connected

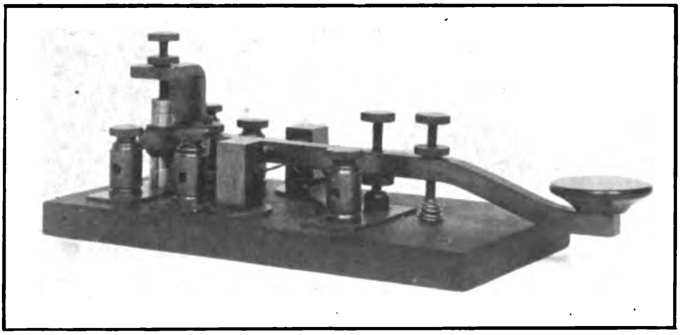

FIGURE 16

to the antenna side of the transmitter; the contact on the base to the antenna side of the receiver. Thus the key acts as an antenna transfer switch when operated. The spring contact, which, unfortunately, is concealed in the photograph behind one of the trunnion posts, opens the starting circuit as described above. This key, which is virtually a break-in key, is made practicable by the low potentials developed with these sets. In one instance, with a current of 3 amperes in an antenna of which the natural period was 540 meters, and which was working at 1,000 meters, the R. M. S. potential, observed with an electrostatic voltmeter between the top of the coupling coil and the ground, was 375 volts. This set will put 2.7 amperes into the

two-wire vertical antenna referred to above, at 500 meters, and nearly 4 amperes into a large flat-top antenna of which the natural wave length is 540 meters, when working at 1,000 meters.

All metal parts of this set are black-oxidized, the box is dead black oak, the name-plate black-oxidized with white lettering.

ONE-SIXTH-KILOWATT AEROPLANE SET

Figure 17 shows a $\frac{1}{6}$-kilowatt aeroplane set. The case is white pine, and is only a temporary arrangement, to be used until we secure something more suitable. On the right will be seen the complete receiving set, consisting of the tuner, variable

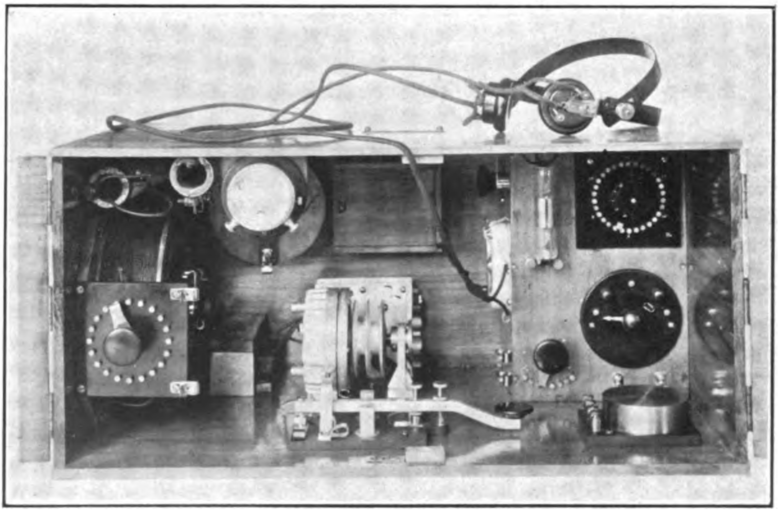

FIGURE 17

condenser, stoppage condenser, audion, audion potential battery with switch, audion heating current rheostat, and audion heating current switch, all mounted as a removable unit. In front of the receiving set will be seen the chopper, which consists of a buzzer, the armature carrying an insulated contact which breaks up the sustained oscillations into audible groups.

The key, of the break-in type described above, is made largely of aluminum, and weighs 12 ounces (0.35 kg.). Details of the key are shown in Figure 18. Behind the key is the gap (Figure 19), embodying the same phosphor-bronze diafram used in the standard gaps, but constructed principally of aluminum

castings, weighing complete 2.5 pounds (1.1 kg.). Behind the gap is a cast aluminum frame, holding two mica condensers (Figure 20), one of which is the primary condenser, the other the starting circuit condenser. The starting circuit inductance is seen behind the radiation meter, the starting circuit resistance in the upper left-hand corner. To the left-hand side of the

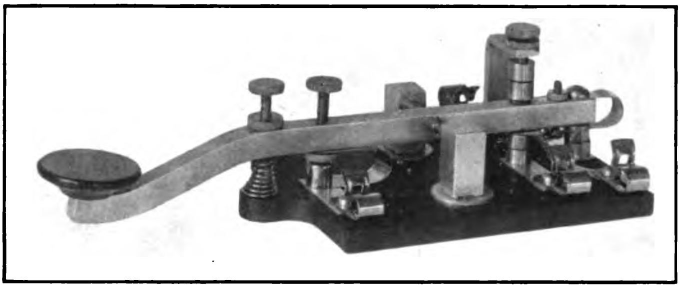

FIGURE 18

box is secured the coupling coil (Figure 21), wound with number 16 double-cotton-covered, shellacked wire* on well varnished wooden forms, and provided with switches to vary both the primary and secondary inductance; which weighs 2 pounds, 6 ounces (1.1 kg.). The small iron-core choke coil is behind this. The small box above the condensers contains the audion heating

FIGURE 19

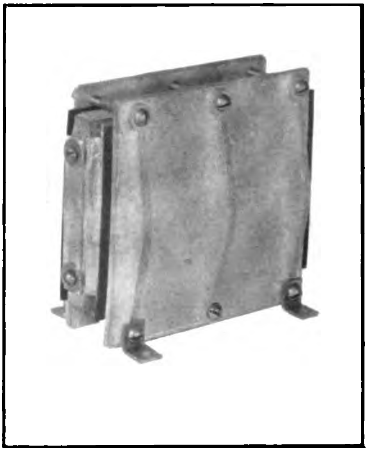

FIGURE 20

* Diameter of number 16 wire = 0.051 inch = 0.129 cm.

battery, which consists of one of the ordinary large 3-cell flash-light batteries. Spring binding posts are used almost altogether, as they would seem less likely to become loose from vibration. The box complete weighs 31.5 pounds (14.3 kg.).

The generator (Figure 22) accompanying this set is a small 3,300 R. P. M. direct current machine, giving 350 volts on no load, and 150 volts, and 1.2 amperes at its working point. It

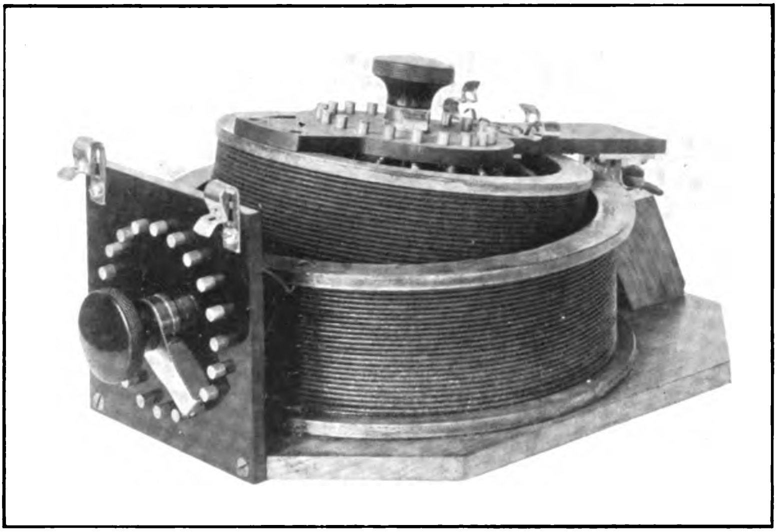

FIGURE 21

weighs 15 pounds (6.8 kg.). It is driven by the wind which operates a small aluminum air propeller mounted on the end of the generator shaft, and connected to the generator thru a centrifugal clutch. The clutch keeps the generator at its proper speed when the aeroplane speed is 50 miles (80 km.) per hour, or beyond. The entire generating unit weighs 20 pounds (9 kg.).

The antenna consists of 50 meters of number 16 phosphor-bronze wire, wound on a ball-bearing wooden reel, and is to be trailed behind. The engine, strut wires, etc., will be used as ground. The weight of the complete outfit is 55 pounds (25 kg.).

We have not as yet made any tests from a plane. The tests of a single-gap set referred to previously in this paper were made with this set. The inductance, capacity, and radiation resistance of the above aeroplane antenna were calculated, the latter by

Dr. G. W. Pierce, and a similar artificial antenna was made up. The radiation current was from 1.5 to 2 amperes, or about the same as in the test referred to above.

The hydrogen for these sets is supplied compressed to a high pressure in steel tanks. The tanks for the panel set are approximately 6 inches (15 cm.) in diameter, 5 feet (1.5 m.) long,

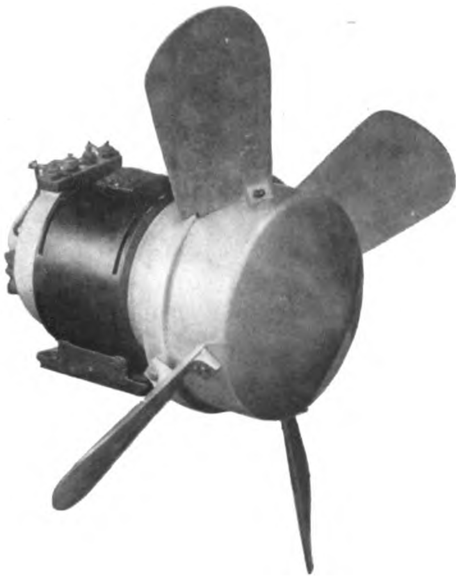

FIGURE 22

holding 50 cubic feet (140 liters) of gas, while those for the aeroplane set are 1.5 inch (3.8 cm.) in diameter and 6 inches (15 cm.) long, holding 1.5 cubic feet (4.3 liters) of gas. Altho the hydrogen plays a necessary part in the action of the gap, it must go thru a complete chemical cycle of some sort, as it does not appear to be consumed; the writer having run a carefully sealed gap well over ten hours without renewal. The tanks are necessary only to supply gas lost thru any small leak that might develop.

The Chaffee Gap seems on the whole to have a wide range of commercial and laboratory applications. In the laboratory its use would include the production of sustained oscillations for the measurement of radio frequency inductance, resistance or capacity, and of dielectric and magnetic hysteresis at radio frequencies. It seems to be eminently fitted for short distance

radio telephony, and its range can undoubtedly be increased when a suitable controlling device or transmitter is found. When used as a radio telegraph transmitter, it has the advantages of lightness, extremely low potentials, absolute quiet, one wave, adaptability for beat reception, and, in the rotary type, exceptionally high efficiency.

Since writing the above portion of the paper, I have discovered that alcohol vapor can be substituted for hydrogen in the stationary gaps with great success, if the alcohol is fed into the bottom of the gap chamber thru a ¼-inch (6 mm.) round cotton wick from a small oil cup of the ordinary wick type. The carbon seems to distil off around the wick, and form a light flaky soot—which can be cleaned out occasionally. The electrodes remain free from soot, which was not the case when alcohol was dropped in from above, and enough carbon did not collect in a twenty-hour continuous run (key down) to interfere with the operation of the gap. The gap sometimes starts with a slight explosion, but operates normally and without waiting. It uses about 2 c. c. of alcohol per hour. This, of course, adds considerably to the commercial value of the apparatus, as hydrogen is sometimes hard to obtain.

In conclusion, I wish to thank Professor George W. Pierce, whose kindness has enabled me to do a large part of the development work connected with these sets in the Cruft Laboratory of Harvard University.

SUMMARY: The construction of a commercial Chaffee gap (copper and aluminum electrodes in hydrogen) is described. The use of such gaps in connection with "starting" and "tone" circuits for the production of sustained waves modified into groups of audio frequency is considered. The phenomena are illustrated by Braun tube oscillograms. Impulse excitation is obtained.

The construction and operation of a ½-kilowatt panel set and a ⅛-kilowatt aeroplane set of this type are given in detail, together with performance data.

DISCUSSION

John L. Hogan, Jr.: It is interesting to compare the two types of heterodyne response described by Mr. Washington and Dr. De Forest. The base note produced in the heterodyne receiver when listening to radiation from a well adjusted Chaffee arc, is stated to be purely and regularly musical, but to have a slight hiss overlaid upon it. The pitch is constant, in spite of the fact that the wave lengths used were comparatively short and that, consequently, an extremely small percentage change of wave frequency would produce a large variation in heterodyne-tone. The sound produced in the heterodyne receiver when listening to radiation from "quenched-arc" set, such as described by Dr. De Forest, is said to be a strong hiss. Presumably the successive wave trains are of approximately the same persistence in the two cases, and presumably the group or discharge frequency may be made approximately the same in either the Chaffee or de Forest arrangements.

This brings us to consideration of an explanation which has been given to describe just such heterodyne-tone phenomena. It appears that when wave trains occur at a group frequency which is a comparatively large fraction of the wave frequency, a pure heterodyne-tone will be produced if the successive groups have the proper phase relation. If the first half-wave of the second group coincides in phase with the third, fifth, seventh, ninth, etc., half-wave of the first group, and if a similar relation exists between the third and second group, and between the fourth and third, etc., the result will be a pure sustained wave having a boundary curve the amplitude of which changes slightly at the group frequency. The main energy of the wave is purely sustained and this portion results in a pure heterodyne-tone. The slight amplitude variation sets up irregularities which cause the overlaid hiss described by Mr. Washington. Phase conditions of this sort seem to be secured in the Chaffee arrangement, when each wave train is "triggered off" by reaction from that which preceded it.

In the case of the "quenched arc," where no provision is made for phasing up successive wave trains, the high group frequency results in groups of waves which overlap each other and which have random phase relations. Sometimes the new group destroys partially or totally the effective energy of that which preceded it, and sometimes the two energies are added.

The constantly varying phase of the resulting antenna current causes an irregularity in the beats produced at the heterodyne receiver, and the signal sound, altho loud, is hissing and not musical.

The comparison outlined above forms an interesting application of the physical principle that regularity of impulse application, as regards time, is essential if true musical tone of the impulse frequency is to be produced. Much the same effect has been noticed in connection with maintaining the tone quality of signals produced by spark transmitters.

Lee De Forest: Concerning the paper by Mr. Washington, it may be of interest to state that I have been working for some time on an arc in air between two flat tungsten electrodes three-quarter of an inch (1.8 cm.) in diameter, the circuits being practically the same as those used by Mr. Washington and Dr. Chaffee. The electrodes are clamped into ordinary aluminum cooling flanges. As far as my measurements go, I found very marked similarity between this arc and the one in the Chaffee gap. The efficiencies were also very much like those Mr. Washington described. In the matter of sustained radiated waves, however, not having had much experience with the Chaffee arc, I was much surprised to learn that one got a clear note from it in the ultraudion receiver. With the tungsten quenched arc transmitter one does not get a clear note, but a very loud hissing noise like escaping steam, or like that from a tikker. This would seem to show that the radiation from the antenna energized from the Chaffee arc is much more nearly continuous than when energized by the quenched arc with tungsten electrodes in air.

Benjamin Liebowitz: Mr. Washington's paper is of special interest to me because it illustrates the fact that, to obtain high efficiencies with arc oscillators, a rapidly responsive generator with a sharply falling voltage characteristic is necessary. The reasons for this were explained in a paper published not long ago in the "Physical Review,"[1] in which I showed that if the arc characteristic "with oscillations" is an equilateral hyperbola, (as is approximately the case, generally), and if the generator voltage is constant, the efficiency cannot exceed 50 per cent. I showed also that this efficiency limitation could be overcome by the use of a generator with a sharply falling voltage-current characteristic, provided the response was sufficiently rapid.

[1] December, 1915.

The truth of these statements can be seen from the following graphical analysis: The relation between mean arc voltage (P) when the arc is oscillating, and mean arc current (x_m) is, as already mentioned, approximately an equilateral hyperbola in most practical cases, i. e., $P x_m = \text{constant}$. But, since the pulsations in the supply current are small, due to the large series inductance, the product $P x_m$ gives the average rate at which power is drawn by the arc.[2] Hence, arc oscillating systems tend to draw power at a constant rate, so far as variations in the mean supply current are concerned. This is illustrated by Curve III of the accompanying curve-sheet. The *useful* power supplied to the system, however, is very variable. Let E be the generator voltage, assumed constant, and R the resistance of the entire supply circuit, exclusive of the arc. Then the average useful power supplied is $E x_m - R x_m^2$, since the pulsations in x are small. For the total power input is $E x_m$ and the losses are $R x_m^2$. It should be noted that the losses in the arc are not included in $R x_m^2$, for the arc resistance drop is included in the mean arc voltage, P. The graph of the total power is Curve I, and of the useful power is Curve II.

For operation, useful power supplied must equal power drawn by arc, i. e., the possible operating points are the intersections r and r', of curves II and III. If we could operate at r', we should have an efficiency of $\frac{s' r'}{s' p'}$, since $s' r' = \text{power}$ drawn by arc, and $s' p' = \text{total power input}$. At r however, the efficiency is $\frac{s r}{s p}$. In other words, a given arc output (including arc losses) can be obtained at two values of supply current, one below $\frac{1}{2} \cdot \frac{E}{R}$, the other an equal amount above $\frac{1}{2} \cdot \frac{E}{R}$; and operation at the low value of current would give high efficiency, at the high value, low efficiency. But operation at the low current is unstable, because if anything should happen to decrease the current ever so slightly, the power drawn would exceed the useful power supplied and the arc would go out.

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[2] Average power $= \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} e x \, dt$. But since x is nearly steady and equal to x_m, this becomes $\frac{x_m}{t_2 - t_1} \int_{t_1}^{t_2} e \, dt$. But $\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} e \, dt = \text{average of } e = P$, hence Average Power $= P x_m$.

It follows that we must operate with mean supply currents equal to or greater than $\frac{1}{2} \cdot \frac{E}{R}$. If by suitable adjustment, the point K were brought into coincidence with n, the equilibrium would be neutral and the efficiency would be a maximum,

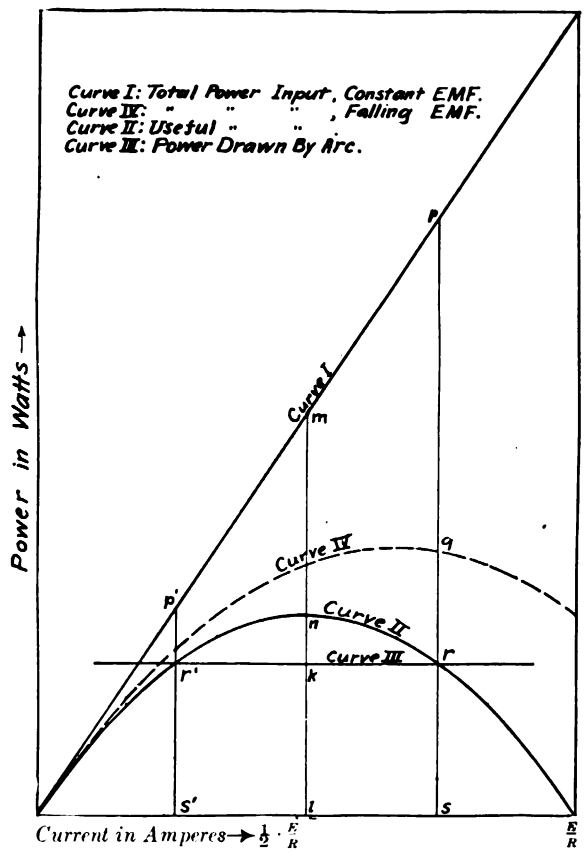

FIGURE 1

viz: $\frac{l\,n}{l\,m} = \frac{1}{2}$. Hence it follows that if the generator e. m. f. is constant and the arc characteristic is an equilateral hyperbola, the efficiency cannot exceed 50 per cent. It should be noted that the losses in the arc itself are not included, a fact which will more than counterbalance such departures from the equilateral hyperbola law which may exist in practice. We are on the safe

side, therefore, in saying that the efficiency cannot exceed 50 per cent.

Suppose, now, that the impressed e. m. f., instead of being constant, falls off linearly with increase of current, i. e., let the generator voltage be $E - h\,x_m$, and let the resistance of the supply circuit now be R'. Then the useful power supplied is $(E - h\,x_m) \cdot x_m - R'x_m^2 = E\,x_m - (R' + h)\,x_m^2$. If we choose R' and h so that $R' + h = R$, the original resistance of the supply circuit, then the useful power supplied will have the same graph as before: viz: Curve II. The power input, however, will now be $E\,x_m - h\,x_m^2$, whose graph is Curve IV, and the efficiency when operating at r becomes $\frac{s\,r}{s\,q}$ instead of $\frac{s\,r}{s\,p}$. By letting h approach R and R' approach zero, curve IV can be made to approach Curve II and the efficiency (exclusive of the arc) can be thus brought as close to 100 per cent. as desired.

Hence, by using a generator with a sharply falling voltage characteristic, the efficiency limit of 50 per cent. can be overcome. The voltage changes of the generator, however, must follow the current changes very rapidly, i. e., the generator must be quickly responsive, for if the current ever falls below the critical value $\frac{1}{2}\frac{E}{R' + h}$, a very short time is sufficient for extinction.

The results of this analysis appear to be contradicted by the high efficiencies obtained in some of the large Poulsen stations, where efficiencies above 60 per cent. have been measured. The discrepancy may be due to a very marked departure of the arc characteristic from the equilateral hyperbola law, or to experimental error. That the departure from said law, however, should be sufficient to more than counterbalance the arc losses, seems very unlikely. The discrepancy, on the other hand, seems too large to be attributed to ordinary experimental error. Hence we are led to seek something peculiar about the circumstances of the measurements.

This peculiarity, I believe, lies in a hitherto unsuspected variation of antenna resistance, which is measured with very small current and is tacitly assumed to be the same at very large currents.

The potential gradient in the soil in the neighborhood of a large antenna is very minute at measuring currents, so that, if the soil conduction is due to moisture, the ground may act as a dielectric at these extremely small gradients, i. e., as an electrolytic condenser. At the higher potential gradients due to

operating currents, however, there can be no doubt that the soil is a fairly good conductor. Thus there may be substantial changes in the effective resistance due to ground losses in passing from measuring currents to operating currents. The same may be true to a certain extent of the surrounding dielectrics, which Miller[3] has just shown to play such a prominent part. I submit, therefore, that the efficiencies of our large stations are, at the present time, unknown quantities, because we cannot assume the antenna resistance measured at very small currents to be the same at very large currents, without further proof.

[3] "Effect of Imperfect Dielectrics," Bulletin Bureau of Standards, March 20, 1916.

(FURTHER DISCUSSION ON "EXPERIMENTS AT THE U. S. NAVAL RADIO STATION, DARIEN, CANAL ZONE". BY LOUIS W. AUSTIN)

ON TELEPHONIC MEASUREMENTS IN A RADIO RECEIVER

COMMUNICATED BY

J. ZENNECK

(PROFESSOR OF EXPERIMENTAL PHYSICS, TECHNISCHE HOCHSCHULE, MUNICH, GERMANY)

Attention has already been called to some difficulties connected with the interpretation of the audibility factor. The following general remarks on the question of telephonic measurement of the received current may be of interest altho I am well aware that they do not point out anything new.

1. THE MEASUREMENT OF THE AUDIBILITY

The current in the detector circuit generally is a pulsating current, the frequency of which is identical with the group-frequency produced in the transmitter or the receiver. The movement of the telephone diafram is determined by the *A. C.* component of this pulsating current; only this component therefore can be determined by any telephonic measurement. This *A. C.* component or audio-current may be assumed at first to be sinusoidal.

The idea in measuring the audibility is to measure the amplitude of this audio-current; but not to measure it in amperes but in an arbitrary unit, namely that amplitude which just permits the individual observer to distinguish between dots and dashes. Accordingly the audibility is defined as the ratio of the amplitude of the actual audio-current to the amplitude just permitting distinguishing between dots and dashes.

This indeterminedness of the basic unit implies a serious difficulty when audibility measurements have to be made by different observers or by the same observer at different times. The unity may be different for different measurements. The amplitude which just enables one to distinguish dots and dashes is different for different individuals, and, for the same person,

depends on physiological and psychological conditions. It will certainly be smaller when, apart from the signals, the telephone is absolutely quiet, than when continuous strong noises are produced by strays in the telephone. In these two cases, if the audio-currents due to the signals were exactly the same, in the latter case a much smaller audibility would be measured than in the former. Therefore the plain statement, that for a special transmitter and receiver an audibility of 200 had been measured, does not mean much for the measuring physicist, important as it may be for the installing engineer.

The following method for measuring the audibility is generally adopted. The telephone is shunted by a non-inductive resistance R, which is so adjusted as to make dots and dashes just distinguishable. How accurately this critical resistance can be measured largely depends on the skill and experience of the observer. The main disadvantage is that the disappearance of a gradually diminishing tone and not a sharply defined minimum, as is obtained with the A. C. Wheatstone bridge, has to be determined.

The critical value of the shunt resistance having been measured, the value of the audibility factor is to be calculated. The amplitude I_o of the audio-current in the telephone is given by the equation

$$I_o = I' \frac{R}{\sqrt{(R+R_t)^2+(2\pi N L_t)^2}} * \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

I' being the amplitude of the audio-current in the branches AA' and BB' in Figure 1, R_t the audio-frequency resistance of the telephone, L_t its audio-frequency inductance and N the group-frequency. The shunt resistance R being removed, the amplitude of the telephone current may be $=I''$. Then according to the definition mentioned above the audibility

$$A = \frac{I''}{I_o} = \frac{I''}{I'} \cdot \frac{\sqrt{(R+R_t)^2+(2\pi N L_t)^2}}{R}.$$

In order to make

$$A = \frac{\sqrt{(R+R_t)^2+(2\pi N L_t)^2}}{R} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and therefore only dependent on the constants of the telephone and its shunt, the condition

$$I'' = I'$$

* Assuming that the reaction of the vibration of the telephone diafram on the telephone circuit can be neglected. If this is not the case, the effective resistance R_t of the telephone as well as its effective inductance L_t may also depend on the amplitude of the audio-current and not only on its frequency.

has to be fulfilled; that is, the current in the unshunted branches AA' and $B'B$ ought not to be changed by shunting the telephone. This implies that

(a) The radio-frequency current flowing thru the detector must not be affected by shunting the telephone, otherwise the

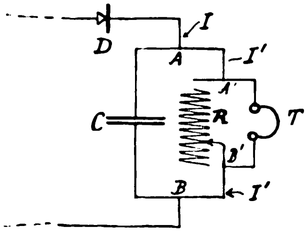

FIGURE 1

audio-current produced by the detector would also be changed. This can be avoided by making the radio-frequency reactance of the condenser C small compared with the radio-frequency impedance of the branched circuit R, T.

(b) The ratio of the audio-currents I'' and I' must be independent of R.

This can be done by making the audio-frequency reactance of the condenser C great compared with the audio-frequency impedance of the telephone. Even in this case, when the telephone is unshunted, and much more so, when it is shunted, practically all the audio-current flows thru the branch $AA'\,B'B$. But if a long wave be used in connection with a relatively high group-frequency, this condition may be contradictory to the condition (a).

Then the arrangement of Figure 2 may be employed supposing, however, the audio-frequency impedance of the telephone to be great compared with any value of the resistance R used. The shunt R, therefore, cannot be disconnected, as this would mean making $R = \infty$. In consequence of this, only the *ratio* of two audibilities A_1 and A_2 can be measured, not the audibility itself. According to equation (2)

$$A_1 : A_2 = \frac{\sqrt{(R_1 + R_t)^2 + (2\pi N L_t)^2}}{R_1} : \frac{\sqrt{(R_2 + R_t)^2 + (2\pi N L_t)^2}}{R_2},$$

if R_1 and R_2 are the values of the shunt resistance R necessary in both cases to make dots and dashes just distinguishable and

if R_1 as well as R_2 are small compared with the audio-frequency impedance of the telephone.

It seems to be very general usage to substitute for equation (2) the relation:

$$A = \frac{R+R_t}{R}.$$

This, of course, is correct only if the audio-frequency inductance of the telephone is very small compared with $R+R_t$, which is

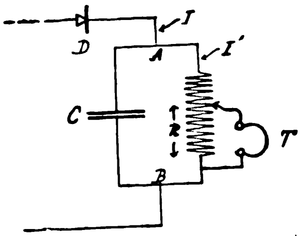

FIGURE 2

not the case in the telephones generally used for radio-telegraphic work. (A. H. Taylor and A. S. Blatterman, PROCEEDINGS OF THE INSTITUTE OF RADIO ENGINEERS, 4, 135, 1916.)

2. THE RELATION OF THE AUDIO-CURRENT IN THE TELEPHONE TO THE RECEIVED ANTENNA CURRENT

It may first be assumed that a quenched gap system be used as a transmitter emitting groups of exponentially damped waves of one frequency.

In the first place the amplitude of the audio-current depends on the characteristics of the detector, on whether the detector response depends on the amplitude of the radio-frequency current or on its square or on any other function of it. It depends further very largely on the sensitiveness of the individual detector at the time of the measurement. Therefore, the detector should be calibrated.

But the calibration ought not to be done by means of a galvanometer. This would mean measuring the $D.$ $C.$ component of the detector current, and not its $A.$ $C.$ component which determines the audibility. This latter is very much affected by the "inertia" of the detector. The inertia of two detectors being different, one of them may give an audibility

twenty times greater than the other and still the deviation of a galvanometer produced by these detectors may be the same for the same radio-frequency current.

Also the capacity of the condenser C shunting the telephone (Figure 1) comes into consideration. As soon as the audio-frequency reactance of the condenser is not great compared with the impedance of the telephone, an increase in the capacity of the condenser diminishes the audibility in the same way as an increase in the inertia of the detector.

All these difficulties, as well as those involved in the uncertainty of the unit of audibility and in the variable sensitiveness of the detector, are met by comparing the audibility produced by the incoming signals with that due to a *standard circuit* of the same group-frequency extremely loosely coupled with the receiving antenna. As a standard circuit, a condenser circuit with an instrument measuring the R. M. S. current (such as a sensitive hot wire ammeter or a thermo-element) may be used. How to excite free oscillations of a known and variable initial amplitude and having the natural decrement of the circuit, using a rotating or vibrating switch (Figure 3) or an interrupter (Figure 4) is shown in the figures mentioned. The condenser C is periodically charged to the potential produced by the battery B and regulated by the potentiometer P, and discharges thru the circuit.

By means of this standard circuit, which, in general, is to be tuned to the frequency of the incoming waves, the entire receiving device can be calibrated. The coupling of the standard circuit with the antenna may be kept constant, and, by varying the voltage to which the condenser C (Figures 3 or 4) is charged, the R. M. S. current in the standard circuit varied. Then the audibility corresponding to each R. M. S. value of the current may be measured. Or the oscillations in the standard circuit may be kept constant and its coupling with the antenna (that is, the coefficient of mutual inductance, M, between the antenna and the standard circuit) may be varied. In this way, the audibility is measured as a function of M and of the amplitude of the oscillations induced in the antenna by the standard circuit, this being proportional to M.

If the constants of the arrangement are known, by this method the amplitude of the oscillations, which are set up in the antenna by a distant transmitter can be measured in absolute units or in amperes (*F. Braun*, "Jahrbuch der drahtlosen Telegraphie," 8, 132, 212, 1914). But for this measurement, the dec-

rement of the standard circuit should be equal to that of the incoming waves. Assuming, for instance, a thermo-detector to be used, the indication of which depends on the square of the current and therefore on the energy of the oscillations, the amplitude of the audio-current (and therefore the audibility) will be different for different decrements even when the energy of the antenna current is the same. Whether or not the decrement of the standard circuit is the same as that of the incoming waves, can easily be checked if, for instance, a resistance is inserted in the antenna, or the antenna is detuned by a certain amount. The decrease of audibility must be the same for excitation by

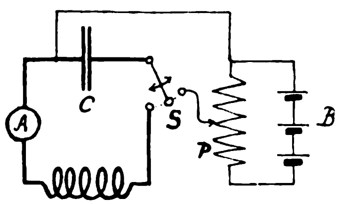

FIGURE 3

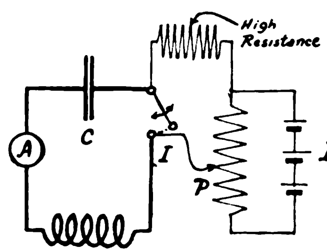

FIGURE 4

the standard circuit and for excitation by the incoming waves.

If the decrements be the same, it is desirable to adjust the amplitude of the oscillations produced in the antenna by the standard circuit so as to give the same audibility as the incoming waves by varying the amplitude of the oscillations in the standard circuit or by varying its coupling with the antenna. Then the E. M. F. induced in the antenna by the incoming waves is the same as that produced by the standard circuit. This amplitude can be easily calculated from the constants of the arrangements and the value of the R. M. S. current in the standard circuit. In this modification, the method is independent of the characteristics of the detector, of the capacity of the condenser in shunt to the telephone, and is also independent of the conditions mentioned in the first section of this discussion. It can be employed regardless of whether or not an amplifier is used.

3. THE RECEPTION OF UNDAMPED WAVES

If the transmitter emits continuous waves and a tone is produced in the receiver by one of the well known methods, generally the amplitude of the audio-current in the telephone is not only

dependent on the amplitude of the incoming waves, but also on the device which produces the tone in the receiver. For instance, it is well known that in the beat method, the amplitude of the audio-current depends just as much on the amplitude of the locally produced oscillations as on that of the incoming waves. Therefore there does not exist any definite relation between the amplitude of the audio-current and the amplitude of the oscillations set up at the receiving antenna.

In this case also the difficulty can be overcome by using a standard circuit. But it would be incorrect to use the same type of standard circuit as that described above. The damped oscillations produced in the antenna by such a circuit would be affected by the receiving device (amplifier, beat reception) in quite a different way from the undamped oscillations set up in the antenna by the incoming waves. It is absolutely necessary to use as a standard circuit a continously oscillating system of exactly the same frequency as that of the transmitter. Such a standard circuit can very easily be had by means of an oscillating electron relay, such as the audion, and its R. M. S. current and therefore also the current amplitude can be easily measured by a sensitive hot wire ammeter or a thermo-element. Otherwise the method for measuring the amplitude of the oscillations set up in the receiving antenna by the incoming waves, is exactly the same as that explained in section 2 of this discussion for damped waves. It is still simpler in that no complications are introduced by the damping.

SUMMARY: If the audibility method is to be used for measuring the amplitude or energy of the incoming waves, useful results can be obtained only by making comparative tests with a properly constructed standard circuit for each measurement, or at least for each set of measurements.

ARC OSCILLATIONS IN COUPLED CIRCUITS*

By

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CHARACTERISTIC CURVES

Before entering upon a description of the phenomena in coupled circuits, it may not be entirely useless to review briefly some of the arc phenomena in single circuits and to recall the usual method of studying them.

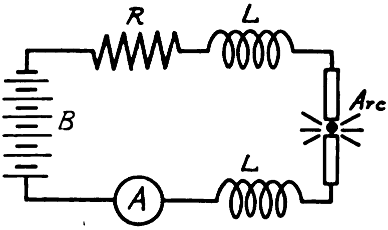

FIGURE 1

The energy is usually supplied from a D. C. source of several hundred volts thru a resistance and a large inductance. The inductance prevents the radio frequency oscillations from going into the battery or dynamo circuit and tends to keep the supply current constant.

If, as in Figure 2, we plot the potential across the arc against the steady current flowing thru the arc for different values of the current, we obtain what is called the "static characteristic" of the arc. The slope of the curve, or the value of $\frac{dv}{di}$, is negative and the characteristic is said to be "falling." The fall of the curve is much more abrupt when the arc is immersed in hydrogen or some hydrocarbon gas.

If, now, we replace the battery of Figure 1 by an A. C.

*Presented before the Boston Section of The Institute of Radio Engineers, January 27, 1916.

source, then the terminal potential and the current are functions of the time, and the instantaneous values of v and i do not have the same relation to one another as the relation found from the

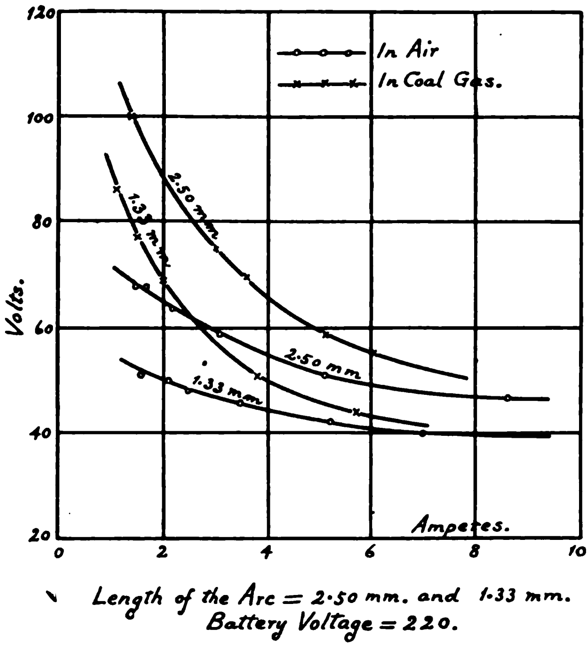

FIGURE 2—Static Characteristics

static characteristic. The curve of v against i with the alternating impressed E. M. F. has been called "dynamic characteristic" by Professor H. Th. Simon.[1]

Professor Simon explored this by means of a Braun tube arranged so that the cathode ray pointer was deflected in the horizontal direction by the current and in the vertical direction by the potential difference. The diagram that he obtained with the alternating impressed E. M. F. is a closed curve of the form of Figure 3, and shows a phenomenon called "arc hysteresis."

While the P. D. is going up from zero to the value corresponding to the point a, there is a small current flowing, but at a the current begins to increase very rapidly while the P. D. drops and the spot on the screen comes to the point b along the curve ab. During the succeeding decrease of the current the spot does not

[1] H. Th. Simon, "Phys. Zeitschr." VI, p. 297, 1905. "Phys. Zeitschr." VII, p. 423, 1906.

follow back the path ba, but the P. D. remains much lower as represented by bc. From the point c the current and voltage drop almost linearly to zero and then reverse, so that a symmetrical curve is traced in the third quadrant.

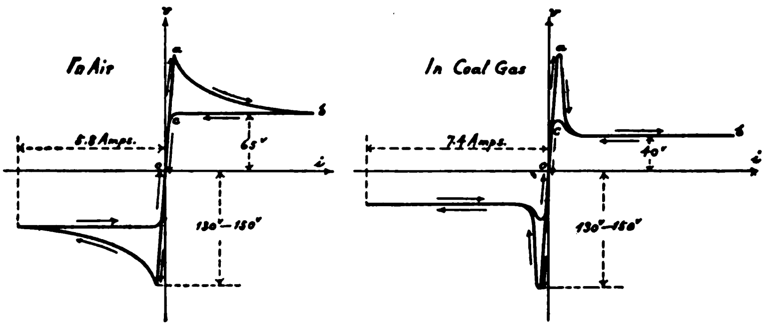

FIGURE 3—Arc Hysteresis Curves

In 1900, Mr. Duddell[2] showed that a D. C. arc gave out a musical note when it was shunted by a condenser and inductance. He gave as one of the necessary conditions of oscillation a negative resistance of the arc or the negative value of $\frac{dv}{di}$.

The most extensive and valuable study of the dynamic

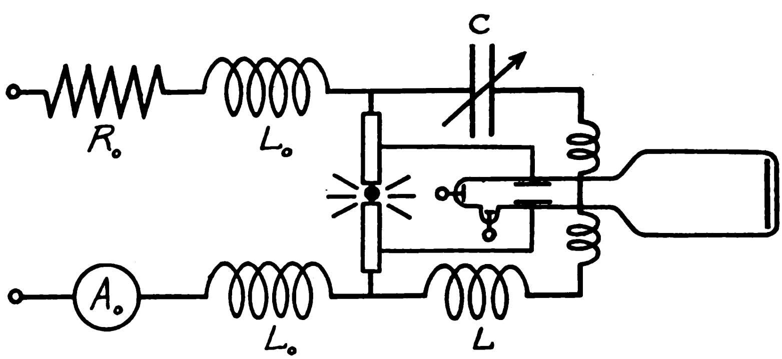

FIGURE 4—Connection of Braun Tube for Taking the Dynamic Characteristics

characteristics of the oscillating arc was made by Simon and his students. The Braun tube arrangement used by him and his followers is as shown in Figure 4.

[2]W. Duddell, "Journal I. E. E.," Vol. 30, p. 232, 1900.

If there were no oscillations, the current thru the arc, i_o, would be nearly constant. The condenser discharge thru the arc tends to superpose a sinusoidal current and make the current pulsating. So long as i_o is larger than the amplitude of pulsation, there is no extinction of the arc and the oscillation is called an "oscillation of the first type." The oscillation of this type is generally obtained in musical arcs.

When the fluctuation becomes larger than i_o, there will be a duration of no current and the arc will be extinguished for a moment. If the arc extinguishes, a constant current i_o will flow into the condenser and charge it up until its potential becomes sufficiently high to cause the next discharge across the arc gap. This is called the "oscillation of the second type" and is most readily obtained in practice at radio frequencies, especially when there is any dissimilarity of the electrode material or other electrode conditions.

Since the condenser is charged by a constant current, the

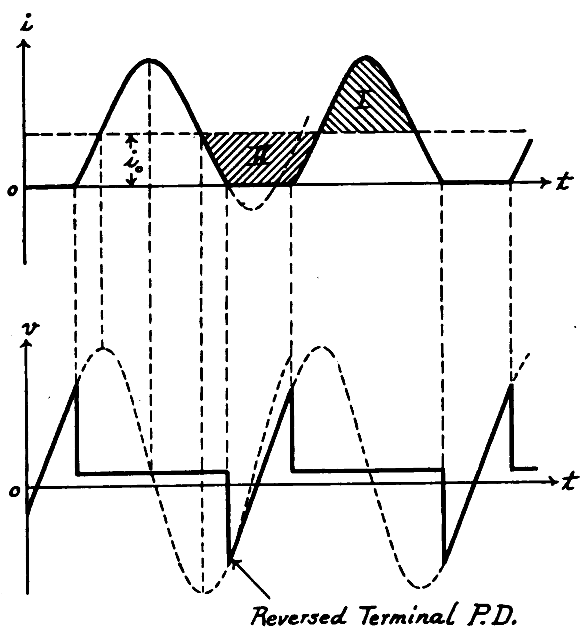

FIGURE 5—Oscillation of the Second Type

current in the oscillation circuit can no longer be sinusoidal with time. The two shaded areas of Figure 5 must evidently be

equal, for the electric quantity discharged thru the arc (I) must have been stored in the condenser during its charging (II).

Altho the condenser potential is to vary in a sinusoidal manner during the discharge, as shown by the dotted lines in Figure 5, the terminal P. D. of the arc is never harmonic, but it remains nearly constant while the arc is burning and becomes equal to the condenser potential at the extinction. During the extinction of the arc, the P. D. goes up along a straight line until the next discharge takes place.

If the supply current i_o is small and the capacity of the condenser comparatively large, then a longer time will be needed to charge up the condenser and the consecutive discharges will take place at longer intervals so that the behavior of the arc discharge will resemble more or less that of quenched sparks of one-half cycle each. (Figure 6.)

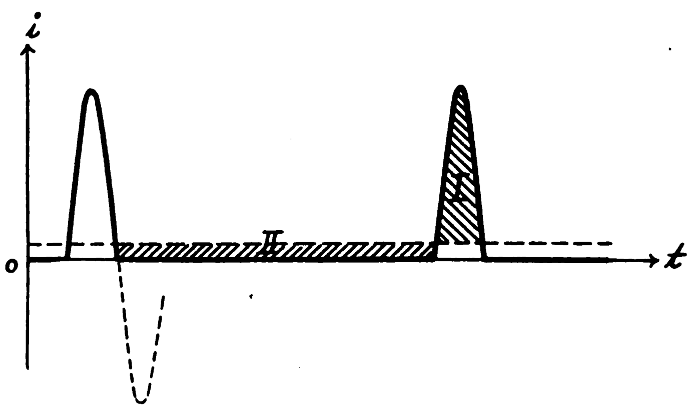

FIGURE 6—Discharges at Long Intervals

If the terminal P. D. which becomes reversed at the extinction (Figure 5) is large enough to cause a discharge across the gap, it will light a small arc in the opposite direction. The oscillation with such a reverse discharge is called an "oscillation of the third type."

The dynamic characteristics observed on the Braun tube screen look as shown in Figure 7.

These diagrams enable us to study the several quantitative relations of the oscillatory arcs and sparks.

The important points which can be noted from the above are that the oscillation is usually non-sinusoidal, and that the frequency cannot be determined by the oscillation constants

alone, but varies because of a slight variation of the supply current or the arc condition.

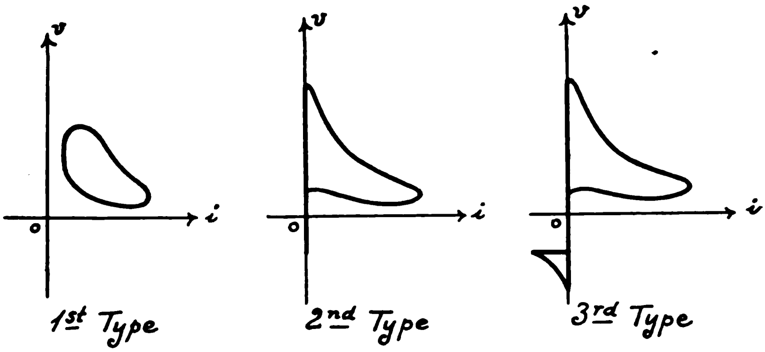

FIGURE 7—Dynamic Characteristics of Arc Oscillations

CYCLIC CURRENT DIAGRAMS

The object of the following experiments is to show, by means of a Braun tube oscillograph, the reacting effect of a coupled secondary circuit upon the oscillation of a carbon arc.

Tho many observations have been made with Braun tube on the arc oscillation, the action of the coupled secondary oscillation has seldom been demonstrated. Jones and Owen's[3] elaborate experiments on musical arcs were made only on the oscillation of the first type with particular combinations of circuit constants, i. e., with a large capacity and a small inductance in the primary and a very small capacity and a large inductance in the secondary, so that the oscillation resistance $\sqrt{\frac{L}{C}}$ was comparatively small in the primary and exceedingly large in the secondary.

Figure 8, Figure 9, and Figure 10 show diagramatically the arrangements of the writer's experiments:

Figure 8 is the arrangement for obtaining the dynamic characteristic with a secondary circuit coupled with the primary oscillation circuit. The arrangements of Figure 9 and Figure 10 are made to obtain what the writer will call "cyclic current dia-

[3] E. T. Jones and M. Owen, "Phil. Mag." 18, p. 713, 1909.
E. T. Jones, "Phil. Mag." 17, p. 28, 1909.

grams," in which the horizontal scale measures the strength of current i, and the vertical scale corresponds to $e = L\frac{di}{dt}$, or the first derivative of the current*.

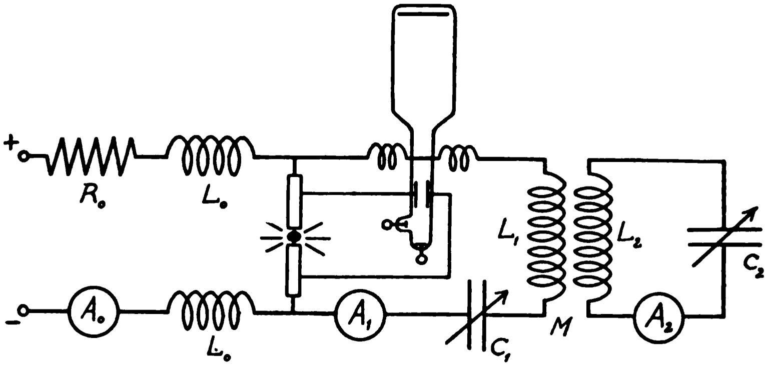

FIGURE 8—Connection for Obtaining Dynamic Characteristic of Arc Oscillations

As this diagram gives us the value of $\frac{di}{dt}$ with respect to i, it is useful for the exploration of the variation of currents and consequently of potentials with respect to time. In order to

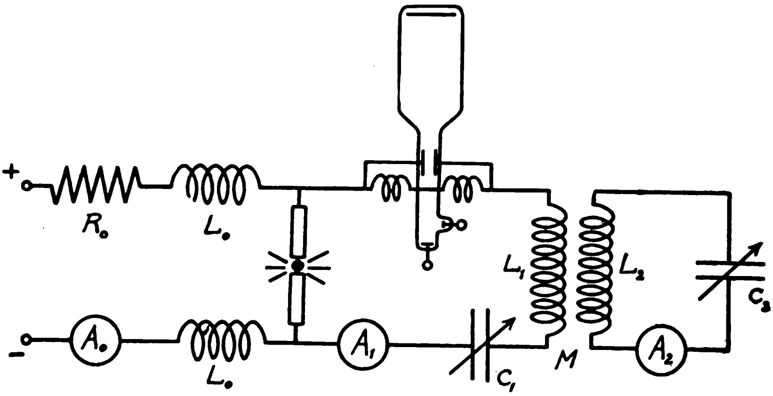

FIGURE 9—Connection for Obtaining Cyclic Current Diagram of the Primary Oscillations

make its use clear, let us take for example the oscillation without the secondary circuit. Figure 11 shows the cyclic current diagrams of the oscillations in single circuits.

*Here L is the small inductance of the current coil of the Braun tube whose terminals are connected to the pressure plates of the tube.

In Figure 12, the curve of i is developed against time by the aid of the corresponding cyclic current diagram of oscillation of the third type. At a the $\frac{di}{dt}$ starts from zero, i. e., the current

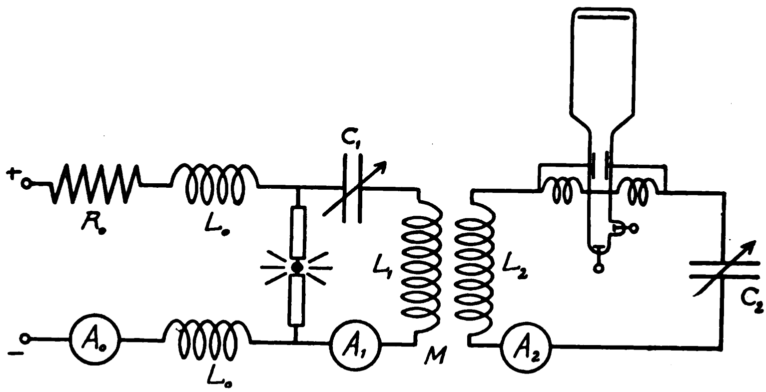

FIGURE 10—Connection for Obtaining Cyclic Current Diagram of the Secondary Oscillations

time curve starts horizontally at a_1 and increases continuously and rapidly. Now $\frac{di}{dt}$ reaches a maximum at b, decreases again to zero at c corresponding to the current-time curve becoming horizontal again at its maximum point c_1. The curve is nearly similar on the other side, but at e, $\frac{di}{dt}$ does not go back to zero,

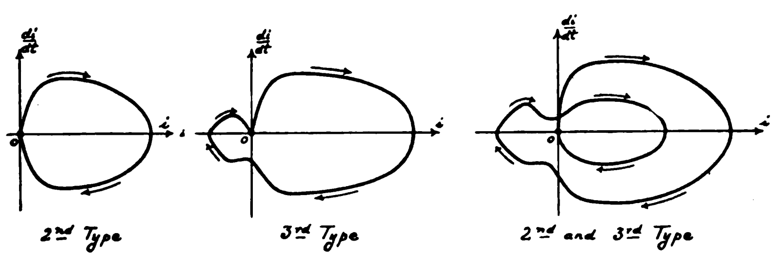

FIGURE 11—Cyclic Current Diagrams of the Oscillations in Single Circuit

which shows that the current-time curve is not horizontal at e_1, but has a certain slope. The curve of i against time becomes again maximum at f_1 and its slope becomes maximum at g_1. At h_1 the i-t curve is once more horizontal. The length of time

between h_1 and a_2 cannot be determined by the cyclic current diagram, but it can be estimated from the relation of the two shaded areas in Figure 12 being equal. Now that the current

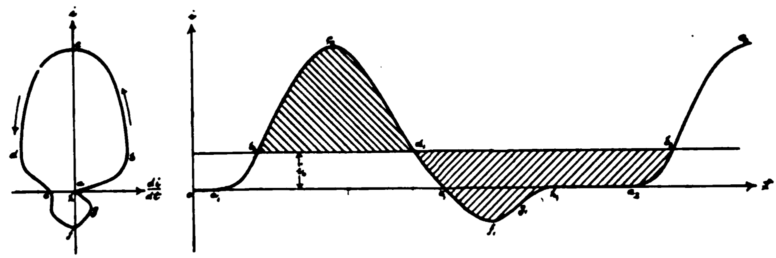

FIGURE 12—Development of *i-t* Curve from Cyclic Current Diagram of the Oscillation of the Third Type

curve with time is obtained, the potential curve with time can also be plotted by means of the dynamic characteristic.

REACTION OF THE SECONDARY OSCILLATIONS

The arc was lighted between solid carbons, in a glass chamber filled with coal gas, from a battery of 440 volts, with large resistance and inductance in series, to keep the supply current at about 4 amperes. C_1 and C_2 were 0.0297 microfarad each and L_1 and L_2 55 microhenrys each.

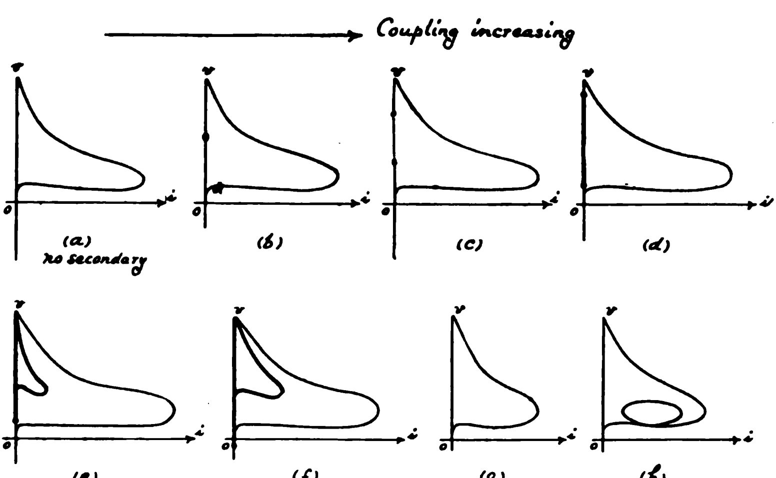

FIGURE 13—Variation of Dynamic Characteristic with Coupling Observed by the Arrangement of Figure 8.

Figure 13 shows the change of the dynamic characteristic of the primary oscil ation obtained by the connection of Figure 8 when the coupling was varied from zero to the closest possible degree.

In (*b*), (*c*) and (*d*) of Figure 13, the brighter portion of the vertical line shows the fluctuation of potential during the charging of condenser, and in (*e*) and (*f*) this fluctuation of potential causes a feeble discharge which is revealed by the appearance of the smaller closed curve. (*f*) changes suddenly to (*g*) and then to (*h*) when the coupling is made very close.

By a slight loosening of the coupling from (*h*), the diagram, especially its loop, shows a tendency to enlarge a little and then changes suddenly to the shape (*f*) without passing the stage (*g*). All three conditions (*f*), (*g*) and (*h*) appeared within a very slight variation of coupling or of the arc condition, and it was sometimes noticed that (*f*) became (*g*) and then (*h*) and suddenly returned to (*f*) again and repeated the cyclic change, while everything was left untouched and unaltered.

Corresponding cyclic current diagrams of the primary and the secondary oscillations were taken by the arrangements of Figure 9 and Figure 10, which are given in Figure 14.

The probable mode of change of E. M. F. and currents with time corresponding to the four cases (*d*), (*f*), (*g*) and (*h*) of Figure

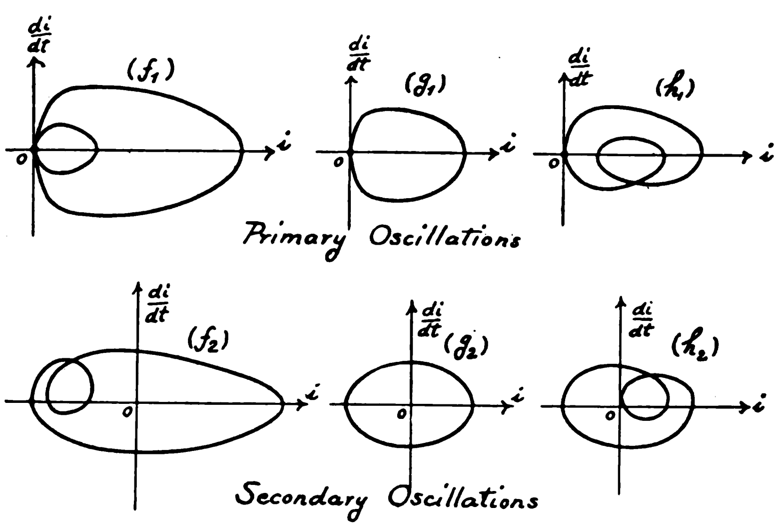

FIGURE 14—Cyclic Current Diagrams of Secondary Oscillations Corresponding to (*f*), (*g*) and (*h*) of Figure 13.

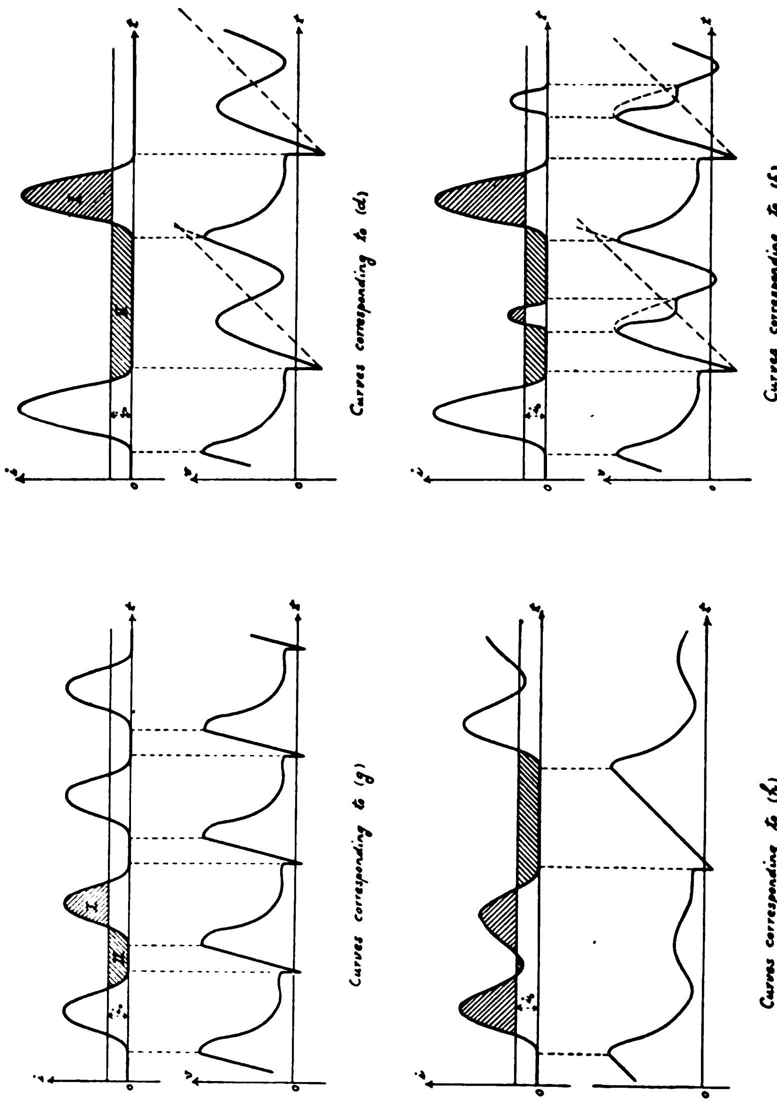

FIGURE 15—Curves of v and i Against Time, Corresponding to (*d*), (*f*), (*g*) and (*h*) of Figure 13

13 is given in Figure 15, which was plotted by the aid of the cyclic current diagrams, Figure 14 and the dynamic characteristics, Figure 13.

In Figure 13 and Figure 15

(*d*) is a case in which a reaction from the secondary oscillation induces a hump of E. M. F. in the primary while the primary condenser potential is going up along a straight line shown by the dotted line in Figure 15, that is to say while the arc is extinguished.

(*f*) is a case in which the reaction not only induces a hump of potential but also causes a separate feeble discharge to take place.

(*g*) The separate discharge is no longer feeble and the reaction makes the similar discharges occur with nearly double frequency.

(*h*) is a case in which a hump is induced by the reaction of the secondary while the current is flowing across the arc and its potential is nearly constant.

EFFECT OF VARYING THE FREQUENCY RELATION

The following is the result of observation with variable secondary capacity. C_1 was kept constant at 0.0297 microfarad and C_2 was varied from zero up to 5,900 micro-micro-farads. The supply current was about 4.2 amperes.

Figure 16 shows the deformation of the dynamic character-

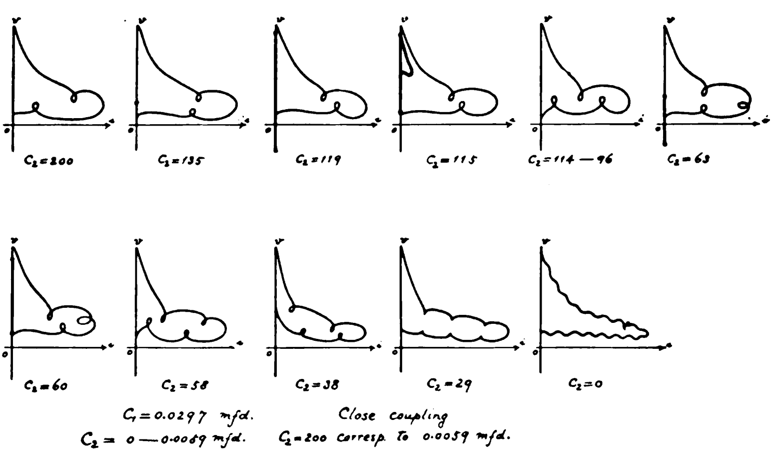

FIGURE 16—Variation of Dynamic Characteristics When the Reaction of the Secondary Oscillations at Higher Frequencies is Inducing Ripples in the Primary Oscillations

istics when the frequency of the secondary circuit was gradually increased.

Figure 17 shows the corresponding variation of the cyclic current diagrams of the primary oscillations.

Following the increase of the secondary frequency, the reaction upon the primary creates many ripples in each oscillation. One of the diagrams of Figure 17 is developed in Figure 18.

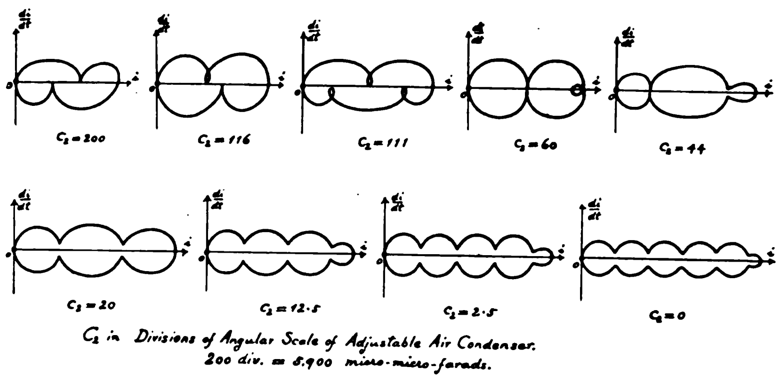

Figure 17—Cyclic Current Diagrams Corresponding to Figure 16

The effect of changing the distance between the primary and secondary coils is shown in Figure 19. There is no change of the number of ripples induced from the secondary, but their intensity and phase relation undergo slight change.

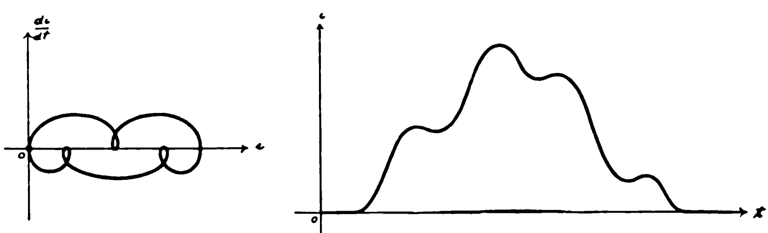

Figure 18—Developed i-t Curve Showing Ripples

Figure 20 shows the variation of the cyclic current diagram of the secondary oscillation, when its frequency is varied by varying C_2. It shows that the oscillation is not always purely

harmonic. It is also plain from Figure 20 that there are several maxima and minima of the intensity of secondary oscillation. This is exactly what the writer experienced with the discharge between metallic electrodes.[4]

Figure 21 gives the results of measurement of the secondary current by means of a hot wire ammeter.

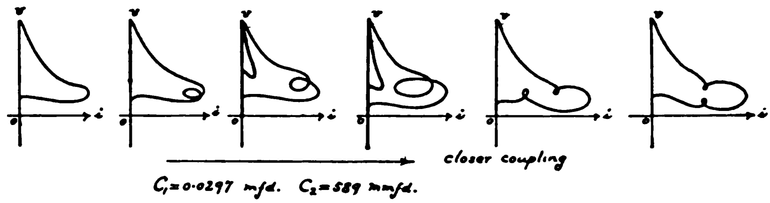

FIGURE 19—Variation of Ripples in Dynamic Characteristic Due to the Variation of Coupling

Another observation, with C_1 varying from zero to 2,750 micro-micro-farads and C_2 from zero to 3,175 micro-micro-farads and a small supply current of 0.43 ampere, proved the existence

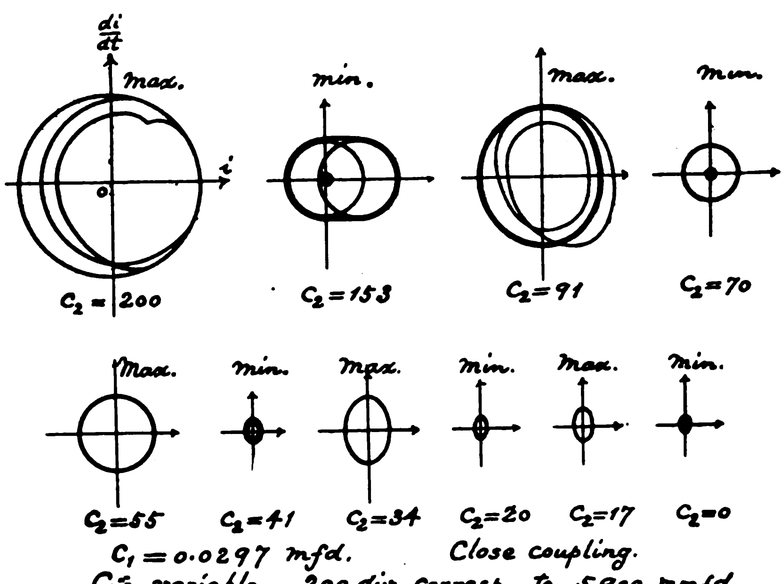

FIGURE 20—Cyclic Current Diagram of the Secondary Oscillation

[4] H. Yagi, "Electrician," Vol. LXXVI, p. 195, 1915.

of the same kind of reaction and also the peculiar fluctuation of the secondary current. (Figure 22).

In Figure 21 and Figure 22 the maxima of the current effect with respect to C_2 occur with regularly increasing intervals.

Let τ_1 be the period of the fundamental oscillation of the primary, or the time interval between two consecutive impulses which are given by the primary to the secondary, and τ_2 the period of the secondary free oscillation; then, when $\gamma = \frac{\tau_1}{\tau_2}$ is an integer, there are maxima of the secondary current. The

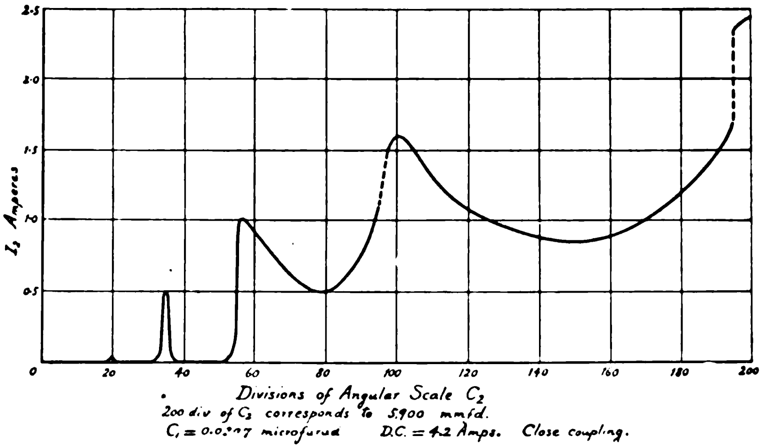

FIGURE 21—Variation of Secondary Current

reasoning is quite analogous to that for the production of sustained oscillation by means of quenched sparks[5], namely, the intensity of the secondary oscillation is the greatest when each fresh impulse from the primary occurs at such time as to be in step with the persistent oscillation already created in the secondary circuit.

As long as the period between two successive discharges in the primary is kept constant, the interval between maxima of Figure 21 and Figure 22 must depend only upon the period of the secondary oscillation which is proportional to $\sqrt{C_2}$.

Figure 22 also shows the dependency of these intervals upon C_1.

It was confirmed by experiments that there can be neither

[5] H. Yagi, loc. cit.

the reaction nor the periodic fluctuation as described, when the primary frequency is larger than the secondary frequency. The dynamic characteristic has shown no ripples and the cyclic

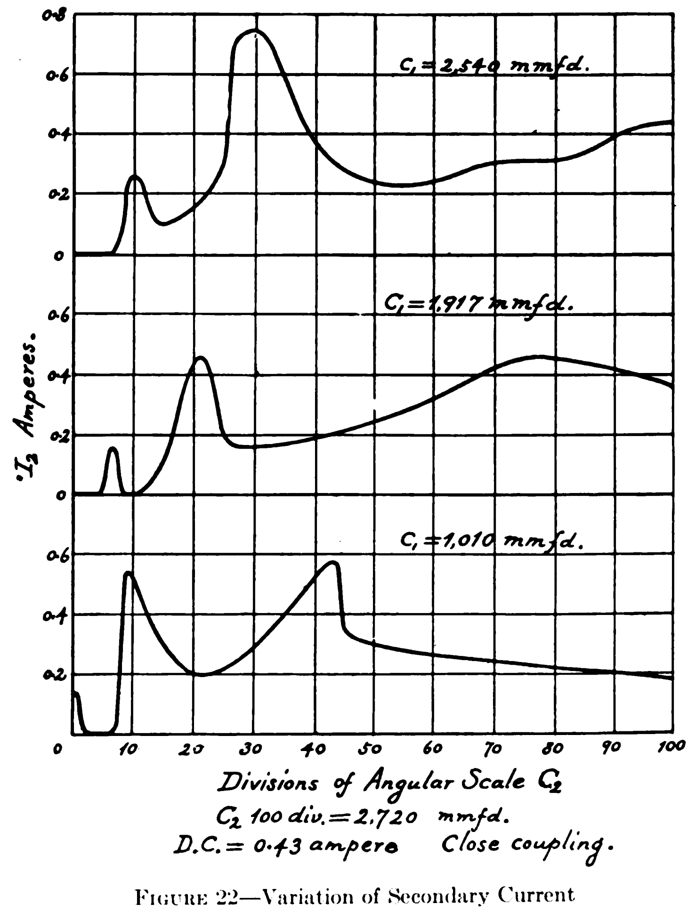

FIGURE 22—Variation of Secondary Current

current diagram of the secondary oscillation was a smooth circle.

When C_1 was further reduced, the arc oscillation of the second type became one of the first type and then ceased to oscillate.

EXPERIMENT WITH SMALL ARCS

In order to bring the carbon arc nearer to a state of spark, a higher resistance was introduced into the supply circuit, which brought the supply current down to less than 0.4 ampere.

C_1 and C_2 were 0.0297 micro-farad each, and L_1 and L_2, 55 micro-henrys each.

Now the arc was very often mixed with sparks, which could be recognized by the following facts.

Figure 23 shows the cyclic current diagrams of the primary oscillation, in which thicker lines correspond to arcs and finer

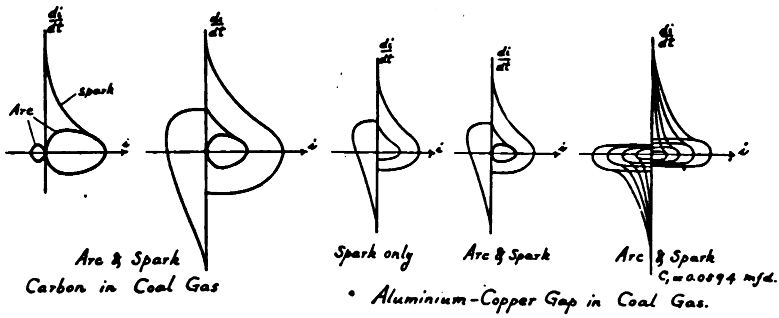

FIGURE 23—Cyclic Current Diagrams of the Primary Oscillation Showing the Mixed Occurrence of Arcs and Sparks

lines to sparks. The diagrams of metallic arcs and sparks are given for comparison to show the similarity. Spark could generally be noticed by hissing sounds, and, when only sparks occurred, there was no glow of the carbon electrode.

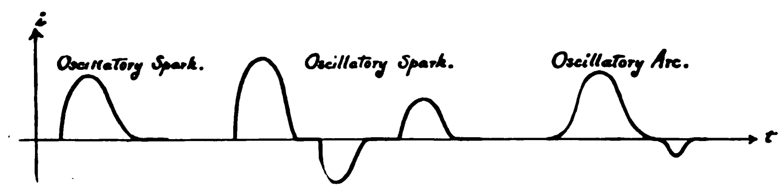

FIGURE 24—Variation of Current with Time in Oscillatory Arc and Spark

In the cyclic current diagram of Figure 23, the initial value of $\frac{di}{dt}$ is very large for a spark and it shows that the current increases very suddenly as shown in Figure 24.

It may be noticed from Figure 23 that, in cases of metalic arcs, the initial value of $\frac{di}{dt}$ is not zero, but has a certain magnitude; in other words, the current curve with respect to time does not start in a horizontal direction.

Figure 25 shows the cyclic current diagram of the secondary oscillation. The cyclic current diagram of a sustained harmonic oscillation is to appear as a circle or an ellipse upon the screen, and Figure 25 shows clearly that sustained oscillations are created in the secondary. The method was found very convenient by the writer[6] for the demonstration of damped and sustained harmonic oscillations.

It may also be seen from Figure 25 that a spark can produce

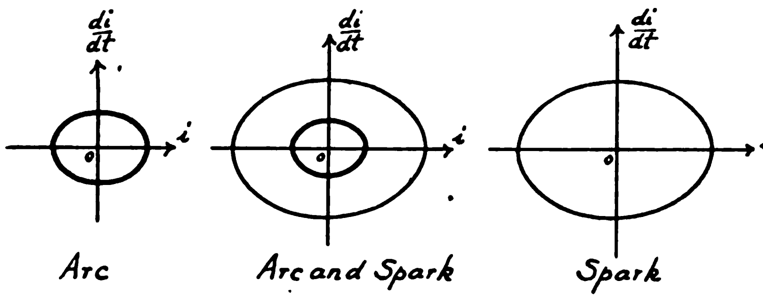

FIGURE 25—Cyclic Current Diagrams of the Secondary Oscillation Produced by Small Arcs and Sparks

oscillations of larger amplitudes in the secondary than an arc under the same conditions.

Many thanks are due to Professor J. A. Fleming for enabling me to carry out experiments in his research laboratory; and to Professor George W. Pierce and Dr. E. L. Chaffee for giving me valuable advice while working out this paper in the Cruft Laboratory, Harvard University.

SUMMARY: 1. When a secondary circuit is closely coupled, its reaction causes variation in the primary oscillations and consequently in the secondary oscillations themselves.

2. The cyclic current diagrams afford a means of studying the variation of current with time, which cannot be easily done by the dynamic characteristics.

3. There are favorable conditions for maximum current effect in the secondary. They depend on the ratio of the frequency of the separate discharges to that of the secondary oscillation.

4. A spark creates more powerful oscillations than an arc of the same current strength, tho the former are not quite as steady, owing to the necessarily higher discharge potential.

The above principles are fully illustrated by a number of experimentally obtained dynamic characteristics and cyclic current diagrams.

---

[6] H. Yagi, loc. cit.

DISCUSSION

Jonathan Zenneck: Some six years ago one of my students, S. Subkis, made an investigation into the behavior of coupled arc generators along the same lines and using almost the same experimental methods as Professor Yagi. His paper has been published in the "Jahrbuch der drahtlosen Telegraphie," Volume 5, 1911, pages 507 and 545.

The work of Subkis was restricted to audio frequency currents. The present paper therefore goes beyond what Subkis did in dealing with far higher frequencies and therefore approaching much more nearly the practical working conditions.

The method of Professor Yagi, which consists in having the cathode ray beam of a Braun tube plot the derivative of the current with respect to the time against the current itself may prove useful for analyzing the form of radio frequency currents. The question of whether a radio frequency current is sinusoidal or not cannot generally be answered by the inspection of instantaneous photographs on a moving photographic plate, since the phosphorescent spot on the screen of the Braun tube is not bright enough. In this case, the method of Professor Yagi may be employed, making use of the well-known principle of having the curve on the phosphorescent screen repeated every period, thereby furnishing a curve bright enough to give a good photograph permitting analysis of the current curve.

Cyril F. Elwell (by letter): I have read the paper with much interest and the author is to be commended on a very painstaking piece of work. However from the point of view of working installations the very variable factor of the strength of the magnetic field is left out of count and there is no doubt that if the author continues his study, taking account of the strength of the magnetic field, speed of rotation of carbon, arc length, etc., he will delve into a very useful and profitable field. The range of secondary capacities taken was good for the size of arc experimented with, but what I would like to see some light on is the problem of the very large arc on large capacities. It is not easy to obtain a large arc on which to experiment, so that the experiments on a small arc should take into consideration all the variables which have to be taken into consideration in large arcs, in order to throw any light on the large arc problem. Then again factors which would disturb the quality of emitted

oscillations in a small arc would be inconsequential in a large one. One of the troubles with the critics of the Poulsen arc is that they base their opinions as to the continuity of the oscillations on home-made arcs, and from the same data decry the possibility of building large arcs, as witness even recent writers who give as their opinion the impossibility of building arcs of 100 kilowatts and upward. In my opinion there is no difficulty in building an arc as large as required, the difficulty being to build an antenna to radiate a useful part of the energy. The radio art is indebted to Mr. Hidetsugu Yagi for his valuable contribution to theory. It is to be hoped that he will continue his investigations and contribute the results in the near future.

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ALFRED N. GOLDSMITH, Ph.D.

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PHYSICAL ASPECTS OF RADIO TELEGRAPHY *

By

JOHN L. HOGAN, JR.†

(CHIEF RESEARCH ENGINEER, NATIONAL ELECTRIC SIGNALING COMPANY)

Very soon after wire telegraphy was first accomplished, conditions were encountered in which the desirability of effecting electrical signaling without connecting wires became apparent. Islands were to be reached by telegraph, and rivers were to be crossed. It was difficult to keep cables in operation in some of these locations, and some means of eliminating the wire connection was therefore sought. As a result of this need, there arose a number of methods of telegraphing without wires, some of which were based on conduction, others on magnetic induction, and still others on electrostatic induction. It was not at all difficult to explain the operation of any of these; in the conduction system the numerous paths of current thru the earth or water could be traced by imaginary lines, and in the induction systems the lines of force could be visualized. The physical mechanism of the transmission was as clear to nearly all the workers in the telegraph art of that period as was the mechanism of the ordinary wire telegraphy.

When radio telegraphy was introduced, however, " wireless " became a mystery to most people. Communication between ships, and from ship to shore stations became common, and the attention of the public was attracted more strongly than it had been during the lives of the older systems. Scientific interest was aroused all over the world as a result of Marconi's first experiments. Scientists, engineers and laymen in all civilized countries attempted to duplicate the early apparatus and to secure similar effects. Not many of the experimenters understood the physics of what they were attempting, and this condition gave rise to a

* Paper presented before the Pan-American Scientific Congress, at Washington, D. C., for The Institute of Radio Engineers, on January 4, 1916.

Presented at Baltimore, Md., under the joint auspices of the Baltimore Section, American Institute of Electrical Engineers and the Department of Engineering, Johns-Hopkins University, on February 11, 1916.

† Alternate to Second Pan-American Scientific Congress, for The Institute of Radio Engineers. Vice-President of The Institute of Radio Engineers, New York.

semi-technical and technical literature, a large part of which it is almost shocking to consider. Such conceptions as the operation of a coherer by the impinging of waves directly upon it, and the deflection of waves from the " ether " to the coherer by means of an elevated wire, were carried into the new longer-wave radio telegraph art from the old laboratory experiments with electromagnetic waves a few centimeters in length. Ideas of radiation from antennas, the production of waves by impulsive spark discharge, the role of the earth in transmission, and related matters were extremely vague.

Of course, the general rule of indefinite and erroneous ideas as to the physical mechanism of radio telegraphy was proven by a few notable exceptions. Some few earnest workers in the new art undoubtedly had good engineering and clear physical conceptions of radio telegraphy, even in the earliest days. Their writings during the few years near 1900, prove definitely that their ideas were clear; in fact, the records of that time are very fair indications of the mental attitudes of the numerous persons then interested in radio signaling.

The veil of mystery which covered radio workings almost completely was not in any way lessened by commercial operations during the years which followed practical applications of the new art. From about 1904 to 1909, the entire radio art fell into commercial disrepute in the United States, thru the illegitimate commercial operations of several notorious stock-selling agencies. This unfortunate condition undoubtedly had a great affect upon the development of radio technology, since it repelled earnest scientific investigators. It is probable that many of the engineering problems of radio telegraphy which have only been solved within the past three or four years, would have been overcome much earlier had not radio telegraphy been so violently exploited commercially.

Within the past five years, the realization has been growing that there is essentially no reason why radio telegraphy should be made a mystery. It is of course true that ultimate causes and ultimate effects are, and will probably remain, mysteries to all of us; nevertheless, we do secure and make use of knowledge concerning immediate causes and their effects. We collect information as to what actions produce certain results, and, by correlating these in quantitative systems, create our applications of the physical sciences.

Radio telegraphy is now, and has been for some little time, at that stage of its technical development where it can be considered

to consist of a series of expected effects, resulting from a series of controllable causes. In other words, radio telegraphy is upon an engineering basis. Its instruments can be designed to meet various needs, and their operation can be predetermined with accuracy. The physical nature of these controllable causes is closely allied with that of the elements of all electrical systems; in fact, apparatus for radio telegraphy consists merely of varied aggregations of the same physical elements which are used in all branches of electrical engineering.

The scope of natural phenomena made use of in radio telegraphy, is, however, considerably greater than that which occurs in any single branch of ordinary engineering. The electrical actions of power transmission, the conversion of current frequencies, the peculiar conditions in transmission lines of distributed electrical constants, the free wave-motion effects of radiation, reflection, refraction, interference and absorption, the delicate engineering problems of small powers, such as occur in telephony, and the physics of sound generation and recording are by no means all of the important fundamental actions involved.

When the application to radio telegraphy of the physical elements using these principles is considered in greater detail, it is found that the units are merely those of every-day engineering. The inter-relations are perhaps peculiar to the conditions of radio signaling, but the elements are the same. Radio telegraphy is merely an additional practical system for the communication of intelligence, and, like all such systems, depends fundamentally upon only three parts; viz., a medium of transmission, a means to excite this medium, and a means for observing the excitation of the medium. For example: in ordinary speech, the vocal system sets into vibration, according to a conventional code called a language, a medium of transmission (which is, in this case, the atmosphere). The air vibrations, like other free waves, travel thru space in all directions, and are intercepted at the receiving point by an ear and auditory system which retranslate them into the language code. Thus intelligence may be transmitted in ordinary conversation, by the interaction of the three fundamental elements above set forth.

These three elements are present in all communication systems. In the wire telegraph, the battery and key are the exciting-source of the transmitter, the transmission medium is the hypothetical ether surrounding the line wire which guides the electromagnetic pulses to the receiver, and the final element is the telegraph sounder which magnetically observes the effects pro-

duced by the transmitter. Similarly, in the telephone, the currents over the line wire are an inseparable part of the electromagnetic disturbance produced by the combined action of the transmitting battery and modulating microphone; these voice-modulated disturbances and their accompanying currents are observed magnetically by the telephone receiver at the distant station.

In radio telegraphy, the three fundamental physical elements are also present. The medium of transmission is the same luminiferous ether which is assumed to carry light vibrations thru space. The transmitter may consist of any one of a large number of forms of apparatus which will serve to vibrate this ether of space according to some pre-arranged code. The receiver, likewise, may be any one of many types of instrument which will serve to detect ether vibrations and to produce appreciable effects, dependent in occurrence or vigor upon the intensity of the ether vibrations.

In the first transmitters for radio, a spark gap was connected across the secondary terminals of an induction coil. one side of the spark gap was connected to earth and the other side to an elevated insulated conductor as in Figure 1. Upon each inter-

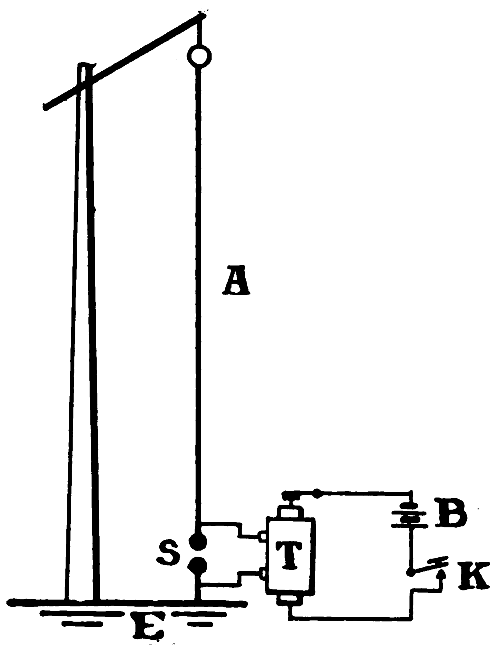

FIGURE 1

ruption of the coil's primary circuit, a surge of potential developed in the secondary and charged the elevated wire to a definite potential with respect to the earth, so storing in it a definite amount of energy depending upon the voltage and capacity of the system. If the spark gap was sufficiently narrow, the potential would reach a value more than high enough to rupture the air between the electrodes, and the energy which had been stored in the aerial wire would discharge across the spark gap in a rapidly damped oscillation. That is to say, during the instant of passing of the spark, an alternating current of very high frequency and having constantly decreasing maxima would exist in the aerial wire. The frequency of this alternating current is dependent upon the capacity and inductance of the antenna system; and it was soon found that by inserting lumped inductance at the base of the antenna, as in Figure 2, the frequency of oscillation

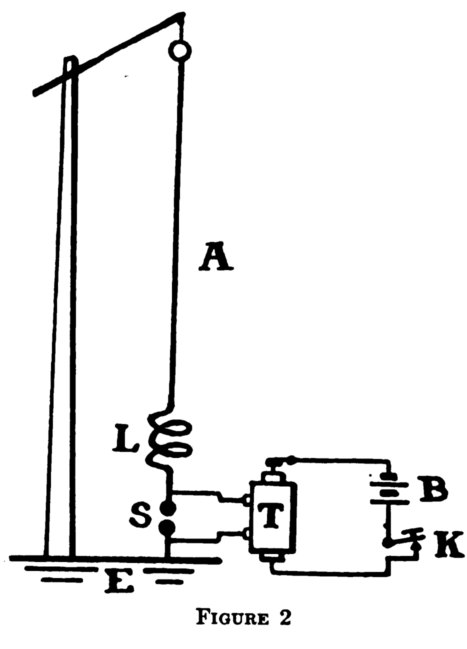

FIGURE 2

could be reduced as far as desired. It was also found that if instead of charging the antenna and allowing it to discharge directly across a spark gap, a condenser in a separate circuit were charged and allowed to discharge thru an inductance, this inductance might be magnetically coupled to the antenna system, as

in Figure 3, and would, if the capacity and inductance relations were correctly adjusted, produce a more intense and less rapidly decadent high frequency current in the aerial wire.

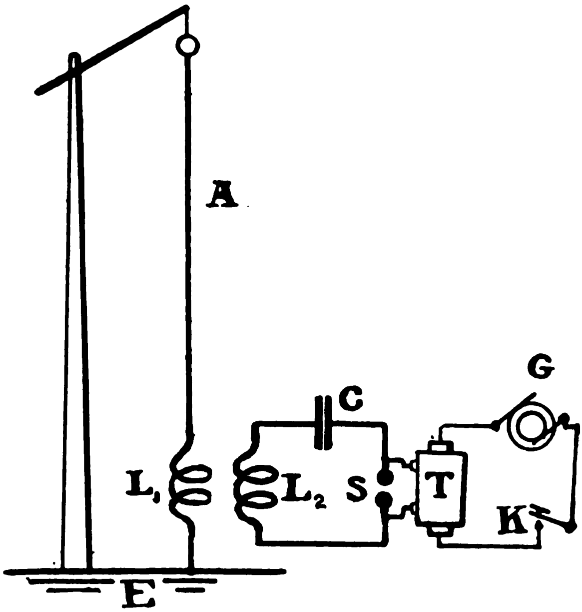

FIGURE 3

It was also found possible to generate high frequency alternating currents directly in the antenna system by making that conductor a part of the circuit supplied from a special high frequency (or so-called radio frequency)* alternator. This arrangement results in an absolutely uniform flow of high frequency current in the antenna system, so long as the alternator is connected and in operation. This is shown in Figure 4.

The effect of any alternating current in any conductor is to produce in the ether about it interlinked magnetic and static fields. These fields whirl, reverse, expand and contract according to the variations in direction and intensity of the current in the conductor. Figure 5 shows the wave field spreading from such a system. If the frequency of the current becomes high, (of "radio frequency"), say beyond 10,000 cycles of alternation per second, a large part of its energy is sent off into space as electromagnetic

* Frequencies above 10,000 cycles per second are arbitrarily called "radio frequencies."

waves, which always have the same frequency as the current. If, as in a radio telegraph antenna, the conductor carrying the radio frequency current is partly or wholly vertical and has one

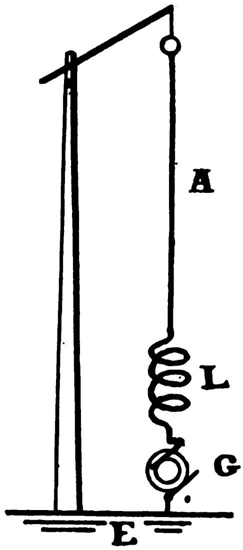

FIGURE 4

end connected to earth, the electromagnetic waves which are sent off will have inseparably associated with them alternating

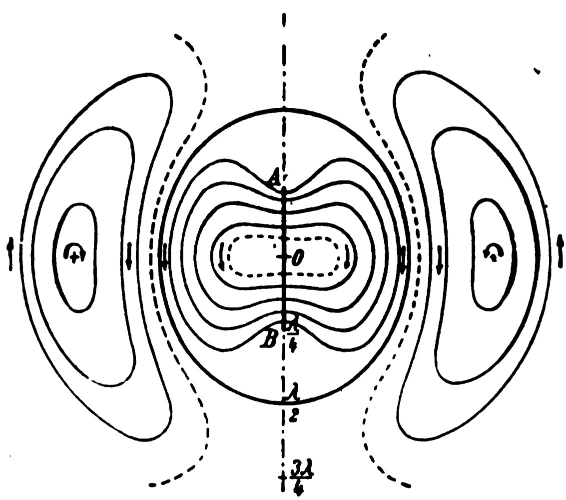

FIGURE 5

currents in the surface of the earth. The waves will spread in all directions from the transmitting aerial, but will always have their bases terminating upon the earth's surface, as in Figure 6.

It is a physical property of such travelling electro-magnetic waves that they set up alternating currents of their own frequency in any conductors upon which they impinge. If we

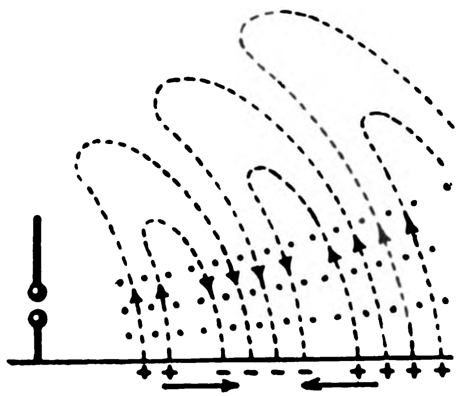

FIGURE 6

project into the air an antenna system consisting of a number of wires, and connect the lower end of this aerial to the ground, (perhaps thru a suitable observing instrument, as in Figure 7), the electromagnetic waves which arrive at this structure will set up in it alternating currents. If these currents are sufficiently intense, their presence may be detected by the indication on a thermo-ammeter placed in series at the base of the aerial wires; if, as is common in radio receiving stations, their amplitude is quite small, some more delicate receiver will be necessary to indicate their presence. Obviously, if this receiver is of such form that it gives a visible or audible signal to show the time of beginning and cessation of the arrival of electromagnetic waves, it may be associated with a distant controllable transmitter for the purpose of communication. Any convenient code of signals may be used, for instance that devised for use on the Morse telegraph. To transmit actual messages, then, it is only necessary to set up short and long series of waves, corresponding to dots and dashes, and, at the receiver, to produce short and long effects in accordance with these short and long series of waves.

The receivers originally used were very crude, and usually consisted of imperfect contacts which were adjusted to be just

on the point of closing. The strong voltage impulse set up in an aerial system by a powerful arriving wave was conveyed to the terminals of such a loose contact instrument, or coherer; and was usually enough to complete the closure of a battery and relay circuit also connected across the coherer terminals, as in Figure 8.

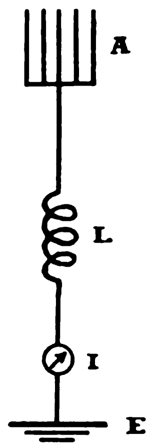

FIGURE 7

After the receipt of each signal this device required mechanical or other agitation, in order to break its internal circuit and prepare it to indicate another signal. In this type of apparatus the intensity of the first impulse received was of the greatest importance; and the operation was not aided in any way by waves received after the first strong disturbance. It was soon found that greater final sensitiveness could be secured by using thermal or electrolytic receivers, which rectified and added together the small effects produced by each individual wave of a signal, and gave a cumulative indication on some instrument such as a telephone. Practical firm-contact solid rectifiers were later discovered, which acted upon the cumulative and proportional principle and also produced indications by conversion of the radio frequency antenna currents into pulsating currents which could operate a telephone. Still later such vacuum tube rectifiers as the audion, which combines unidirectional conductivity and amplifying action and thus gives great sensitiveness, were developed. In connection with ordinary spark transmission all these receivers have proved useful; for operation with the continuous

waves produced by direct operation of a radio frequency alternator other and more complicated forms of receiver have been devised.

Electromagnetic waves produced by radio telegraph transmitters are not the only disturbances which vibrate the medium of transmission which we call the ether. This same hypothetical body conveys those extremely rapid vibrations which produce the sensation of light, and those, somewhat lower in frequency, known as radiant heat. The range of vibration frequencies for these two classes of electromagnetic waves lies well above ten

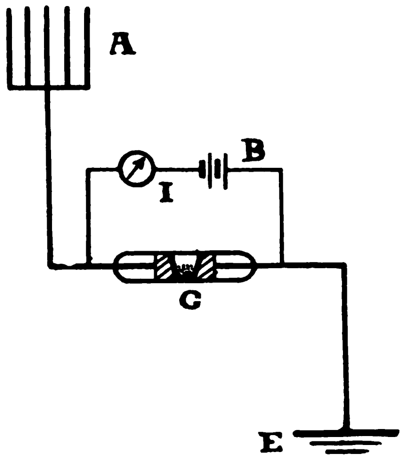

FIGURE 8

million per second, and hence they produce no appreciable direct effect upon radio telegraph receiving stations. The wave frequencies used in radio telegraphy are in general from one million down to as low as twenty thousand per second. Even within this range, however, there are natural ether waves, caused by various forms of electrical disturbances in nature, which produce unfortunately violent effects at radio telegraphic receivers. The natural atmospheric electrical disturbances may be divided into two main classes, those produced by "static" and those produced by "strays." Static effects appear, in general, as currents flowing between the aerial system and earth as a result of the discharge into the antenna of the electrification of dust and moisture particles drifting thru the atmosphere. Ordinarily these discharges create little or no difficulty in radio telegraphy, and they are

usually not concerned with electromagnetic radiation, except when they are sufficiently violent to set up waves travelling outward from conductors in which they pass to earth. Strays, however, are looked upon as electromagnetic impulses, (often of great violence), which are set up in the ether by some distant and usually powerful electrical discharge; e.g., lightning. These heavy impulses ordinarily produce, in receiving aerials, damped alternating currents of the natural or tuned frequency of the aerial wire system.

In order to appreciate more fully the practical inter-linkage of these various physical characteristics, it is desirable to consider a little more closely what occurs during the operation of the several main types of transmitting and receiving apparatus.

In the plain aerial instrument of Figure 1, the total antenna resistance is very large; and as a result only a few electrical oscillations take place before the amplitude of the radio frequency current reaches a practical zero. A discharge of this sort is shown in Figure 9, in which antenna oscillating current is

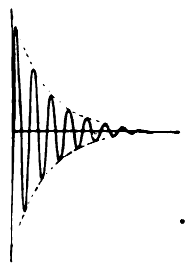

FIGURE 9

plotted against time. Assuming the aerial to be 250 ft. (76 meters) in height, the frequency of the oscillating current will be about 750,000 per second, and the length of the radiated wave about 400 meters. When series inductance is inserted, as in Figure 2, the wave length and persistence of the system are increased, and the discharge is more nearly that represented in Figure 10. Assuming the same aerial, and a loading inductance of 0.5 millihenry, the frequency is decreased to 330,000 per second, which makes the radiated wave length about 900 meters. If the coupled circuit of Figure 3 is used, the oscillating currents in the closed circuit X induce similar oscillating currents in the open circuit Y; these two condenser circuits are adjusted to have

practically the same natural period of vibration. The antenna current in this case is still less strongly damped, and, when the spark gap circuit is properly designed, may be represented by the graph of Figure 11. It will be noted that the persistence of antenna current increases in each of the successive types, and

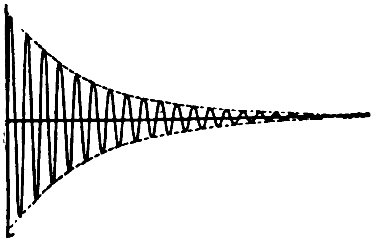

FIGURE 10

that in Figure 11 the decrement (an indication of the rate of amplitude decrease with time) is comparatively small. For the sustained wave alternator sender of Figure 4, the radio frequency current has a constant amplitude, as shown in Figure 12, and the

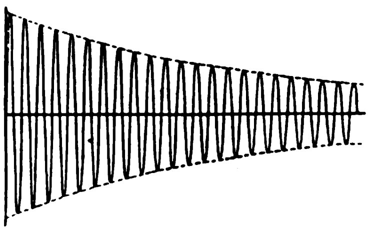

FIGURE 11

decrement is therefore zero. These four last figures, 9 to 12 inclusive, represent not only the amplitude variation with time of the current in the antenna, but also that of the electromagnetic wave in space. This wave upon reaching a receiving aerial sets up in it currents of identical frequency and (except in the case of sustained waves) a somewhat greater decrement, the latter depending upon the constants of the receiving apparatus.

The illustrative receiver of Figure 7 is shown as comprising an antenna, an inductance coil, and a thermo-ammeter connected in series to earth. These elements form an oscillating electrical system having a definite resonant period dependent upon its

capacity and inductance. The effective sharpness of resonance in the circuit will depend upon the persistence of the incoming wave and the resistance of the receiver as a whole. Greatest selectivity is secured when the receiver resistance and the decre-

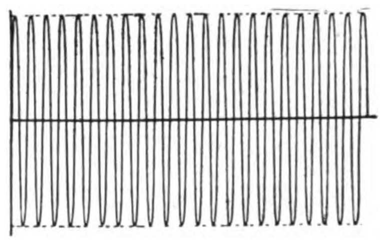

FIGURE 12

ment of the incoming waves are a minimum. Evidently the later types of transmitter, having a maximum persistence, give the greatest freedom from interference between stations. Re-

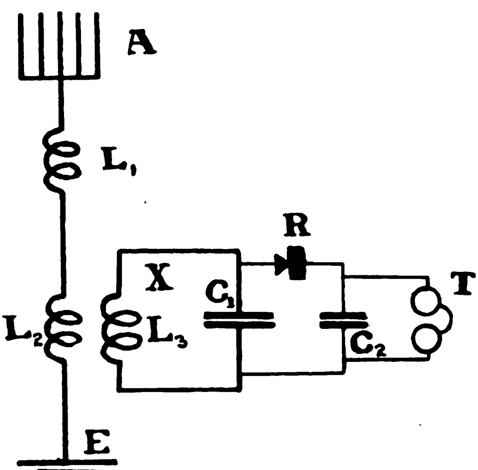

FIGURE 13

ceivers designed for continuous waves are in general least affected by impulsive disturbances; therefore the harmful effects of strays or atmospherics are minimized when this type of apparatus is used.

A modern form of receiver for grouped-wave operation is

shown in Figure 13, and consists of an antenna and ground having connected between them suitable inductances to make the resonant period of the aerial system agree with that of the incoming wave. Magnetically linked to one of the antenna inductance coils is a secondary coil L_3. This secondary has connected across its terminals a condenser C_1, which permits the resonant period of the closed circuit X to be brought into accord with that of the incoming wave and the antenna system. The rapidly alternating potentials developed across the condenser C_1 are applied across the rectifying detector R and condenser C_2, and, because of the asymmetric resistance characteristic of the detector R, the condenser C_2 is charged in one direction during the receipt of a wave train. The charge which it takes is then discharged thru the telephone T and produces a movement of the diaphragm.

The operation of the entire transmitting system may become more clear by reference to Figure 14. Here the axis P shows the

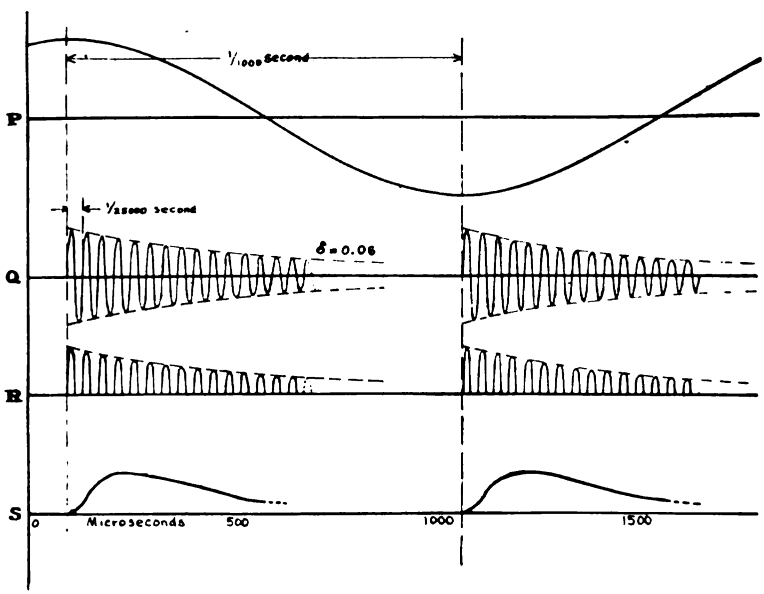

FIGURE 14

secondary potential of the power transformer at the transmitting station, plotted against time. Assuming a frequency of 500 cycles per second, which is common for spark telegraphy, the transmitting condenser is charged first in one direction and then

in the other, at intervals of 1/1000th of a second. At the instant of charge to maximum potential, indicated by the vertical lines, the condenser discharges across the spark gap (see Figure 3), with oscillations, as shown on the Q axis of Figure 14. These oscillating currents have a frequency dependent upon the capacity and inductance of the closed circuit X, and for this graph are assumed to oscillate at 25,000 cycles per second, corresponding to a 12,000 meter wave length. The wave train is quite persistent, consisting of some 30 oscillations before reaching a small value, and therefore lasts for about 0.00075 second. Such groups of oscillations follow each other at intervals of 1/1000th of a second; when repeated for each condenser discharge, they result in practically identical currents in the antenna circuit Y, practically identical waves in the ether between sender and receiver, and practically identical currents in the receiving antenna circuit and in the receiver's closed circuit X. This same graph, along the Q axis of Figure 14, represents the alternating voltage impressed across the detector R and condenser C_2; the current thru the rectifier is indicated by the half waves shown on the R axis of Figure 14. These charge the condenser which discharges thru the telephone in current pulses such as shown at S, one pulse for each group of waves. If the transmitting sparks occur one thousand times per second, one thousand current pulses pass thru the telephone windings in one second, and the telephone gives off a tone having a sound frequency of one thousand per second or corresponding approximately to the second C above middle C on the musical scale. By holding the sending key down for a short time, say one-twentieth of a second, fifty of these wave groups are sent out, a short tone is heard in the telephone receiver, and a Morse dot is signalled. By holding the key down approximately three times as long, one hundred and fifty trains are emitted, a longer tone is heard in the receiver, and a Morse dash is indicated. Thus short and long pressures of the key at the transmitting station may be translated, according to the Morse code, at the distant receiver.

The above explanation applies to any of the grouped-wave transmitters, and shows how a tone is produced at the receiving station. If the wave trains are uniformly spaced, and occur at a fairly high rate, the response of the telephones is musical; if the sparks are irregular in occurrence, hissing or scratching sounds will be produced in the telephone. Atmospheric impulses or irregular disturbances of any sort produce such irregular sounds; therefore, by making the spark rate high and definite, it is pos-

sible for the receiving operator to distinguish easily between the musical signal tone, carrying the message which he desires to interpret, and the interfering noises from strays. This method of reducing the disturbing effects of atmospheric has been found in practice to be most effective.

When sustained waves are emitted, as by the transmitter of Figure 4, the stream of current in the antenna is ordinarily constant and uniform during the times the key is held down. At the receiver there is consequently no tone-effect, such as that just described, unless an interrupter or its equivalent is placed in some one of the circuits. A rotary circuit breaker may be placed in series with the antenna at the transmitter, or at the receiver, and will produce an action indicated in Figure 15. In these graphs the sustained radio frequency current of 25,000 cycles per second, is shown along the axis T. At U it is shown broken up into groups succeeding each other at the rate of one thousand per second. This graph represents the potentials which are applied to the detector R in Figure 13, and which result in rectified potentials such as shown on axis V, and pulsating currents thru the telephone winding as shown on axis W. Morse signaling is effected by short and long pressures of the key, as before. This interrupter method of using sustained waves gives a pure note in receiving, but is not notably efficient. A somewhat analogous interrupter method of receiving sustained waves involves the use of a rapidly vibrating contact called a tikker, at the receiving station; this contactor replaces the rectifier R, in Figure 13, and operates quite sensitively. It does not produce a musical tone in the telephone, however, and so telegraphy in which it is used is unnecessarily subject to interruption because of atmospheric interference. One modification of this device varies its resistance, instead of actually opening the circuit; another, by relating properly the wave and interrupter frequencies, produces a whistling signal sound.

The most satisfactory method of receiving sustained waves does away with all interrupters and vibrating contacts, and produces pure musical signal tones by the interaction of two radio frequency alternating currents or potentials. The apparatus which is used in a preferred form of this heterodyne receiver is shown in Figure 16, where the left hand side represents the ordinary receiver as shown in Figure 13, having added to it, however, a generator of feeble radio frequency currents in the circuit F, G, H. It is a well known physical principle that the addition of sine waves of slightly different frequencies results

in a composite wave of a mean frequency which varies in amplitude periodically at a rate equal to the numerical difference in the component frequencies. The musical beats noted in the

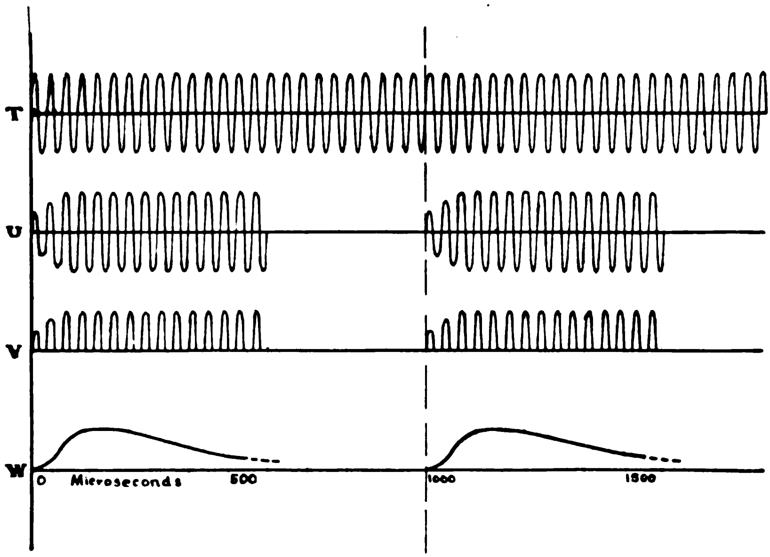

FIGURE 15

tuning together of mandolin strings, and the flickering of synchronizing lamps in power generating stations, are familiar examples of beats occurring according to this principle. The

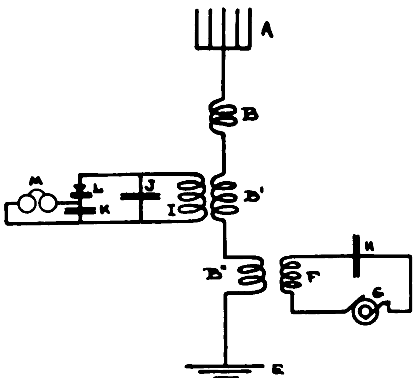

FIGURE 16

application to radio receivers becomes clear from a reference to Figure 17. Here the antenna current generated by the incoming sustained wave is indicated along the *A* axis. The current produced in the receiving antenna system by the local generator, having a slightly different frequency, is shown along the *B* axis. These currents add together algebraically, and, by their inter-

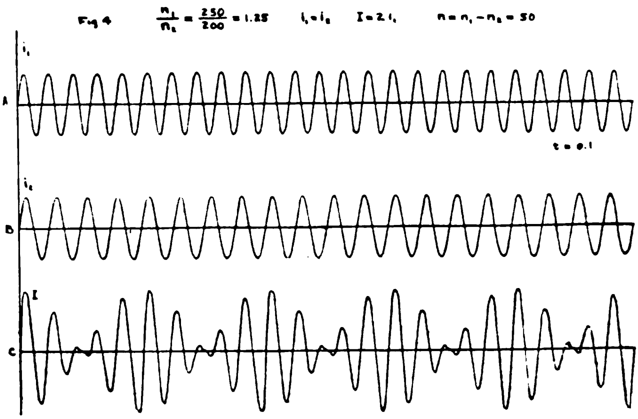

FIGURE 17

ference, produce beats at a rate equal to the difference between them, as shown along the *C* axis. If the incoming wave is of a frequency of 100,000 cycles per second and the locally generated current is of frequency 101,000, the beat frequency is one thousand cycles per second, the numerical difference of the components. Referring to Figure 18, the graph along the *C* axis may be taken to represent the potentials applied across the detector *L* of Figure 16; these are rectified as shown on the *D* axis, Figure 18, and result in current pulses at the rate of one thousand per second, passing thru the telephones, as shown by axis *E*. In this heterodyne receiver the response depends upon the interaction of two sustained radio frequency currents, and is its maximum when both of these are of zero decrement. The signal tone is purely musical, and may be varied in pitch to any point desired merely by altering slightly the frequency of the local oscillator. The circuits used are designed for maximum effect at the maximum persistence, since they depend for their action upon receipt of sustained waves. These three points co-operate to eliminate largely disturbances due to strays; and by taking advantage of

the three features it has been found possible to secure the maximum freedom from interruption by atmospherics.

Having considered the qualitative physical relations involved in practical radio telegraphy, some features of the quantitative study may now be outlined. Before this is done, however, it is desirable to determine what requirements must be met by radio in order to give commercial service. There are, of course, all degrees of "commercial" service; radio should be expected, however, to perform as well or better than could a wire telegraph or submarine cable interconnecting the points served by the radio stations. Operation of this character may be expected if the following main requirements are met:

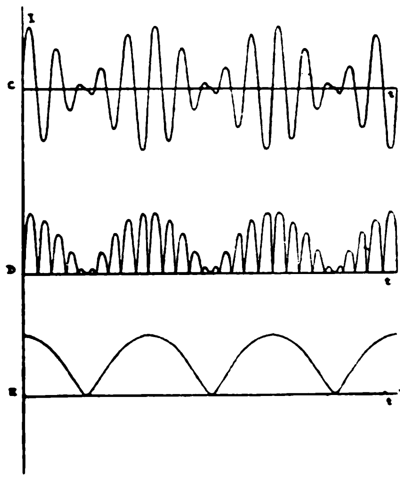

FIGURE 18

(1) The stations should be duplexed, so that messages may be sent in both directions at the same time, in order to save delays enforced by simplex sending.

(2) Spare transmitting and receiving apparatus should be installed, and the units should be so designed that twenty-four hours' service can be furnished per day of operation.

(3) The received signal should be musical, and of intensity sufficient to permit the operator to copy messages directly upon a typewriter, day or night, winter or summer.

Taking these up in order, it is evident that duplex transmission can be effected if the two senders operate on somewhat different wave lengths and if at each end of the link the trans-

mitter and receiver use well separated antennas. Referring to Figure 19, if *A* and *B* represent transmitters at New York and San Juan, Porto Rico, 1,400 miles (2,200 km.) apart, *A* and *B* may be considered to send on wave lengths of 5,000 and 6,000 meters, respectively. If then, about fifteen miles from each of the transmitters, there is installed a receiving station, it will be possible for the receiver *A* near New York, to copy signals from the San Juan transmitter *a* on a wave length of 6,000 meters

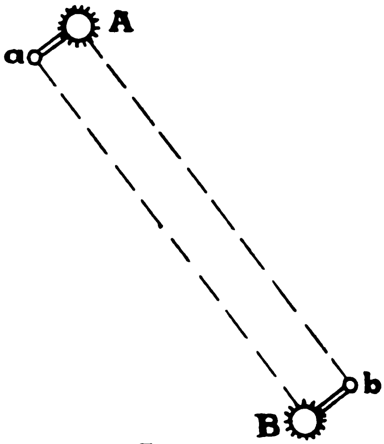

FIGURE 19

without interference from the local transmitter *A*. Simultaneously the Porto Rican receiving station *b*, will be able to read messages from New York *A* on 5,000 meters, without interference from *B*. By making *a* and *b* the operating stations, and controlling the transmitters at *A* and *B* by relays operated thru land lines from the operating to the power stations, genuine duplex transmission can be effected.

As to the second point, reliance must be placed entirely upon the designers of the apparatus to be used. No instrument should be installed which has not given evidence of its ability to perform satisfactorily over long periods, without requiring undue attention from experts.

Intensity of received signals, the third point above, is to be secured in several different ways. It has been demonstrated, however, that the use of very sensitive detectors or amplifiers cannot be depended upon, for the reason that atmospheric dis-

turbances are magnified as much or more than the desired signals. In the absence of severe stray disturbances, it is possible to signal over long distances with small transmitted power, by using intensifying and extremely delicate receiving instruments. However, such apparatus is not satisfactory for commercial radio telegraphic service, in the present development of the art, for the reason that even moderate stray interference will overwhelm the sensitive receiving apparatus and make translation of messages impossible. It is therefore necessary to instal transmitters of sufficient power to produce an easily readable signal with receivers of moderate sensitiveness and rugged characteristics. Reduction of strays is then effected by taking advantage of persistence-selection and musical tone, as outlined above.

The numerical relations between the elements which govern certain of the above stated requirements have been the subject of many researches from almost the first years of radio telegraphic practice. A great many experiments have been made to interrelate the severable variables, and as a result there have been deduced some quite well verified relations between transmitted power, antenna height, wave length, distance, and audible intensity of received signals on various types of receiver. Such numerical data is, of course, an essential for the predetermination of station constants to meet specific requirements. Without going into great detail, it may be stated that if a current of 100 microamperes is set up in the receiving antenna by the incoming signal-carrying wave, a sufficiently loud response will be had, on the heterodyne receiver, to permit separation of the signals from severe atmospheric disturbances; altho it is true that occasionally, when lightning storms are near, strays will become so violent that 150 microamperes or perhaps even more would be desirable. Practical operation, in the absence of strays, can be carried on with perhaps one-tenth this antenna current, but the additional signal intensity should be available if really commercial service is to be secured. As a result of long trials made jointly by the United States Navy and the National Electric Signaling Company in 1910, 1911, 1913 and later,* the following expression has been quite well confirmed:

$$I_r = \frac{392\, I_s\, h_1\, h_2}{\lambda\, d}\, \varepsilon^{-\frac{0.0474\, d}{\sqrt{\lambda}}}$$

* Reported in "Some Quantitative Experiments in Long Distance Radio Telegraphy" by L. W. Austin, "Bulletin Bureau of Standards," Vol. 7, Number 3, 1911, page 315; and "Quantitative Results of Recent Radio Telegraphic Tests Between Arlington, Va., and U. S. S. *Salem*," by John L. Hogan, Jr., "Electrical World," January 21, 1913, page 1361.

where I_r equals receiving antenna current in microamperes (thru 25 ohms, a fair receiver effective resistance), I_s equals sending antenna current in amperes, h_1 and h_2 equal sending and receiving antenna effective heights in feet, λ equals wave length in meters, and d equals distance of transmission in kilometers. If h_1 and h_2 are expressed in meters, the 392 above should be replaced by 36.3. The constants 392 and 0.0474 are reasonably accurate for daylight transmission over sea water or almost perfect ground; for night time transmission signals are almost invariably louder than indicated by the mathematical expression.

It has been assumed that, with the best receiving apparatus obtainable, $I_r = 100$ will give commercial transmission under unfavorable conditions. On attempting to solve the equation for this value, it is at once seen that for any given distance several combinations of transmitter power, antenna height, and wave length are possible. The balance between height of sending aerial and power of sending equipment is one which the natural economy of location most determine; in some places it is less expensive to build a low aerial and use large power, while in others, the reverse is true. The received signal is also dependent upon the height of the receiving aerial, but the amount of interference from strays is increased when tall aerials are used. It is therefore preferred to restrict antenna heights at the receiving station. The wave lengths used should be in general the longest at which the proposed antennas operate efficiently, since this will give as a rule the minimum attenuation. Data on the most desirable and economical types of transmitting and receiving antennas are not available, but it is to be expected that practice in this direction will become more nearly standardized in the future.

It may be interesting to consider the types of installation which are necessary to fulfill the above stated physical conditions for practical radio telegraphy over distances of 2,000, 3,000, 4,000 and 5,000 kilometers. The following tabulation gives a group of numerical values of the computed physical constants for each of these distances, and represents what may be considered good engineering practice of the present day. The assumptions are based upon the use of rugged heterodyne receivers and sustained wave transmitters; the transmitting antenna power (and consequently the sending antenna current) would have to be largely increased if spark transmission or other types of receivers were used:

Distance in Kilometers..	2000	3000	4000	5000
Distance in Statute Miles	1240	1860	2480	3100
Wave Length in Meters.	4000	7000	10000	12000
Antenna Heights in Feet (=3.28 height in meters)				
Sender...............	450	700	850	1000
Receiver.............	300	400	450	500
Antenna Currents				
Receiving in Micro-amperes..........	100	100	100	100
Sending in Amperes...	64	105	170	265
Sending Antenna Resistance in Ohms				
Radiation Component.	1.9	1.5	1.07	1
Total.............	3.5	3	2.5	2.5
Sending Antenna Power in Kilowatts........	14.5	33	72	175

The comparatively large powers which are necessary in order to transmit regularly over long distances thru strays should be particularly noted.* When the bad effects of atmospheric disturbances are reduced far beyond the point in commercial practice of the present day, it will be possible to signal at all times with perhaps one-tenth the transmitted power now used. This saving of transmitting cost will give radio telegraphy a further and tremendous advantage over wire or cable signaling, by reason of its economy; therefore the atmospheric disturbance problem is receiving the most serious attention of radio engineers all over the world.

It may be that some of the methods which have already been devised and used in laboratories, but which are not yet applied commercially, will solve the problem of permitting continuous operation on small power. It may be that the methods which seem promising at the moment will be as great disappointments as have been a large number of others attempted in the past ten years. In whatever way this one remaining obstacle to the greatest economies in radio telegraphy may be overcome, the fact remains that at the present time radio communication may be depended upon for entirely commercial service over long distances; it is necessary only that the stations should be designed with a full engineering understanding of the large number of physical problems involved.

* (See page 451 of this issue of the PROCEEDINGS for comparative data.—EDITOR.)

SUMMARY: The development of radio telegraphy, as a mysterious and little understood physical art, from well known inductive and conductive methods of telegraphy is briefly stated. It is shown that radio telegraphy is now subject to engineering treatment and consists of a series of expected effects resulting from a series of controllable causes. The large scope of natural phenomena involved is outlined, and the general physical basis of all communication systems stated. The fundamental operation of transmitters and receivers, from those first used to the most modern sustained-wave-heterodyne apparatus, is described. Difficulties produced by atmospheric disturbances, and their effects upon the requirements of commercial radio telegraphy are discussed. Following the qualitative considerations, some of the quantitative physical relations involved in practical radio telegraphy are outlined. Important constants for transmission over distances of 2,000, 3,000, 4,000, and 5,000 kilometers are given, and the paper concludes with a brief outline of future development.

ARLINGTON RADIO STATION

AND ITS ACTIVITIES IN THE GENERAL SCHEME OF NAVAL RADIO COMMUNICATION*

By

CAPTAIN WILLIAM H. G. BULLARD, U. S. N.

(SUPERINTENDENT OF THE UNITED STATES NAVAL RADIO TELEGRAPHIC SERVICE)

The naval radio station at Radio, Va., known and referred to generally as the Arlington radio station, was the first high power radio station constructed for the Navy Department, and was intended as the primary link in a chain of high-powered radio stations, whereby naval ships within the vicinity of our continental or insular coasts could always be reached directly or by relay.

Many sites were examined around Washington and in its vicinity before the present one was finally selected; and it may be mentioned here that the site of a radio station involves many considerations. First of all, the electrical conditions in the way of ground connections must be good, and the possibility of the absorption of electromagnetic waves by high mountains, land, or buildings in the immediate vicinity must be considered. It must have sufficient area on which can be erected towers and buildings; be near a source of power, if not self-sustaining; must be accessible by at least some of the ordinary means of communication, and as far as possible must be protected from assaults by possible enemies. If near the coast, it should be located sufficiently inland to insure safety from gun-fire from ships, and sufficiently distant not to be subjected to assault by raiding parties. It should preferably be near a base of supplies; and should have land wire, telephone or cable communication. One of the necessary considerations in the determination of a site near Washington was that it should be on government owned land.

All of the sites examined in the vicinity of Washington

* Presented for the Washington Section of The Institute of Radio Engineers before a joint meeting of the American Institute of Electrical Engineers and The Institute of Radio Engineers, February 29, 1916.

were government owned; among others, these included ground near the Naval Observatory, sites on the grounds of the Soldiers' Home, and on ground near the site of the St. Elizabeth Hospital. All of these sites met with more or less objection from

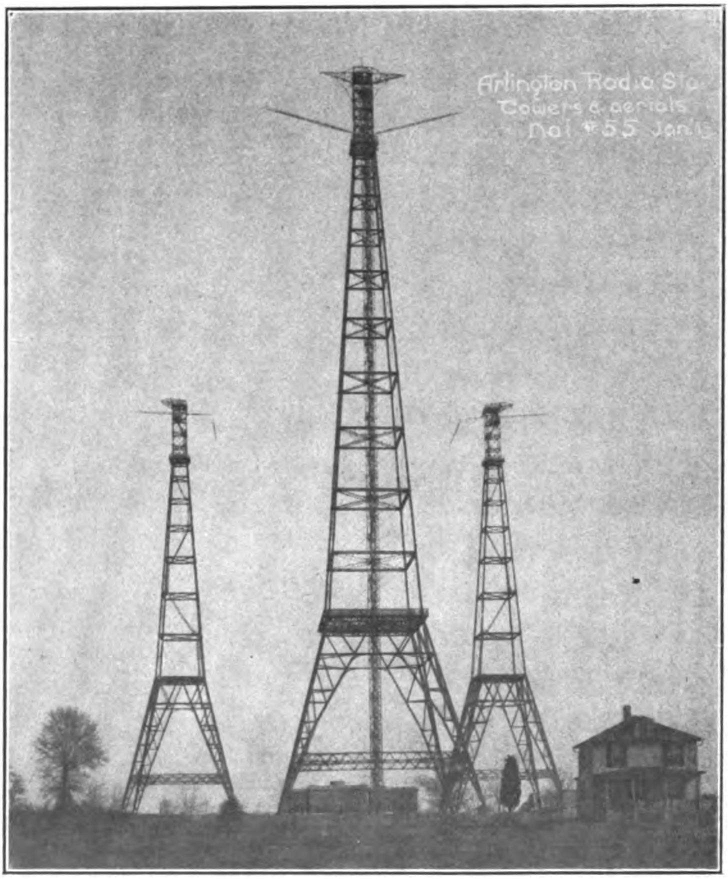

FIGURE 1—Towers and Station, Arlington

various outside interests. The site finally selected, being the present one, was formerly a part of the Government Reservation known as the Fort Myer Military Reservation, and the ground, 13.4 acres in extent, was transferred from the War to the Navy Department by act of Congress. A general view of the towers of the station after completion is shown in Figure 1.

The average elevation of the space on which the towers are

built is about 190 feet (58 m.) above sea level. The view shows three skeleton steel towers, one 600 feet (183 m.) high from the ground, the other two each 450 feet (137 m.) high. The centers of the towers form an isosceles triangle, the base of the triangle being 350 feet (107 m.) long and the altitude 350 feet. Figure 2 shows one tower leg with its insulation switch and short-

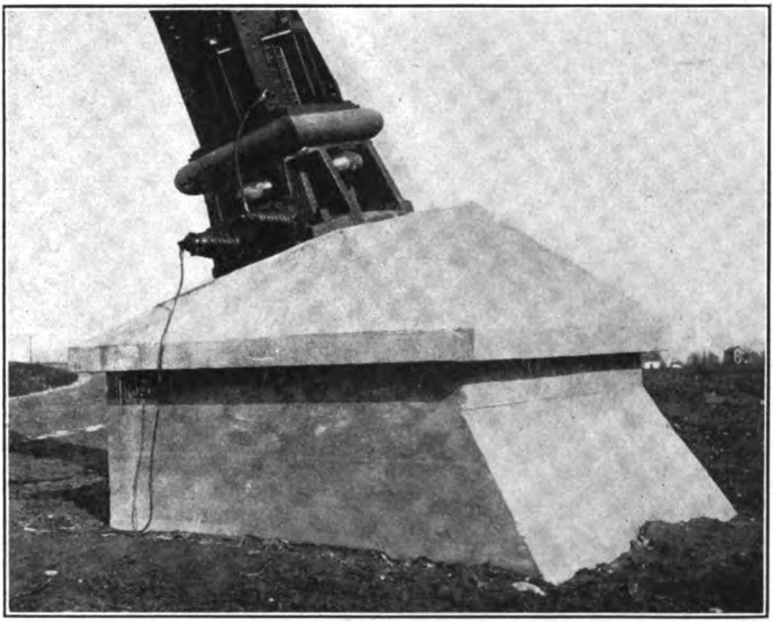

FIGURE 2—Base of Tower and Ground Switch, Arlington

circuiting switch. The base of the triangle, the distance between the two shorter towers, runs approximately magnetic north and south, and this is shown in Figure 3, which shows also in general the outline of the ground occupied by the station and the approximate dimensions. Two hundred and seventy-five tons (25000 kg.) of steel were used in the construction of each of the smaller towers, and 500 tons (45,000 kg.) in the larger. These towers were supplied and erected by the Baltimore Bridge Co., and the steel was furnished by the Carnegie Steel Co.

The current as supplied is 3 phase, 25 cycle, 6,600 volts; and after entering the basement it is transformed to 220 volts.

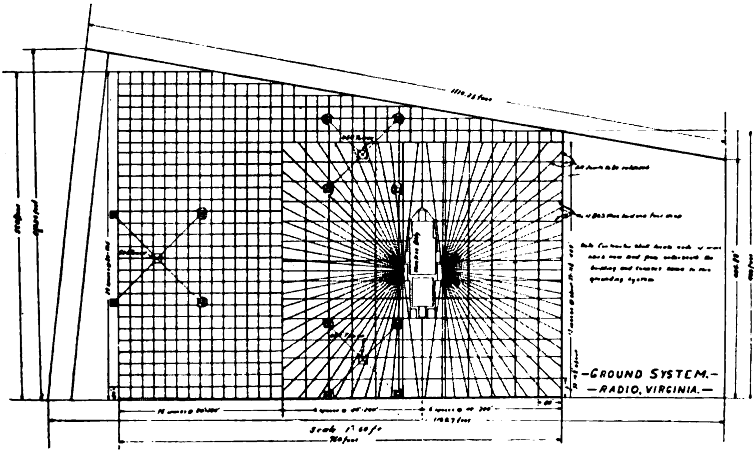

FIGURE 3

SPARK SET

The first set installed in the Arlington station was a spark set constructed on the Fessenden system, and a general view is shown in Figure 4. The main driving unit is a Westinghouse

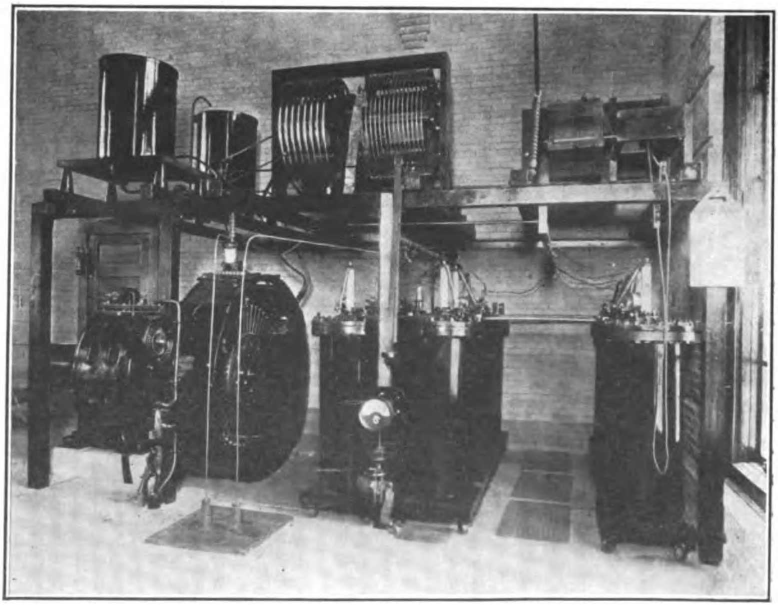

FIGURE 4—100 K.W. Spark Set, Arlington

200 horse power, 220 volt, 25 cycle, 3 phase synchronous motor, 300 revolutions per minute, and is controlled by means of an oil switch with auto starter. On this motor shaft, and driven by it, is an 8 kilowatt, 110 volt direct current generator, which is used to excite the fields of both the 200 horse power driving motor and the driven 100 kilowatt generator, which furnishes the energy for the transmitting apparatus of the radio set.

The 100 kilowatt generator is a General Electric, 220 volt, 500 cycle machine, and is driven at 1,250 revolutions per minute thru a leather belt by the 200 horse power motor. On the generator shaft is the rotor, or moving portion of the synchronous rotary spark gap, as shown in Figure 5, which consists of a fiber

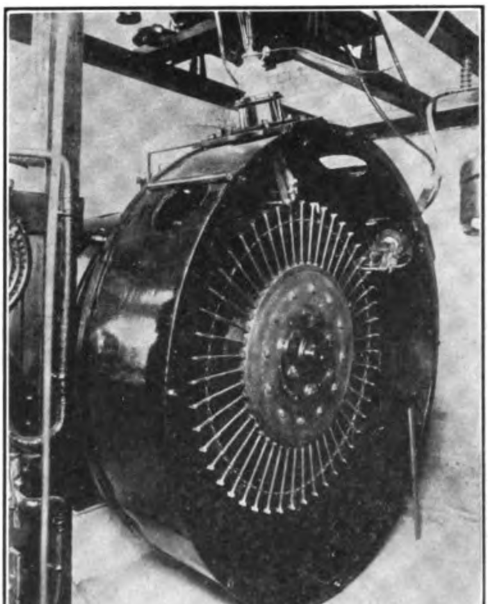

FIGURE 5—Rotary Gap of 100 K.W. Spark Set, Arlington

wheel with a heavy brass ring on its outer circumference from which protrude 48 copper tractors about ten inches long. The bearing at this end of the generator is especially constructed with a large flange 70 inches (1.78 m.) in diameter which supports the casing for the rotor. The casing, which carries the stationary electrodes of the spark gap, is fitted so that it can be moved

backward or forward by a worm gear. Provision is made to cool the stationary electrodes by running water thru them. The requirement that the stationary electrodes may be moved is very essential as any small changes in the variable factors which produce changes of wave length tend to cause the sine wave of the alternator to lead or lag, and it is necessary to move the electrodes so that the sparking will occur at the peak of the wave in the condenser charging current.

The main leads of the generator run to a panel on the switchboard, and after passing thru a circuit breaker carry current to the primary of the transformer. The wiring diagram is shown in Figure 6. One of these leads is broken by a large relay key,

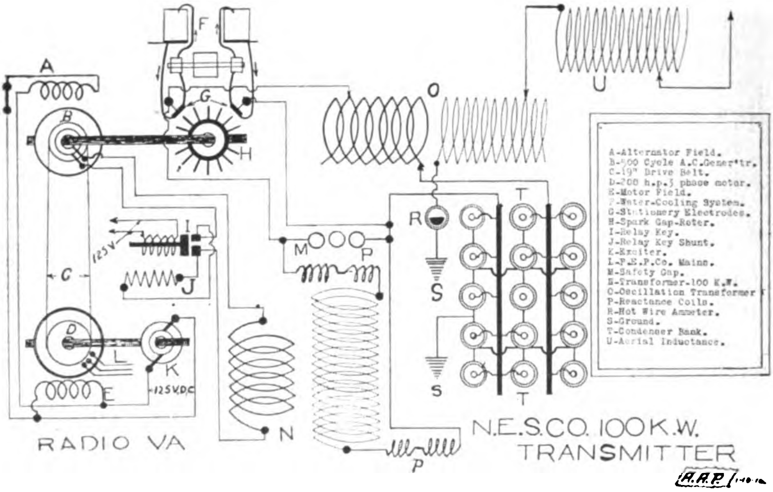

FIGURE 6

and shunted around the relay contacts is a large, variable current capacity, resistance grid which takes care of the considerable current in the primary from the time the circuit breaker is closed until the condensers are almost up to the point of discharge. When the key is closed and opened the greater part of the current is taken up by the grid, and this serves the double purpose of protecting the contacts from wear and of keeping the condensers constantly up to the sparking point ready for instant discharge. This relay key is operated by a small sending key in the operating room or at any other distant point.

The secondary leads carry current from the transformer

at 25,000 volts to the stationary electrodes, and shunted across the electrodes is the closed circuit containing the condensers and primary inductance of the oscillation transformer in series.

The primary inductance is a special helix made of ten turns of one inch (2.54 cm.) copper tubing about four feet (1.2 m.) in diameter, fitted with suitable spring clips by means of which the leads can be clamped to any turn for varying the sending wave lengths.

The condensers were furnished by the National Electric Signaling Company, and are of the compressed air type. An open view of one of these condensers is shown in Figure 7. Each consists of a large metal cylindrical tank in which the plates (about 200) are suspended; one set being connected to the tank itself, and the other set connected by a rod thru an insulator running thru the center of the cover. A lead washer under the rim of the cover and a lead bushing around the insulator insures the tank being air tight. The plates are placed one-eighth inch (3.2 mm.) apart, and at that distance would not stand the high voltage were it not for the compressed air. After the plates have been properly placed and the condensers assembled, the air is compressed to a pressure of 250 pounds per square inch (17.6 kg. per square cm.), and a special treatment is given the plates to increase the dielectric strength between them. A safety gap is set on the outside of the tanks between the rod thru the insulator and a terminal on the tank cover, this gap being slightly longer than the distance between the plates. The primary current is then turned on intermittently, allowing the sparking to take place inside until the small particles of dust in the air are burned out, after which the spark will jump the safety gap. The latter is then lengthened and the operation continued until the safety gap is enlarged to one inch(2.54 cm.). In this operation, known as "burning out," the generator voltage must be reduced to as low value as possible. By this treatment a spark can be made to jump a one inch gap in air before it will jump the one-eighth inch gap in compressed air. Each condenser has a capacity of 0.036 μ f. (microfarad), 14 units being used in multiple series, (two sets of seven in parallel and two sets in series).

The secondary of the oscillation transformer is made up in the same manner as the primary but is of three-eighth inch (9.5 mm.) copper tubing and has twice the number of turns. One lead is taken off to a hot wire ammeter and from there to the ground and the other lead has a spring clip and can be con-

nected to any turn of the loading coil or antenna inductance. The adjustment of the oscillation transformer is made as nearly correct as possible before the spark is turned on; then the loading coil, which has similar contacts, can be revolved while the spark is in operation so as to bring the antenna (and secondary

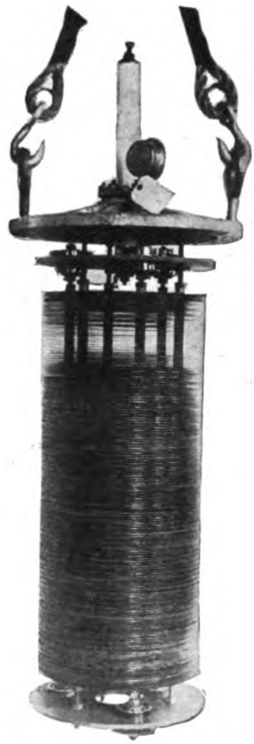

FIGURE 7—Compressed Air Condenser

circuit) into resonance with the primary. This is done by watching the reading of the hot wire ammeter and moving the loading coil until maximum antenna current is obtained.

The primary of the oscillation transformer has a screw attachment by means of which the primary can be moved farther away or nearer to the secondary so to obtain the proper amount of coupling to ensure a sharp or pure wave.

The antenna lead is taken from the loading coil to a switch on short pole mast outside the building, the lead passing thru an electrose insulator fitted in a plate glass window one inch

(2.54 cm.) thick and five feet (1.58 m.) square. The switch on these masts is controlled by a lever and sprocket chains from the sound proof operating room. A view of this outside switch is shown in Figure 8.

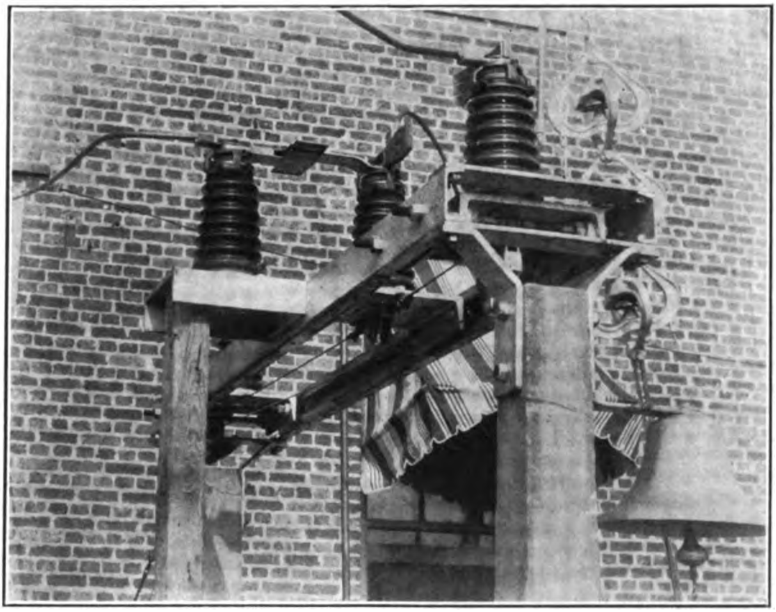

FIGURE 8—Antenna Switch, Arlington

The antenna is made up of three sections, 23 wires in each section, each wire consisting of 7 strands of number 20 phosphor bronze.* These wires are attached to spreaders made up of three inch (7.6 cm.) pipe, 88 feet (23.2 m.) long, reinforced by trusses; and the spreaders are attached to the towers by 10 electrose insulators between them and the towers. A general view of the construction of the antenna is shown in Figure 9. It is open at the highest end, at the 600 foot (183 m.) tower, and two sections are brought down to the 450 foot (137 m.) towers, and there joined to the main section by jumpers made up of 23 wires bunched in the form of a rope. The main section is what is known as a "T" antenna, and the vertical part ("rat tail") is taken from the middle. The 23 wires of the "rat tail" are brought down in the shape of a fan for 300 feet (92 m.) and

* Diameter of number 20 wire = 0.032 inch = 0.081 cm.

then in the form of a large cage the rest of the way to the switch on the pole mast.

The above antenna arrangement gives a fundamental of 2,100 meters with a capacity of 0.0094 μ f., and can be readily swung over by means of a switch on the pole mast for use either in transmitting or receiving. In the latter case, it forms a very

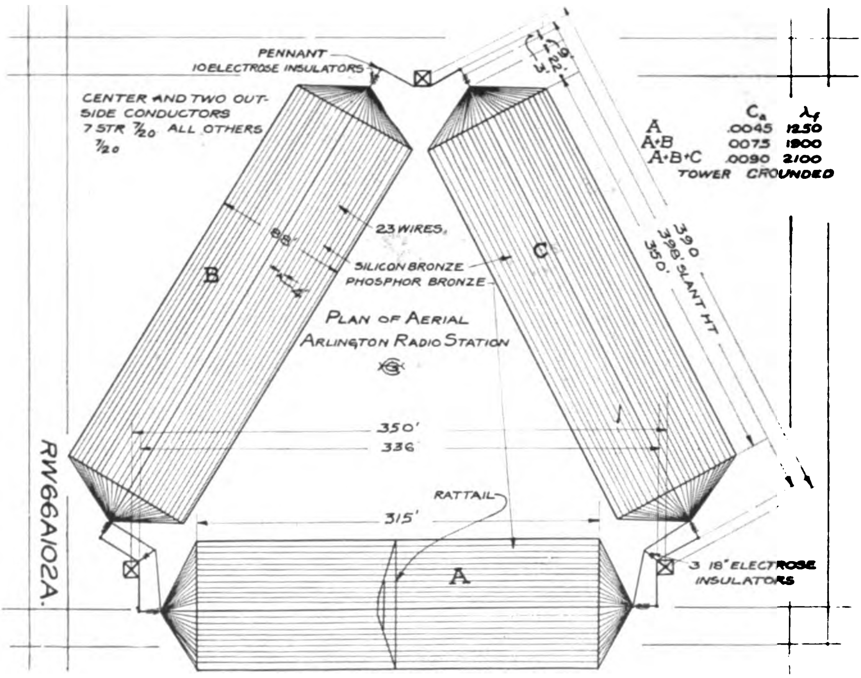

FIGURE 9

efficient arrangement for receiving long waves, as it is not necessary to insert an inductance of necessarily high resistance. For receiving shorter waves, a small antenna has been put in place at a height of 300 feet (92 m.) and is used also in transmitting with a 5 kilowatt transmitter which is installed for local work, such as that with Philadelphia, New York, and Norfolk.

The ground connections consist of many lengths of wire buried in the ground at various depths, in the space near the towers, and laid in a checker-board pattern with soldered junction. A general plan of the ground wire connections is in Figure 3. Miles of this wire were laid making a large net-

work; and finally wire leads are run down the slopes ending in a small stream that flows near by. The ground connection between the antenna and this network is thru a large copper strip 6 inches (15.2 cm.) wide and ¼ inch (6 mm.) thick run to the ground wires and permanently soldered to them.

The receiving or operating room at the Arlington station was built to be sound proof and is constructed somewhat like a refrigerator with double doors and walls 20 inches (61 cm.) thick. Before the plastering was put on, the ceiling, walls, and floor were covered with ¾ inch (1.9 cm.) "linafelt" for sound proofing, and then a layer of chicken wire of ¼ inch (6 mm.) mesh was secured over the linafelt. The meshing was carefully electrically connected together, and then several strips of copper were soldered to it and taken to the ground connection outside the building to make a screen for the receivers so that any induction effects from the generator would be absorbed by the screen.

The room is ventilated by two small fan motors, 220 volt, 25 cycle, 3 phase and the air ducts have baffle plates lined with felt on the same principle as a muffler or Maxim silencing device so that the air is silent when it reaches the room. In the air duct is a radiator from the heating system to heat the air for the room in winter months.

5 KILOWATT SPARK SET

The second set installed in the Arlington Station was a 5 kilowatt spark set sonstructed on a system developed by the Wireless Improvement Company, and shown in Figure 10.

The motor generator of this set, which was especially designed for this station, consists of one 15 horse power, 3 phase, 25 cycle Wagner motor, one 10 horse power direct current, General Electric motor, and one 5 kilowatt, 500 cycle, Crocker Wheeler inductor type generator. These machines are directly connected on a common bed plate, with the alternator between the two motors. Three different makes of machines were necessary, so as to get symmetry, and the three machines when lined up are exactly the same height.

When driving from an outside source of power, the 15 horse power motor is used, and the direct current motor then becomes a generator, supplying current for the field of the generator, and is connected thru a station panel for direct current supply to other auxiliaries. In case of failure of the outside source of power, current is obtained from a Diehl, direct current generator

driven by a 30 horse power Remington crude oil engine (Figure 11), which supplies current to the 10 horse power, direct current motor.

Both motors are controlled from the one panel, which is supplied with the set; arrangements being made on the board so that by throwing two switches, either alternating current or direct current supply can be used.

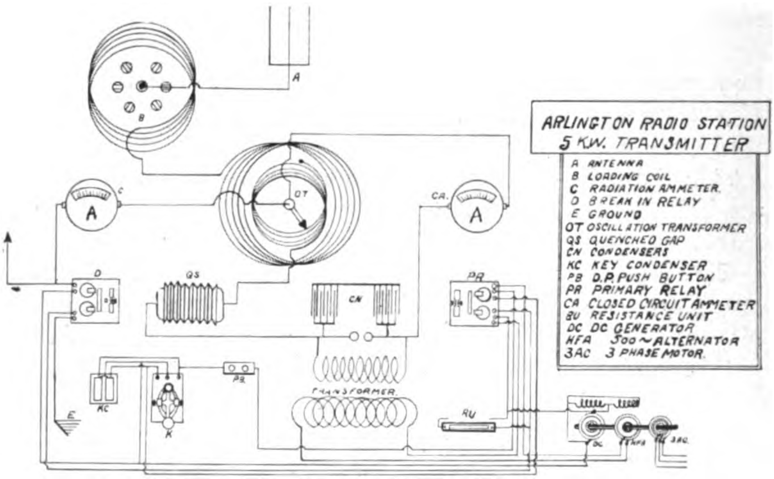

FIGURE 10

The whole set is mounted on two panels, one being for automatic control of the power end, while the other has all the radio apparatus on it. The control is accomplished by a push button, bringing the current to the starter and the set is started in about twelve seconds.

The transmitting apparatus, which is mounted on the other board, differs slightly from other quenched gap sets of the Navy, in that it has fixed secondary and continuously variable primary circuits.

The wiring plan is shown in Figure 10. The primary inductance is in the form of a helix made up of 26 turns of flat rib copper ¼ by 3/32 of an inch (6 by 2.4 mm.) built into a micarta frame. A spiral grooved shaft runs thru the center, carrying an arm on the end of which is a copper wheel, held against the ribbed copper by a spring. As the handle on the front of the board is turned, the arm swings around the helix,

the wheel travelling on the ribs, while the arm is forced along the grooved shaft, in this way covering the length of the helix from zero to its maximum length. The capacity of the primary or closed circuit is made up of standard Leyden Jars of 0.002 μ f., 28 jars being used in a bank of series parallel, forming a total capacity of 0.014 μ f.

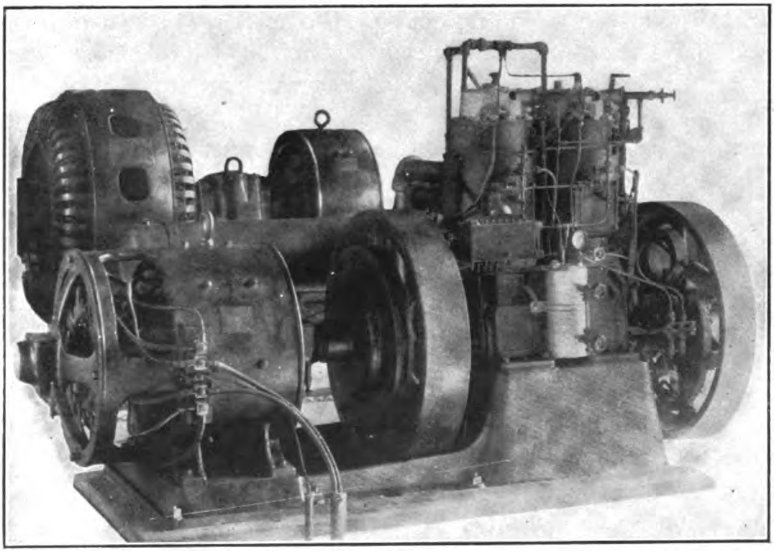

FIGURE 11—Oil Engine and Generators, 5 K.W. Spark Set, Arlington

The secondary helix is made up of same material as the primary, and is fitted on slides, fitting closely around the primary. This helix can be operated from the front of the board by a wheel which moves the helix backward and forward, to vary the coupling. In series with this and the antenna is the loading coil, made of the same material, and containing 48 turns. From certain predetermined places on this coil, leads are taken to the swinging switch on the front of the panel.

The set was originally intended for a range in wave length of 300 to 3,000 meters, but as only two wave lengths, 952 and 2,400 meters are used, the swinging arm in the primary helix has been changed and two points permanently fixed, so that the operator can change the wave in a few seconds to that desired.

From the center of the swinging switch at top of panel is

taken the antenna lead, which is led thru a window in suitable insulation to a four wire antenna. Originally this antenna, consisting of four wires about four feet (1.2 m.) apart, was secured between the north and west towers, forming an inverted "L" under the north wing of the large antenna. This proved to be a very poor location, and other stations complained of weak signals, even when radiating 23 amperes. Apparently this small antenna was blanketed by the larger one above it. A stay was then rigged up between the north and south towers, and the vertical lead was suspended from it by suitable insulators. Then the free end was taken due east, away from the large antenna, the end being secured to a convenient tree by a stay and insulators. This proved a much better arrangement and is in use now.

This arrangement gives a natural period of about 900 meters, with a capacity of 0.00199 μ f. The antenna is somewhat on the style of an inverted "L," with the vertical 150 feet (46 m.) in length, and the horizontal 250 feet (76 m.) in length. The end leads down slightly, so that it is not a true "L."

100 KILOWATT ARC SET

The third set installed at Arlington is a 100 kilowatt, Federal Telegraph Company, arc set, manufactured by the Federal Telegraph Company of Palo Alto, California, and shown in Figures 12 and 13.

This set consists of a motor-generator, arc chamber, magnet poles, magnet coils, inductances and necessary panels.

The motor-generator is manufactured by the General Electric Company, the motor being a 160 horse power, 3 phase, 25 cycle induction motor, and the generator for 500 volts direct current and 100 kilowatt, both mounted on a common base and direct connected. The motor shaft has an extension, whereby a pulley can be mounted and the set run by an engine or other driver by means of belting. The control for the motor is mounted on a panel and is controlled from a position near the arc by means of a small switch, which operates the contactors of the panel, starting the machine on low voltage and automatically bringing it up to full voltage as the starting current is reduced. This machine is brought up to speed in four seconds from the time the switch is closed. The wiring plan is shown in Figure 14.

There is a special pair of panels for the generator, these having been installed with a view to running two arcs and two

generators at the same time, as experiments may be tried with machines in series and parallel. The generator leads are taken to these panels, a circuit breaker being in the line for safety, and from the panel they are taken to a small operating panel near the arc. Before going to this small panel the positive

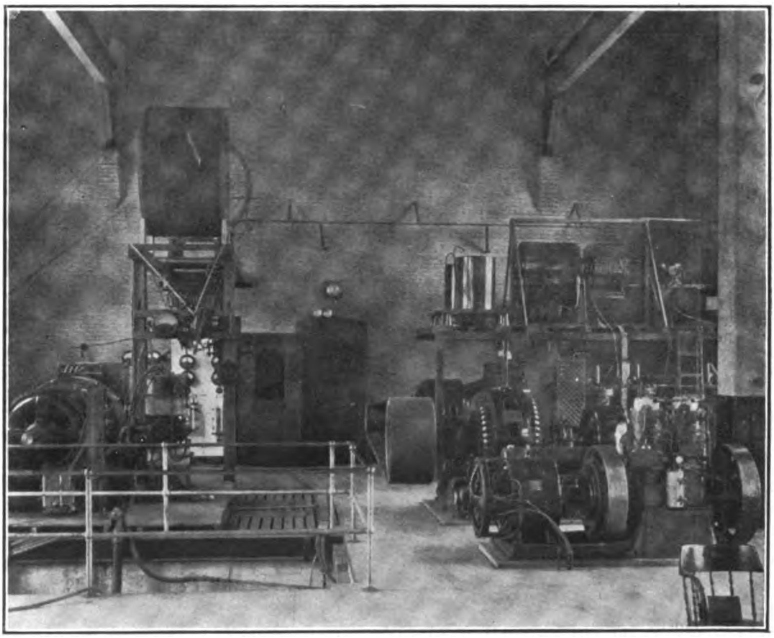

FIGURE 12—Arc and Spark Transmitters, Arlington

lead is connected to three inductive choke coils, while the negative passes to one coil. These coils are to choke back the oscillatory current generated by the arc, and localising the oscillations in the antenna and thus excluding them from the generator.

From the small panel, the positive lead runs direct to the copper electrode, which must always be positive. The negative lead is taken, thru a resistance, to the magnet coils, and then to the carbon, which must always be negative. If the polarity is reversed, the copper electrode would not last two minutes; for if the positive lead were taken to the carbon and the flame thus blown toward the copper, the arc would act like an acetylene torch, causing the metal in the electrode to boil; whereas if

the opposite is the case, the flame is blown toward the carbon, which being rotated by a small motor, does not permit the formation of a crater on the carbon.

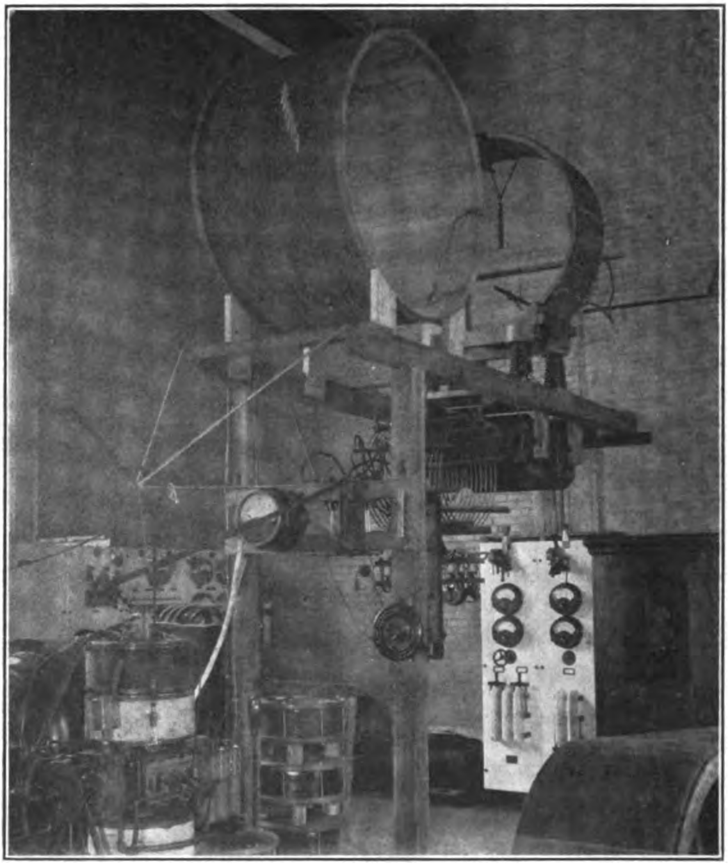

FIGURE 13—Arc Set, Arlington

The resistance in the negative lead is controlled by single pole, knife switches, and allows the arc to be struck on a low voltage. As the arc current increases, these switches are used to cut out portions of the resistance until full voltage is being used.

The arc chamber is water cooled thruout, as are the electrode holders also. Feeding into the chamber is a small flow of alcohol, which, when ignited by the arc, generates a gas, or conducting, ionized vapor, the action being to facilitate re-ignition of the arc after it has been blown out by the action of the electro-magnets.

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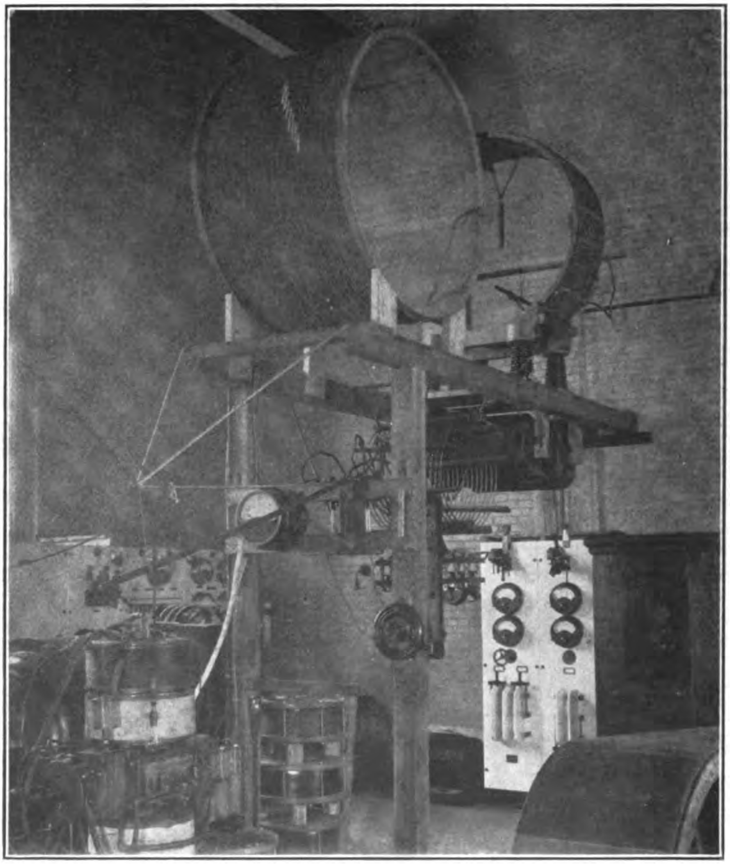

FIGURE 13—Arc Set, Arlington

The resistance in the negative lead is controlled by single pole, knife switches, and allows the arc to be struck on a low voltage. As the arc current increases, these switches are used to cut out portions of the resistance until full voltage is being used.

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From the copper or positive electrode, a lead is taken to the helix, and thence to the antenna. In series with the helix, is a smaller helix of twelve turns, giving wave length change of about two hundred meters; and from each turn of this helix

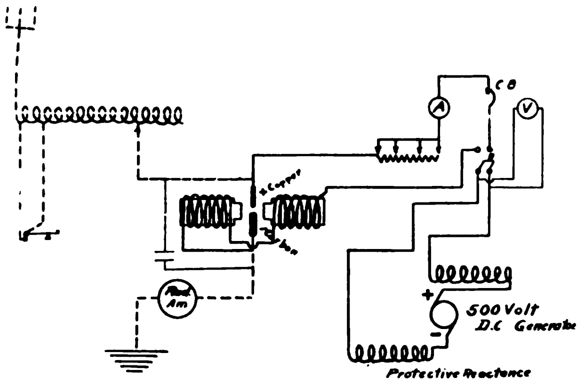

FIGURE 14

a lead is taken to a twelve point relay, which is operated by 110 volt direct current. The resulting action when the contacts are closed, (which happens when the hand key is released), is that the wave is shortened by the twelve turns of inductance being short-circuited. When the hand key is pressed, as in operating, the contacts are opened, thus lengthening the wave. Thus there are two distinct waves sent out, one when the key is pressed, of say 6,000 meters, and the other when the key is released, which would be about 5,900 meters.

From the negative electrode a lead is taken thru the hot wire ammeter, and then to ground.

The operation consists in first striking the arc at reduced voltage, and by means of a fine threaded screw arrangement, bringing the carbon back, thus lengthening the arc, at the same time increasing the voltage by cutting out resistance. This operation is repeated until full voltage is on, when the length of the arc is regulated by the radiation meter, there being a maximum setting, from which opening or closing the arc causes a drop in the antenna current.

Figures 15, 16, and 17 show a reproduction of the log of the Washington station for one day, February 1, 1916; and an examination of it will show the stations daily communicated with. They further show the character of the log required to be kept

by every Naval shore radio station. In twelve months ending June 30, 1915, Arlington handled 78,921 messages. In January of this year, Arlington transmitted 2,737 messages and received 3,452, a total of 6,189 messages and an average of 200 per day.

INTERNATIONAL RADIO COMMUNICATION

In line with the general desire to promote closer business and social relations between the United States and countries of Latin America, as developed by the several Pan-American Scientific Congresses and particularly the last one recently held in Washington, preliminary steps have been taken to ensure an interlocking radio communicating system between the radio stations of the various countries of the western hemisphere.

The plan of communication has been developed by the Naval Radio Service and has been presented to representatives of the various countries thru the co-operation of the State Department, and it is expected there will shortly be an international committee appointed to meet in Washington to consider the suggestions advanced by the Naval Radio Service. This committee will be charged with the duty of preparing the necessary regulations to combine the radio services of all American republics into one homogeneous system for the transaction of government and commercial business, to arrange traffic regulation, to designate regular and alternate routes of transmission, to assign wave lengths to the various stations with a view to eliminating interference, to establish rates for the service rendered and in general to standardize and systematize the administration, operation, material, and personnel features of radio communication in the entire western hemisphere.

It is proposed to divide the territory embraced in the Pan-American republics into zones of radio communication with one control radio station for each zone, in a manner similar to that on which the Naval Radio Service is organized for the transaction of United States Government business.

These zone stations will receive and relay radiograms to their destination in accordance with the regulations provided. It is proposed to have one main station for the entire hemisphere, located in as nearly a central position, geographically, with reference to all American republics, as may be practicable. Such a main station should be capable of direct communication with central stations in each of the proposed zones, covering the

territory of the interested governments, and Darien is suggested as this main station.

The plans drawn up provide for zone central stations at the following places tentatively; changes may later be found to be desirable.

Buenos Aires, Argentine	Guantanamo
Para, Brazil	Washington
Guatemala	Possibly Tela, Honduras

Each of these zone center stations will serve as a receiving and distributing station for the stations in their respective zones, and each will be capable of direct communication with the main station. In each country, preferably at the capital, there will be a central controlling and distributing station, which would be capable of direct communication with the appropriate zone center station and local stations of low power.

The diagram shown in Figure 18 represents graphically the ideas advanced, and it will be noticed that the scheme of communication covers the whole of the United States, Central and South America, and that the zones represent the United States, West Indies, Central America, Northern South America, and southern South America. From each of the zone stations, communication is possible to other distributing stations, and each of these distributing stations can communicate with local stations in various countries. Thus, a message from Washington to Paraguay in the vicinity of Concepcion could be routed via Darien (zone station), Buenos Aires (zone station), Ascuncion, Paraguay (distributing station), Concepcion and thence to its destination by land lines. A study of this scheme will show that any place in these countries that has telegraphic connection can be reached, and the whole system is mutually interlocked.

SOME REMARKS ABOUT NEW HIGH POWER STATIONS

The high power stations in course of construction at San Diego, Pearl Harbor, Cavite, and Guam are well under way, and Figures 19, 20, and 21 show a view of San Diego as it appeared in January, 1916. The steel towers, three in number, of the self-supporting type, are each to be 600 feet (183 m.) high in the form of a triangle, 1,100 feet (336 m.) along the base, each of the other sides being 1,000 feet (305 m.) between towers. The power input will be 200 kilowatt with 150 amperes in the antenna, this current being furnished by apparatus of the Poul-

sen system as developed by the Federal Telegraph Company. The power will be supplied with the following characteristics, viz.: 3 phase, 60 cycle, 11,000 volts. The station is located on a site specially selected and purchased for the purpose and which contains about 27 acres, being about twice the size of the Arlington reservation.

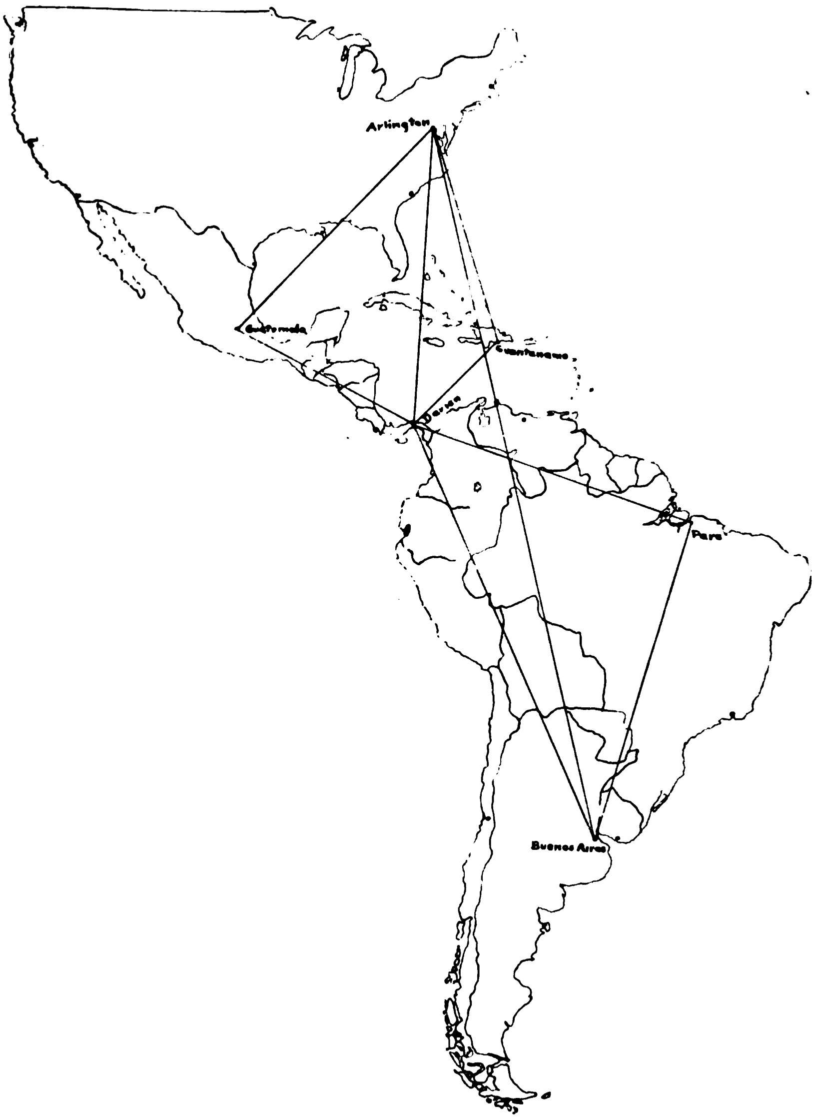

FIGURE 18

The station at Pearl Harbor, generally referred to as Honolulu, will have three steel, self-supporting towers, erected in the form of an equilateral triangle, 1,100 feet (336 m.) between

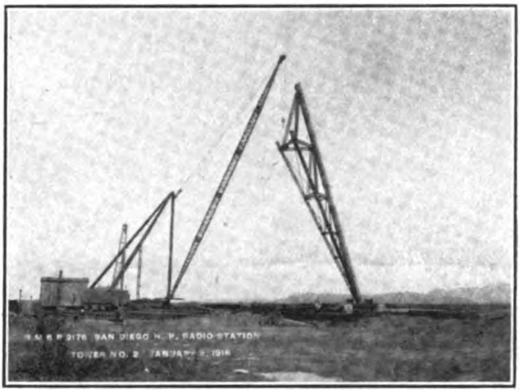

FIGURE 19—Erection of Tower Leg.
San Diego

each two towers. The system used is the Poulsen with 350 kilowatts input power, with 200 amperes in the antenna. Power is obtained from the Naval station plant, 3 phase, 60 cycle, 2,200 volts. The station is built on government property and is not limited in area.

FIGURE 20—350 K. W. Station Buildings,
San Diego, Cal.

The station at Cavite, in the Philippines, will have three towers similar to those at Pearl Harbor, but only 1,000 feet (305 m.) between towers. Apparatus of the Poulsen system,

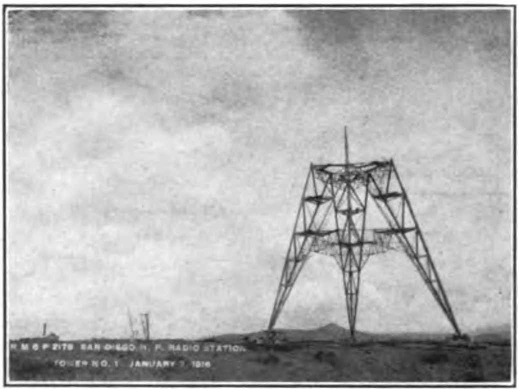

FIGURE 21—Tower Base, 350 K. W. Station, San Diego

with 350 kilowatts input power and 200 amperes in the antenna will be supplied. The power will be from a 250 volt, direct current, oil engine driven generator and storage battery. The

FIGURE 22—Station, Arlington

FIGURE 23—Administration Building, Arlington

towers are built on the station reservation and the area is practically not limited.

The station on the Island of Guam will have two steel self-supporting towers, each 400 feet (122 m.) high, with a distance between towers of 700 feet (214 m.). Apparatus will be of

the Poulsen system, with 35 kilowatts input and 30 amperes in the antenna. Power will be obtained from the Naval reservation site and will be 3 phase, 60 cycle, 2,200 volt. This station is erected on government property and has all space desired.

Details of the antennas have not yet been developed except that it is proposed to have the spreaders a permanent part of the towers, and each wire supported by independent insulators.

FIGURE 24—Superintendent's Office, Administration Building, Arlington

FIGURE 25—Accounting Room, Administration Building, Arlington

As each wire for the large station will weigh something over 300 pounds (136 kg.), some arrangement is desirable whereby it will be possible to handle but one wire at a time.

(Thru the courtesy of Capt. Bullard and Messrs. Pannill and Clark, the Editor is able to lay before the readers of THE PROCEEDINGS OF THE INSTITUTE OF RADIO ENGINEERS photographs of the administrative offices and a practically complete group of illustrations of the Naval radio stations of the Atlantic coast, Porto Rico, the Canal Zone, the Great Lakes, the Pacific Coast, and China. These illustrations, which are suitably labelled, give an admirable idea of the magnitude of this system.—EDITOR)

SUMMARY: The Arlington station of United States Naval Radio Service is described in detail. The towers, large antenna, ground, power supply, 100 kilowatt spark set, receiving room, 5 kilowatt spark set, small antenna, and 100 kilowatt arc set are fully considered as to design and operation. The traffic statistics of Arlington are given.

The question of Pan-American radio communication is considered and a comprehensive zone system is proposed and explained. The routing of a typical message is followed out.

The new high power stations at San Diego, Pearl Harbor, Cavite, and Guam are considered, with some details as to their equipment.

A series of illustrations of the Naval radio stations follow.

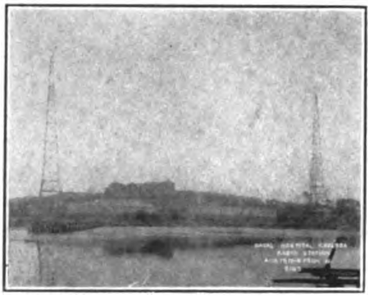

FIGURE 26—Chelsea, Mass.

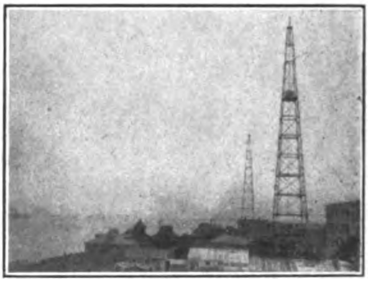

FIGURE 27—Newport, R. I.

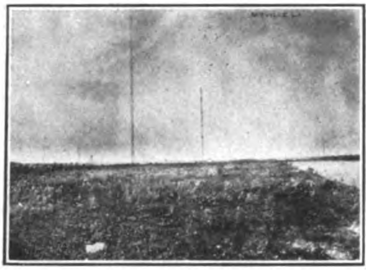

FIGURE 28—Sayville, N. Y. (Station under Naval Control)

FIGURE 29—Tuckerton, N. J. (Station under Naval Control)

FIGURE 30—Naval Radio Laboratory, Washington, D. C.

FIGURE 31—Washington, D. C.

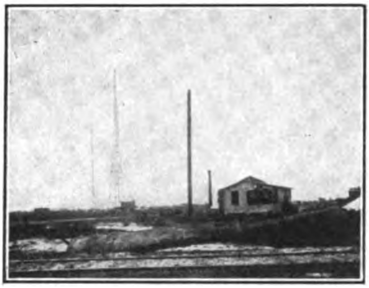

FIGURE 32—Charleston, S. C.

FIGURE 33—San Juan, Porto Rica

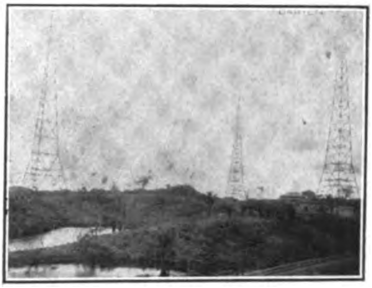

FIGURE 34—Darien, Canal Zone

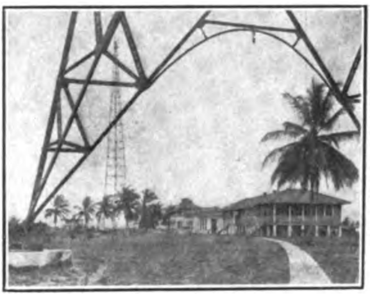

FIGURE 35—Colon, Panama

FIGURE 36—Balboa, Canal Zone

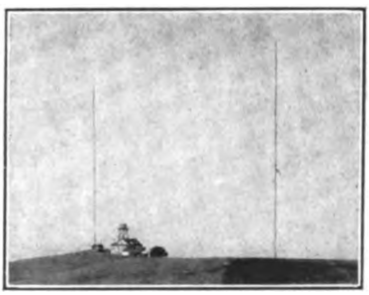

FIGURE 37—Mare Island, Cal.

FIGURE 38—Great Lakes Station

FIGURE 39—Cordova, Alaska

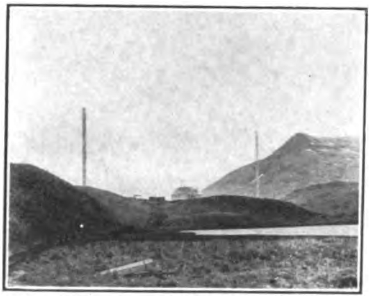

FIGURE 40—Dutch Harbor, Alaska

FIGURE 41—North Head, Alaska

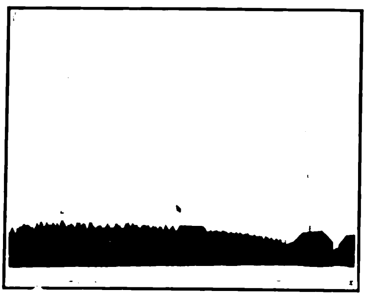

FIGURE 42—Kodiak, Alaska

FIGURE 43—Pekin, China

DISCUSSION

Charles J. Pannill: Few people outside of our own service fully appreciate the size and scope of the Naval Radio Service of which Arlington is the central station. The time is not distant when it will be very necessary for Arlington to be able to have three or four transmitting circuits and the receiving office to be able to take at least five messages, all of which transmitters and receivers will have to be in operation at the same time. This is not only practicable, but can be done with a very small outlay. In addition to the Government traffic for all departments handled by our service, we also handle a large volume of commercial traffic both in our ship-to-shore and point-to-point service, including our service in Alaska. The operation and control of the Sayville-Nauen and Eilvese-Tuckerton circuits are handled by our office, which is as well as the clearing house for all international ship-to-shore business in which American naval and merchant vessels are concerned. It is very gratifying to see such communication as that of Arlington working direct with Darien in day light and Darien working with a naval vessel a thousand miles (1,600 km.) south, thereby effecting communication nearly 3,000 miles (5,000 km.) over land and sea with a ship with only one relay. This is quite different from conditions at the time I entered radio, when communication between Fort Monroe and Ocean View, some four miles (6 km.) was considered wonderful. Captain Bullard has covered the subject so well that I cannot add anything further.

NOTES ON RADIATION FROM HORIZONTAL ANTENNAS[1]

By

Charles A. Culver

(Professor, Beloit College, Cambridge, Massachusetts)

Continuing the work done by the author,[2] Kiebitz, and others, we have recently carried out a series of experiments on the relative radiation efficiency of low horizontal antennas. These tests were conducted at the Cruft High Tension Laboratory, Harvard University, during the spring months of 1915.

A low antenna was erected on the plan shown in Figure 1. The mean height of $A\ B$ was 3.7 meters; $C\ D$, 4 meters; $D\ E$, 1 meter. The wire composing the system was copper, 7 strands of number 22[3], and insulated from the ground. The particular

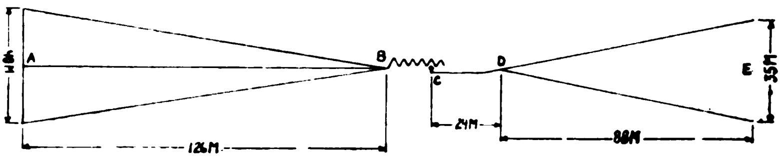

Figure 1—Arrangement of Horrizontal Radiating System at Harvard University

dimensions chosen were determined by local physical conditions and hence have no special significance. The complete system AE had a capacity of 0.00738μf. The capacity of CE was 0.00313 μf., and of CE 0.00355 μf. The decrement of the complete system was approximately 0.3. The radiation resistance of $A\ B$ was 10.6 ohms at 810 meters, and 8.4 ohms at 1,100 meters. Provision was made for utilizing $A\ E$, $A\ B$ or $C\ E$ as radiating units. The water system in the building served as a ground in a number of the tests, while in others a radial earth system was utilized. The latter consisted of a number of stranded

[1] Received by the Editor, February 2, 1916.

[2] "Physical Review," N. S., Vol. III, No. 4, April, 1914. "Electrical World," Vol. 65, No. 12, March 20, 1915.

[3] Diameter of number 22 wire = 0.025 inch = 0.064 cm.

copper wires buried a few centimeters beneath the surface of the ground and extending radially more or less directly beneath the section *A B*. The natural period of the part *A B*, when utilizing the radial earth just referred to, was 740 meters; when using the water system as an earth the period was found to be 665 meters. Power was supplied to the radiating system from a 2-kilowatt air-core transformer operating at 500 cycles. A quenched gap and loose coupling was employed. *B C* represents the secondary of the coupling transformer.

Tests were carried out between the above described station at the Cruft Laboratory and the points shown in Figure 2.

On May seventh and eighth a series of comparative tests were conducted between Cambridge and a temporary receiving station erected at Plymouth. The antenna at the latter place

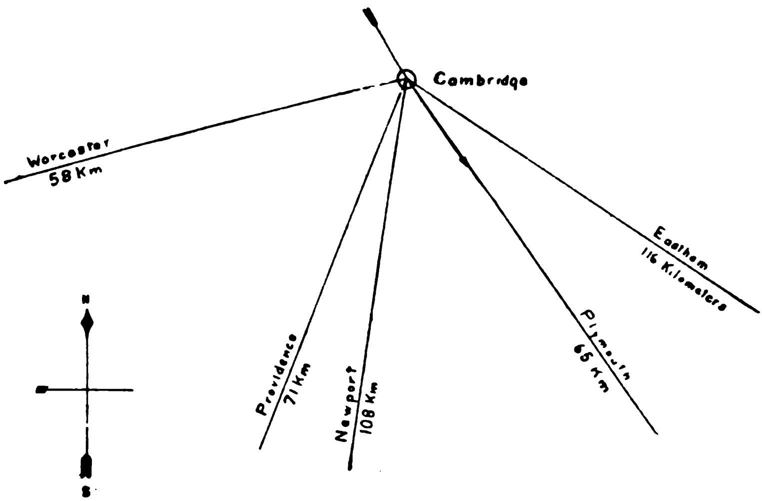

FIGURE 2—Relative Position of Stations. The arrow at the Harvard Station shows the direction of the horizontal radiating system.

consisted of two nearly vertical wires approximately 1 meter apart and 21.3 meters long. The pipes of a steam system served as earth. The receiving equipment comprised a loosely coupled audion with shunted receivers for determining the audibility.

At the Cruft Laboratory, the large vertical antenna was used as a standard of comparison for radiation. This will be referred to as the "standard antenna." It consisted of 16 nearly vertical wires having a mean height of 152 feet (46 m.). Its natural period was 370 meters when using the radial earth,

and its capacity was 0.00208 μf. The radiation resistance was found to be 5.4 ohms at 890 meters, and 8.3 ohms at 644 meters.

The results of these tests appear in the following table. It will be noted that alternate readings were made between the standard antenna and the special type under test.

Table Showing Conditions and Results of Harvard-Plymouth Radiation Tests. Power: 1.9 kilowatts. Wave length: 800 meters.

Test Number	Radiating System	Antenna Current Amperes	Nature of "Earth" at Harvard	Audibility at Plymouth	Remarks
1	Standard	4.65	Radial	23+	7:30 P. M.
2	*AB* Figure 1	4.5	Radial	0	7:50 P. M.
3	Standard	4.65	Radial	24	8:30 P. M.
4	*AE* Figure 1	3.8	None	19+	8:45 P. M.
5	Standard	4.7	Radial	37+	9:30 A. M. Heavy rain during night.
6	*AB* Figure 1 Grounded At *A*	1.8	Radial	4+	9:50 A. M.
7	Standard	4.8	Radial	23+	10:30 A. M.
8	Standard	4.1	Water System	14+	10:45 A. M. Doubtful values owing to resonance conditions at *H*.

It was originally intended to make test number 8 a comparison of *A B* when using the water system as a ground; but owing to an error in arranging the schedule and inability to repeat the experiment, this comparison was omitted. However, in this connection, it should be mentioned that the Harvard station was heard on April tenth by 1*QC*, Eastham, Mass., when using the arrangement just indicated.

Several points of special interest are apparent in the above table. First it is evident that the horizontal system *A E* is comparable in radiating efficiency to the standard vertical antenna—at least this is true for a direction more or less in line with the plane of the horizontal antenna. When one considers the cost of erection and maintenance of a large vertical system such as that on the Cruft Laboratory, the comparison is even more striking.

A second point worthy of note is the result obtained when

using a system made up of $A\,B$ grounded at the end remote from the coupling transformer and the radial earth. The oscillation of this system gave rise to some radiation as will be seen by the audibility at Plymouth, tho it did not radiate appreciably when the end was not earthed. (See test number 4.) We propose to investigate this point further.

When one comes to investigate the radiation from a low horizontal antenna in directions other than that of its own plane, the results are found to be materially different. The following-described tests were arranged to secure data on this latter aspect of the question.

Qualitative tests were made between the Harvard station when using $A\,E$ as a radiating system and a well equipped private station at Worcester, Mass. The Worcester station was unable to hear the Harvard signals.

Thru the courtesy of the United States naval authorities, a series of tests were carried out between the Harvard station and the United States naval station ($N\,A\,F$) at Narragansett Bay, Newport, R. I. This series of experiments was conducted on May thirteenth to nineteenth inclusive, between the hours of 8:30 and 8:45 A.M. Much interference was encountered, particularly on the last two days, but it was found possible to carry out several fairly satisfactory tests.

When radiating an 800-meter wave from the standard antenna at Harvard, the audibility at Newport was found to be 300 ohms. The Newport station was, however, unable to detect signals from the Harvard station when the latter radiated an 800-meter wave from the horizontal system $A\,E$.

With the kind coöperation of Professor Watson, it was possible to attempt a series of tests between the Harvard station and Brown University, Providence, R. I. For reasons which we have been unable to discover neither the radiation from the standard nor horizontal antennas could be detected at Providence.

It thus becomes apparent that the low horizontal antenna has a very low radiation efficiency in directions which make substantial angles with its own plane. In this connection, however, it is interesting to note that this system would respond to incident radiation from the directions in which it would not radiate. For example Arlington ($N\,A\,A$), could be heard at night with an audibility of 4 to 5. Many experiments made with the Harvard ground antenna, however, showed that it is comparatively inefficient as a receiving system in directions other than that of its own plane. This is in line with the results

obtained by Mr. Riner and the author in previous experiments. Incidentally it might be of interest to note that last spring Mr. Riner, while experimenting at Madison, Wis., was able to hear the Arlington time signals and read the weather report when utilizing a ground antenna similar to the one described in our recent paper. Mr. Riner informs the author that his wires were supported about 2 feet (60 cm.) above the surface of the ground, and that a crystal detector was employed.

The Harvard experiments above described are more or less preliminary in character. Owing to local physical and electrical conditions it was impossible to determine the optimum wave length to be employed with an antenna of given length. It is more than probable that a materially higher efficiency would have resulted if it had been possible to adapt the wave to the electrical dimensions of the radiating system. The experiments show, however, that a low horizontal antenna may have for certain directions a radiation efficiency comparable with that of the conventional vertical systems; and also that the horizontal form, as is to be expected, probably has a decidedly asymmetrical radiation curve.

A series of experiments are now being arranged whereby it is hoped to be able to determine the polar radiation curve for a horizontal system placed within a few centimeters of the ground. It is also proposed to determine the optimum arrangement of the component parts of such a ground antenna when used as a radiating system.

In conclusion we wish to thank Professor G. W. Pierce of Harvard University, for his cordial and helpful coöperation in carrying out the investigation described in this paper.

Beloit College, January 1, 1916.

SUMMARY : Experiments on the radiation from various types of low horizontal antennas are described. A 2-kilowatt, 500-cycle quenched spark set is used at a wave length of 800 meters. The influence of the ground is investigated. Directional radiation is found, and also an apparent non-reciprocity between directions of non-radiation and those of non-reception.

A FEW EXPERIMENTS WITH GROUND ANTENNAS*

By

LEONARD F. FULLER

(CHIEF ELECTRICAL ENGINEER, FEDERAL TELEGRAPH COMPANY)

The experimental data described herein were taken early in 1912. Recent articles on ground antennas appearing in the PROCEEDINGS OF THE INSTITUTE OF RADIO ENGINEERS have again brought up the subject, and it is felt that a description of these rather incomplete experiments may be of interest inasmuch as they show polar curves of received current taken with a d'Arsonval galvanometer and crystal detector.

The work was carried on in all kinds of weather and frequent cross-checking of data showed no variation from this cause.

The topography of the country was rough with low hills and both transmitter and receiver were approximately 800 feet (250 m.) above sea level.

The transmitter consisted of a non-synchronous rotary gap, spark set, radiating 4 amperes in a flat-top antenna of ordinary design at a wave length of 1,000 meters and a high decrement.

The receiver was located 2,100 feet (640 m.) from the transmitter and was equipped with a silicon detector and galvanometer for the quantitative work in addition to the usual receiving apparatus. In the actual observations the detector was connected directly in the antenna circuit and no coupling was used. This gave the receiver a high decrement also.

It is to be regretted that only a little over half a wave length separated transmitter and receiver and also that the decrements were high thruout. The theoretical laws governing the performance of ground antennas were followed reasonably well by the experimental observations, however, and it is believed the above conditions did not seriously vitiate the observed results.

The ground antenna consisted of two number 16 B. & S. gauge,† rubber covered stranded copper wires, each 264 feet (80.5 m.) in length measured from the center of the house in which the receiving apparatus was located.

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*Received by the Editor May 15, 1916.

†Diameter of number 16 wire = 0.0508 inches = 0.129 cm.

These wires were laid flat on the earth with the far ends insulated therefrom.

In the FIRST TEST the two wires were kept diametrically opposite, i. e., 180° apart, and were rotated thruout 180°. Readings were made of their angular position and of the galvanometer deflection, the square root of which was proportional to the received current.

The observed readings were as follows:

TEST 1

Angle	Current
θ	I_R
0°	4.84
15°	4.57
30°	4.39
45°	3.55
60°	2.65
75°	1.54
90°	1.14
105°	1.54
120°	2.65
135°	3.55
150°	4.39
165°	4.57
180°	4.84

Wires 180° apart rotated thru 180°. Length of each antenna 264 feet (80.5 m.).
Outer ends ungrounded.

Plate 1 is a polar graph of these data, showing bi-lateral reception, a true cosine curve, with a maximum received current when the antenna lay in the plane of the arriving wave.

The equation of this curve is

$$I_R = K\, l \cos \theta$$

where I_R = received current,
l = length of each wire,
θ = angle between arriving wave and antenna,
K = a constant depending upon the units chosen for I_R and l.

In the SECOND TEST, the antennas were 90° apart and the resultant system shaped like a "V" was rotated thruout 180° with the base of the "V" as the center.

The observed readings were:—

TEST 2

Angle	Current
θ	I_R
0°	2.90
15°	3.80
30°	4.40
45°	4.85
60°	4.40
75°	3.80
90°	2.90
105°	2.49
120°	1.15
135°	0.00
150°	1.15
165°	2.49
180°	2.90

Wires 90° apart rotated thru 180°.
Length of each antenna 264 feet (80.5 m.).
Outer ends ungrounded.
At 0°, one wire was pointing toward the transmitter and the other at 90°.

Plate 2 is a polar graph of these data. It is somewhat similar to Plate 1, but rotated 45° with respect thereto. The main

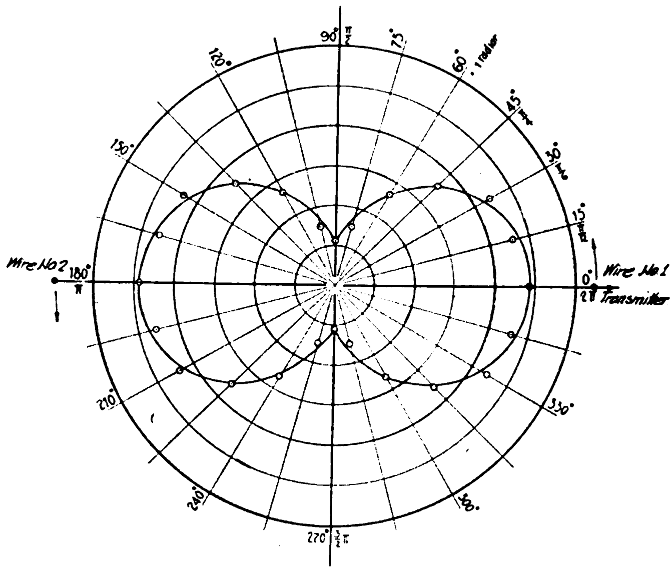

PLATE 1—TEST 1—Wires 180° Apart

point of interest is the excellent way in which the two halves of the ground antenna "buck" at 135° and 315°. No deflection of the galvanometer was visible at these angles.

The equation of this curve is

$$I_R = K\,l\,\cos\theta - K\,l\,\cos(\theta - 90°).$$

In the Third Test one antenna was kept fixed, *pointing toward the transmitter*, and the other rotated.

The observed data were:—

TEST 3

Angle θ	Current I_R
0°	0.00
15°	0.00
30°	0.00
45°	0.00
60°	0.63
75°	1.66
90°	2.53
105°	3.05
120°	3.47
135°	3.61
150°	3.74
165°	3.96
180°	3.94

One wire fixed. Other rotated.
Length of each wire 264 feet (80.5 m.).
Outer ends ungrounded.

Plate 3 shows the polar graph, a cardioid in form. This is similar to curves obtainable with the Bellini-Tosi method under certain conditions.

The equation of this curve is approximately

$$I_R = K\,l\,(1 - \cos\theta).$$

Test Four involved the use of one wire only. An earth connection was substituted for the other. In this way a Marconi directive antenna of extreme length for its height was obtained. The characteristic shown on Plate 4 is of the usual form for this type of antenna altho the zero received current at 90° and 180° is an extreme case of the usual reduction at these points, found in the commercial forms of this type of directive antenna.

The observed data were:—

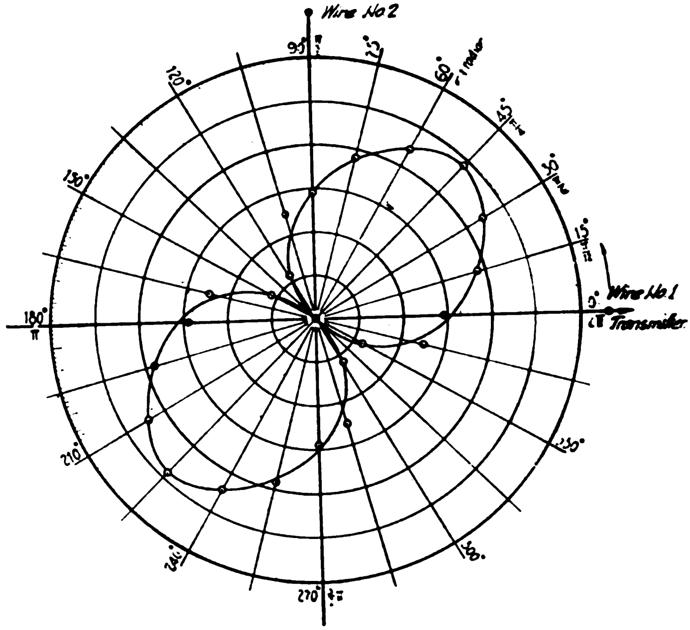

Plate 2—Test 2—Wires 90° Apart

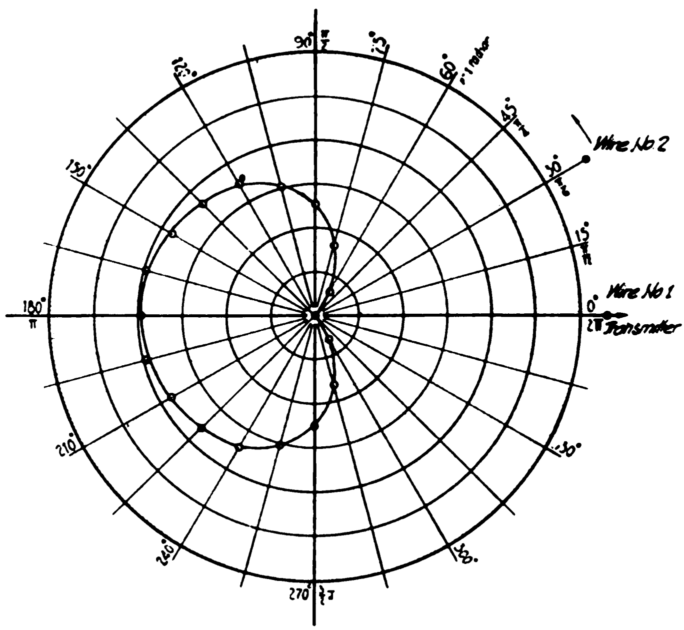

Plate 3—Test 3—One Wire Fixed, Other Rotated

TEST 4

Angle	Current
θ	I_R
0°	3.4
15°	3.2
30°	3.1
45°	2.49
60°	1.45
75°	0.78
90°	0.00
105°	0.78
120°	1.70
135°	3.07
150°	3.40
165°	3.55
180°	3.94

Single wire. One end grounded thru receiver. Other end rotated.
Length 264 feet (80.5 m.).

TEST FIVE involved observation of the effect of wire length upon received current. This current was found directly proportional to the wire length within the limits of the observations.

The observed data follow:

TEST 5

Length	Current
l	I_R
56	0.95
106	1.93
156	3.12
206	4.11
264	4.85

Length (l) = length of each wire in feet.
Outer ends ungrounded.

Plate 5 shows these results in rectangular co-ordinates.

TEST SIX was a short comparison of ground and vertical antennas. The former were the data of Test 5. The latter consisted of a harp of four vertical wires, each 42 feet (13.8 m.) in length on approximately 10-inch (25.4 cm.) centers. These could be cut in or out at will.

The observed results follow:

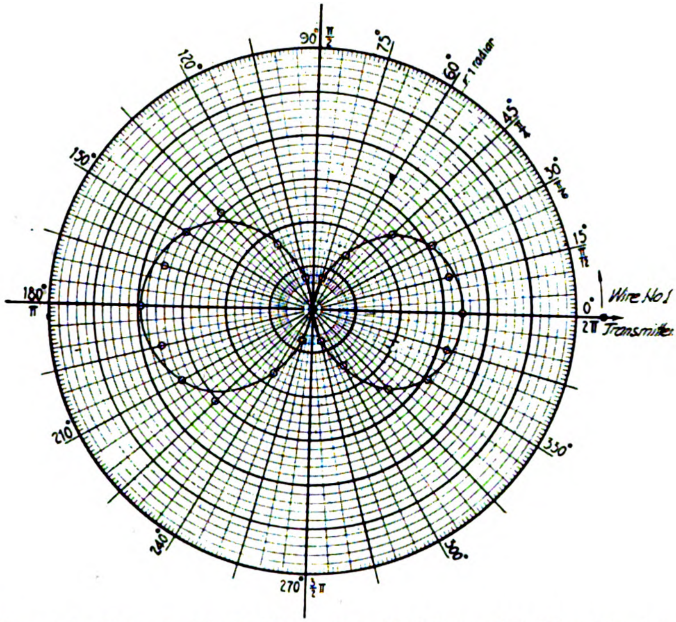

PLATE 4—TEST 4—Single Wire. One End Grounded Thru Receiver. Other Rotated

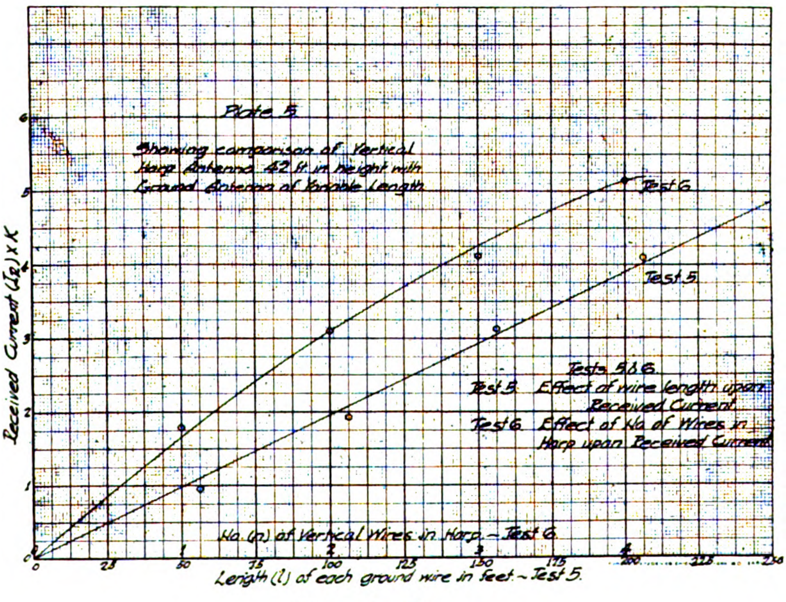

PLATE 5—TEST 5

TEST 6

Vertical Antenna	
Number of Wires	Current
n	I_R
1	1.78
2	3.10
3	4.12
4	5.15

These data are combined with those of Test 5, Plate 5, where the length (l) of each half of the ground antenna is compared with the equivalent number (n) of 42-foot (13.8 m.) vertical antennas required for the same received current.

Atmospheric disturbances were heavy and of about equal intensity on both the vertical and ground antennas.

At the date these data were taken neither continuous wave high powered plants nor oscillating tube receivers were available. Hence none of the present-day long distance reception was possible on the very small antennas used.

SUMMARY: The polar energy distribution curves of various types of low antennas are given for of slightly less than the wave length from the transmitter. The measurements were made with silicon detector and galvanometer. The influence of wire length of receiver, and vertical vs. ground antennas, were studied.

THE EFFECT OF THE SPARK ON THE OSCILLATIONS OF AN ELECTRIC CIRCUIT*

By

JOHN STONE STONE

(CONSULTING ELECTRICAL AND RADIO ENGINEER, PAST PRESIDENT OF THE INSTITUTE OF RADIO ENGINEERS)

When a condenser is permitted to discharge thru a coil under such conditions as not to produce a spark or under such conditions that the resistance of the spark is completely negligible compared to the conductor resistance of the circuit, the phenomena observed accord well with the mathematical theory advanced in 1853 by Professor William Thompson, the late Lord Kelvin.[1] When, on the other hand, the discharge of the condenser is accompanied by the spark and the conditions are such that the conductor resistance of the circuit is negligible compared to the resistance of the spark, then the Thompson theory no longer applies, even approximately, and such oscillations as occur in the circuit are of a character, the mathematical theory of which, I had the honor to present to this Institute in 1914.[2]

The Thompson theory, which we may best describe as the logarithmic decrement theory, is therefore seen to be the mathematical theory of one extreme or limiting case of electrical oscillation. This extreme case I have shown to be characteristic of very low frequency oscillators. The theory which I presented to you last Spring and which may best be described as the linear decrement theory, is seen to be the mathematical theory of an equally extreme case of electrical oscillation—a case which I have shown to appertain particularly to very high frequency oscillators.

Tho most oscillating circuits used in radio telegraphy are of very high frequency, nevertheless, there are many in which the conductor resistance is by no means negligible compared to the resistance of the spark and such electric oscillators therefore fall into the class for which there is as yet no competent theory upon

*Presidential Address presented before The Institute of Radio Engineers, New York, February 3, 1915.

[1] "Phil. Mag.," Series 4, Volume V, page 393.

[2] PROCEEDINGS OF THE INSTITUTE OF RADIO ENGINEERS, Volume 2, Number 4, page 307.

which reliable quantitative measurements may be based or from which reliable deductions may be made. It has seemed to me that it might therefore be desirable to carry the investigation of the effect of the spark upon the oscillations in an electric circuit further than was done in my latest paper and to consider the more general theory in which no limitation is placed upon the relative magnitude of the conductor and spark resistances. Such a general theory, of course, includes the logarithmic decrement theory and the linear decrement theory as special and oppositely extreme limiting cases.

It might well be expected that the deduction of this general theory would involve grave difficulties and that the resulting expressions deduced for the oscillatory charge, voltage, and current would be of considerable complexity. Neither of these sources of inconvenience has, however, been found to exist. The great simplicity both of deduction and result flows directly from the fact that the resistance of the spark is in general inversely proportional to the amplitude of the oscillating current flowing thru the spark, provided the oscillations be of radio frequencies and the spark gap electrodes be not composed of the so-called non-arcing materials. This fact was shown, in my before-mentioned paper, to be one of the necessary consequences of the experimentally observed linear mode of decay of the oscillations, noted when the spark resistance is the dominant resistance of the circuit.

As this paper is practically a second installment of my latest paper on this subject before the Institute, it is not necessary to repeat here the numerous collateral statements which it would otherwise be expedient to make by way of explanation, definition and limitation, and we may therefore proceed directly to the solution of the circuital equation:

$$0=\frac{1}{C}\int i\,d\,t+R\,i+L\frac{d\,i}{d\,t} \tag{1}$$

where C is the capacity of the condenser, i is the current at any moment, R is the total resistance of the circuit at that moment and L is the inductance of the circuit.

Let R_1 be the constant conductor-resistance of the circuit and R_2 be the spark resistance.

By dividing (1) thru by L, we may write the equation as:

$$0=\omega_o^2\int i\,d\,t+2\,(\alpha_1+\alpha_2)\,i+\frac{d\,i}{d\,t} \tag{2}$$

where $\omega_o^2=\frac{1}{CL}$, $\alpha_1=\frac{R_1}{2L}$ and $\alpha_2=\frac{R_2}{2L}$

We know that the current is of the form $i = I \sin(\omega t + \psi)$ where ω and ψ are constant and I only is a function of t because the most careful resonance analysis of these oscillators fails to reveal the presence of a second frequency. If the spark introduced an oscillation of a second frequency, its presence could be detected even tho its amplitude were less than 1 per cent. of that of the principal or fundamental oscillation except in the case of extremely high damping. Furthermore, since the current must be zero when t is zero, if $t=0$, be chosen as the time at which the oscillations begin, then

$$i = I \sin \omega t$$

It follows from what has been said above and from what was shown in my last paper, that

$$R_2 = \frac{d}{I}$$

where d is a constant.

Substituting these values for i and R_2 in the third term of the right hand number of (2) gives

$$-\frac{d}{L} \sin \omega t = \omega_o^2 \int i\, dt + 2 a_1 i + \frac{di}{dt} \tag{3}$$

We see immediately that the effect of the spark in the circuit is merely to introduce a simple harmonic counter electro-motive force $-\frac{d}{L} \sin \omega t$ into the circuit at the beginning of the oscillations and to maintain it there at constant amplitude $-\frac{d}{L}$ as long as the oscillations last.

The solution of (3) therefore resolves itself into the solution of two equations:

$$0 = \omega_o^2 \int i_1\, dt + 2 a_1 i_1 + \frac{di_1}{dt} \tag{4}$$

and

$$-\frac{d}{L} \sin \omega t = \omega_o^2 \int i_2\, dt + 2 a_1 i_2 + \frac{di_2}{dt} \tag{5}$$

where $i_1 + i_2 = i$.

The complete solution of (4) with all constants determined by the conditions, $i_1 = 0$ and $\int i_1\, dt = Q_o$ at $t=0$ is:

$$i_1 = I_1 \varepsilon^{-at} \sin \omega t \tag{6}$$

where

$$I_1 = Q_o \frac{a^2 + \omega^2}{\omega} = Q_o \frac{\omega_o^2}{\omega},$$

$$a = a_1$$

and

$$\omega^2 = \omega_o^2 - a^2.$$

The solution of (5) is:

$$-I_2 \sin(\omega t+\theta)+I_3 \varepsilon^{-\alpha_1 t} \sin(\omega t+\phi) \tag{7}$$

where $I_2=\frac{2\alpha_o \omega}{\alpha_1\sqrt{\alpha_1^2+4\omega^2}}$, $\theta=\tan^{-1}\frac{\alpha_1}{2\omega}$ and $\alpha_o=\frac{d}{2L}$

I_3 and ϕ are determined by the conditions that at $t=0$, $i=0$ and $\int i_2\, dt=0$. These give:

$$I_3=\frac{2\alpha_o\sqrt{\alpha_1^2\omega^2+(\alpha_1^2+2\omega^2)^2}}{\alpha_1(\alpha_1^2+4\omega^2)}$$

and

$$\phi=\tan^{-1}\frac{\alpha_1\omega}{\alpha_1^2+2\omega^2}$$

The complete expression for the current is therefore:

$$i=Q_o\frac{\omega_o^2}{\omega}\varepsilon^{-\alpha_1 t}\sin\omega t-\frac{2\alpha_o\omega}{\alpha_1\sqrt{\alpha_1^2+4\omega^2}}\Bigg\{\sin\left(\omega t+\tan^{-1}\frac{\alpha_1}{2\omega}\right)$$

$$-\frac{\sqrt{\alpha_1^2\omega^2+(\alpha_1^2+2\omega^2)^2}}{\omega\sqrt{\alpha_1^2+4\omega^2}}\varepsilon^{-\alpha_1 t}\sin\left(\omega t+\tan^{-1}\frac{\alpha_1\omega}{\alpha_1^2+2\omega^2}\right)\Bigg\} \quad . . . \tag{8}$$

This expression might not, at first inspection, seem to support fully my earlier statement that the expressions for the oscillatory current voltage and charge are simple and convenient. When, however, this expression is examined in the light of the fact that the ratio $\frac{\omega_o}{\omega}$ is almost exactly unity for slightly or moderately damped oscillators, and is very nearly unity even in the case of very heavily damped oscillators and when further account is taken of the fact that the ratio $\frac{\alpha_1}{2\omega}$ is a very small quantity, even in the case of heavily damped oscillators, it is seen that the expression for the oscillatory current reduces to:

$$i=\left\{Q_o\omega_o\varepsilon^{-\alpha_1 t}-\frac{\alpha_o}{\alpha_1}(1-\varepsilon^{-\alpha_1 t})\right\}\sin\omega_o t \tag{9}$$

In the limiting case of $\alpha_1=0$, or no damping except that due to the spark this expression reduces to

$$i=(Q_o\omega_o-\alpha_o t)\sin\omega_o t \tag{10}$$

which is the case of linear decrement treated in my first paper on the effect of the spark on the oscillations of an electric oscillator.

In the other limiting case, namely that in which α_o is negligible compared to α_1, the expression for the oscillatory current reduces to:

$$i=Q_o\frac{\omega_o^2}{\omega}\varepsilon^{-\alpha_1 t}\sin\omega t \tag{11}$$

which we immediately recognize as the well-known case of logarithmic decrement, first treated by Lord Kelvin and in which there is no damping due to the spark.

Before proceeding further with the specific subject of this paper, I shall call attention to the peculiar relations between ω_o, ω and a_1 which resulted in the marked simplification of the expression (9) for the oscillatory current.

If we define the logarithmic decrement per cycle of the oscillator in the absence of spark resistance as $\delta = \frac{\pi R}{L \omega}$, then the fact that $\frac{\omega_o}{\omega}$ is practically unity for all values of δ is exhibited in Table I.

TABLE I

$\delta =$	0.2	0.25	0.3	0.35	0.4	0.45	0.5
$\frac{\omega_o}{\omega} =$	1.00050	1.00078	1.00114	1.00155	1.00202	1.00257	1.00317

The fact that $\frac{a}{2\omega}$ and $\frac{a\omega}{a^2+2\omega^2}$ are both small compared to unity and practically equal to each other over a wide range of values of δ including even cases of high damping is exhibited in Table II.

TABLE II

$\delta =$	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
	.0080	.0120	.0160	0.020	0.0240	0.0280	0.0320	0.0360	0.040
$\tan^{-1} \frac{a}{2\omega}$	0° 27′	0° 41′	0° 54′	1° 9′	1° 22′	1° 35′	1° 49′	2° 3′	2° 17′
	0.0080	0.0120	0.0160	0.020	0 0240	0.0280	0.0320	0.0360	0.040
$\tan^{-1} \frac{a\omega}{a^2+2\omega^2}$	0° 27′	0° 41′	0° 54′	1° 9′	1° 22′	1° 36′	1° 50′	2° 4′	2° 18′

This digression is made and these tables are here introduced because the relations involved are of such universal application in the theory of oscillators and are capable of producing such marked simplifications of the mathematical expressions and computations therefrom, that it is well to have on record for convenient reference the numerical relations of these functions over a fairly wide range of values of the logarithmic decrement δ.

The expression for the quantity of electricity involved in the oscillations, when this expression is simplified in the same way as that for the current, becomes:

$$q = \left\{ \left(Q_o + \frac{i_o}{a_1 \omega_o} \right) \varepsilon^{-a_1 t} - \frac{i_o}{a_1 \omega_o} \right\} \cos \omega_o t \qquad (12)$$

The voltage is, of course, merely $v = \frac{q}{C}$, where C is the capacity of the condenser.

The expression for the current enables us to determine one for the resistance R_2 of the spark in terms of its initial resistance R_0 and time for, save in the very exceptional cases to which this theory does not apply, the resistance of the spark is inversely proportional to the amplitude of the current.

If we designate this amplitude by I, we have:

$$I = Q_o\,\omega_o\,\varepsilon^{-a_1 t} - \frac{a_o}{a_1}\left(1 - \varepsilon^{-a_1 t}\right) \qquad (14)$$

and

$$R_2 = R_o \frac{I_o}{I} \qquad (15)$$

where I_o is the initial amplitude of the current or $Q_o\,\omega_o$.

These expressions shed considerable light on the quenching of the oscillations of a wave train after a certain time T. In the logarithmic decrement theory there was no indication of the time of quenching, the current merely indefinitely diminished, and tho experience taught us that after a relatively definite time for any particular oscillator, the oscillations positively ceased and the circuit automatically opened at the spark gap, yet there was no recognition of this fact in that theory.

An important result of the fact that this theory which we have just developed, takes cognizance of the definite finite length of an oscillation train, is that it enables us to express in a Fourier's series any isochronous succession of similar damped oscillation trains, provided the oscillation trains do not overlap. The expression in a sine series is effected in this case, as in the case of the linear decrement theory.[3]

In determining the co-efficients of the series it is sufficient to take into consideration only the amplitude of the successive oscillation trains and not the instantaneous value of the oscillations, so that the series is of the form:

$$i = \sin \omega_o t \sum_{1}^{\infty} a_m \sin \frac{m\pi}{T_1} t \qquad (16)$$

where T_1 is the periodic time of recurrence of the oscillation trains or the reciprocal of the group or spark frequency.

An examination of the expression (9) for the current shows that after a time

[3] PROCEEDINGS OF THE INSTITUTE OF RADIO ENGINEERS, Volume 2, Number 4, page 318.

$$T_o = \frac{1}{a_1} \log_{\varepsilon}\left(Q_o \frac{a_1 \omega_o}{a_o} + 1\right) \tag{17}$$

the amplitude of the current becomes zero and therefore by (15) the resistance of the spark gap becomes infinite. This time T_o, is obviously the time of the quenching of the oscillations.

If T is the time of a complete oscillation, then $\frac{T_o}{T}$ is evidently the number N of complete oscillations in an oscillation train, so that

$$N = \frac{\omega_o \log_{\varepsilon}\left(Q_o \frac{a_1 \omega_o}{a_o} + 1\right)}{2 \pi a_1} \tag{18}$$

Concerning the condition of non-oscillatory discharge, evidently this occurs when $T_o = \frac{T}{2}$ so that the condition is reached

when $$\frac{a_1}{a_o} = \frac{\varepsilon^{\frac{\pi a_1}{\omega_o}} - 1}{I_o} \tag{19}$$

The first of the last two expressions shows that a_o, a_1, and ω_o remaining the same, the persistency of oscillations increases with the quantity of electricity discharged across the gap.

This expression (18) also shows that Q_o and $\frac{\omega_o}{a_1}$ remaining the same, the persistency of the oscillations decreases with the ratio $\frac{a_o}{a_1}$.

Similarly, the expression (19) shows that a circuit which is just aperiodic may become oscillatory thru an increase in the quantity of electricity discharged across the gap. This increase of Q_o of course means increase in the ratio $\frac{C}{L}$.

Table III shows the variation in the number of complete oscillations per oscillation train for varying values of $\frac{a_o}{a_1}$.

In this table the remaining quantities are taken as

$$Q_o = 10^{-15},\ \omega = \omega_o = 10^6 \text{ and } \delta = 0.2.$$

TABLE III

$\frac{a_o}{a_1}=$	0.1	0.5	1	2	3	4	5	6	7	8
$N=$	23.1	15.0	12.0	8.96	7.33	6.27	5.49	4.91	4.44	4.05

Figure 1 shows the decay of the amplitude of the oscillations

of this same oscillator for various values of the ratio $\frac{a_o}{a_1}$. The outermost curve of this figure shows the decrescence of amplitude for the case of $\frac{a_o}{a_1}=0.1$, the next curve corresponds to the ratio $\frac{a_o}{a_1}=0.5$, etc., to the innermost curve which corresponds to a ratio $\frac{a_o}{a_1}=9.0$.

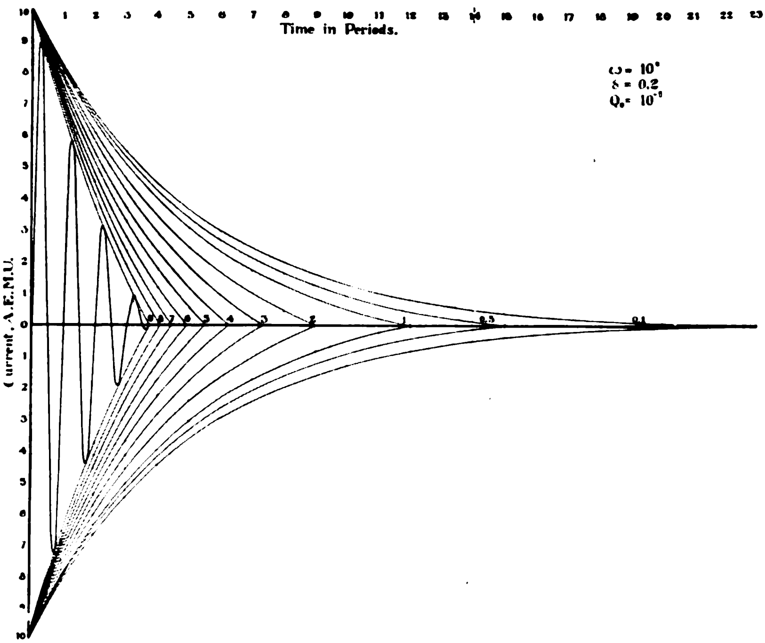

FIGURE 1

Table 4 shows the variations in the number of complete oscillations per oscillation train for varying values of the quantity Q_o of electricity involved in the oscillations. In this table the other constants of the oscillator involved are $\omega=\omega_o=10^6$, $\delta=0.2$ and $\frac{a_o}{a_1}=1$.

TABLE IV

Q_o =	0.0001	0.0002	0.0003	0.0004	0.0005
N =	12.0	15.2	17.2	18.6	19.7

Figure 2 shows the decrescence of the amplitude of the oscillations of an oscillator having these constants, for varying values of Q_o.

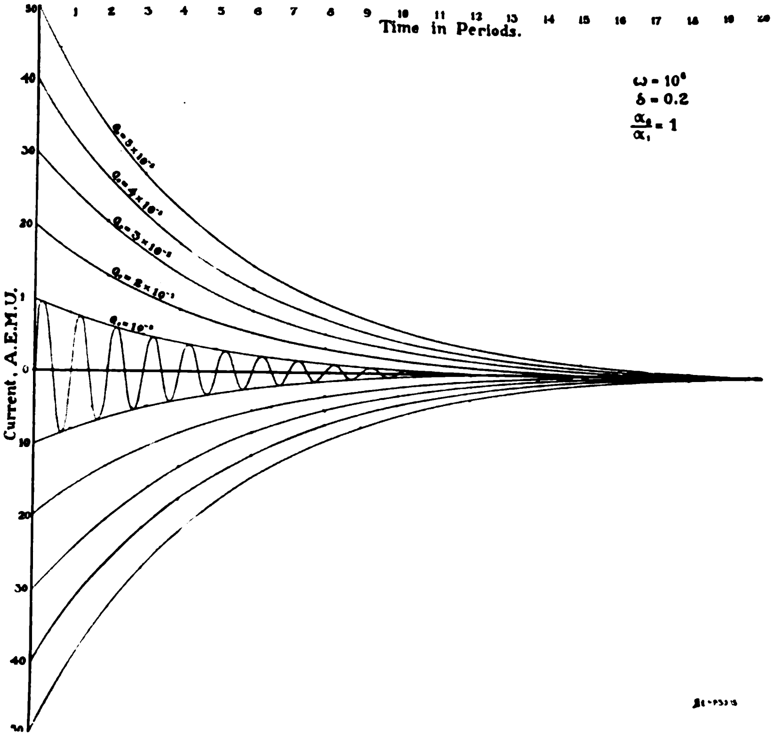

FIGURE 2

The increase of resistance of the spark with time for different values of $\frac{a_o}{a_1}$ and for different values of Q_o is exhibited in Figure 3. The curves marked 0.1, 0.5, 1, 2, etc., to 8, each represent the spark resistance for the corresponding ratio of $\frac{a_o}{a_1}$, the remaining constants of the oscillator being $Q_o = 10^{-5}$, $\omega = \omega_o = 10^6$ and $\delta = 0.2$. The curves marked $Q_o = 10^{-5}$, $Q_o = 2 \times 10^{-5}$, $Q_o = 3 \times 10^{-5}$, etc., represent the spark resistance for corresponding values of Q_o, the other constants being $\omega = \omega_o = 10^6$, $\frac{a_o}{a_1} = 1$ and $\delta = 0.2$.

In my earlier paper on the subject of the resistance of the spark, the mode of variation of the resistance of an arc with the

instantaneous value of the current flowing thru it and the hysteretic lag in the resistance of an alternating current arc is

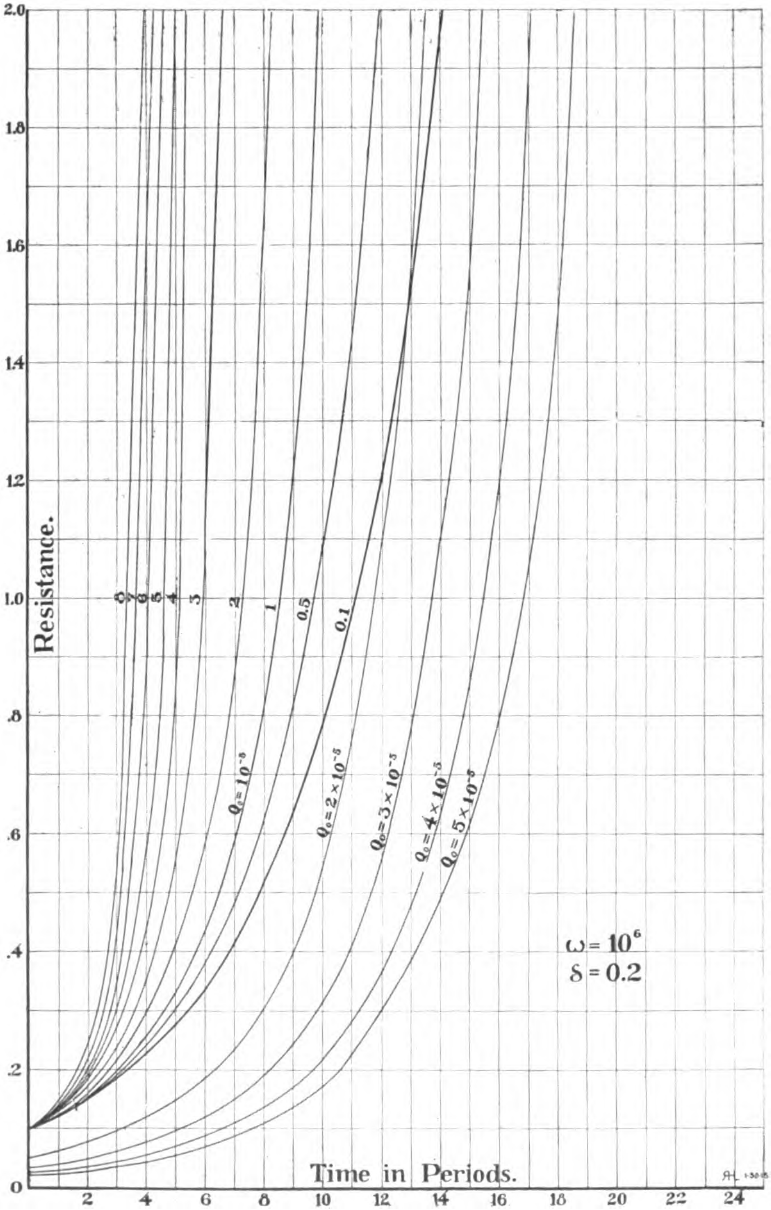

FIGURE 3

discussed.[4] The effect of these factors upon the shape of the curve of the instantaneous values of the voltage at the terminals of the arc is illustrated in Figure 8 of that paper. It is there shown that as the frequency of the alternating current is progressively increased, the instantaneous value of the resistance of the arc or spark becomes more and more nearly independent of the instantaneous value of the current and progressively tends to become more nearly inversely proportional to the amplitude of the alternating current so that when radio frequencies are reached, the simple law of inversed ratio to the amplitude of the current, in general, obtains.

In order to determine, in any particular case, to what extent, if any, there is a deviation in the resistance of the spark from the law of simple inverse ratio to amplitude of current, it is sufficient to examine the oscillators, by means of an extremely loosely coupled exploring resonant circuit, for the first odd harmonic. The relative magnitude of this odd harmonic, if such be found, to the fundamental oscillation will be an index of the departure, if any exists, from the law of simple inverse proportionality of the spark or arc resistance to the amplitude of the current.

Figure 4 of this paper shows how the peculiar dissymmetrical curve of instantaneous voltage across an alternating current arc may be synthetically constructed by the addition to the fundamental sine curve, of the two first odd harmonics. In this figure, curve 3 is the sine curve of the fundamental. Curve 4 is the sine curve of the first odd harmonic, curve 5 is the sine curve of the second odd harmonic, curve 6 is the cosine curve of the fundamental, curve 7 is the cosine curve of the first odd harmonic, the symmetrical curve 1 is the sum of curves 3, 4, and 5, and the dissymmetrical curve 2 is the sum of 3, 4, 5, 6, and 7.

Curve 1 corresponds very closely in shape to that of the voltage across an alternating current arc in which the instantaneous values of the resistance varies according to the well-known law

$$\rho = \frac{a}{i} + \frac{b}{i^2}. \qquad (20)$$

which obtains for unidirectional arcs and for arcs in which the current varies so slowly as not to exhibit the effects of the hysteretic lag of resistance behind current variation.

In this expression, which is the same as that given on page 320 of my previous paper, ρ is the instantaneous value of the

[4] Proceedings of The Institute of Radio Engineers, Volume 2, Number 4, pages 320 to 322.

resistance of the arc or spark in question, a and b are constants and i is the current. This expression for the resistance of an arc or spark may be said to give its "static resistance."

Curve 2, on the other hand, corresponds very closely to the curve of the instantaneous voltage across an alternating current arc of ordinary lighting frequencies in which the effect of the resistance hysteresis is well defined.

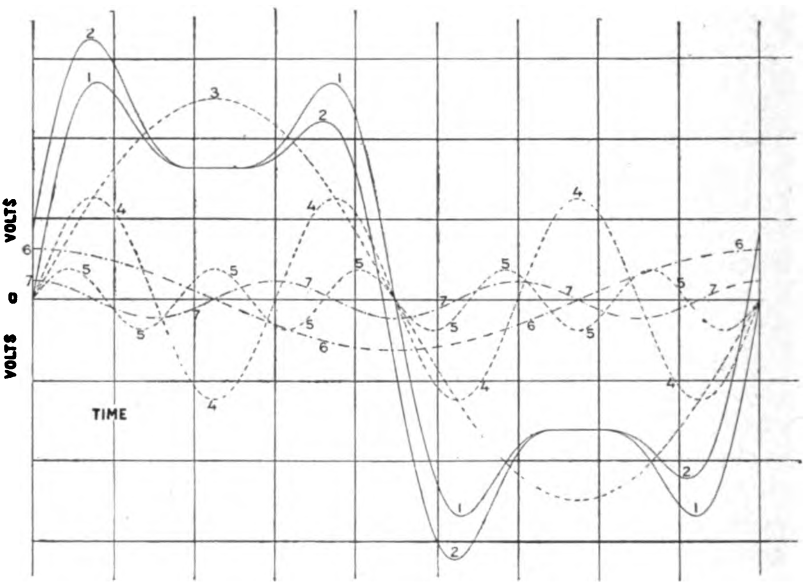

FIGURE 4

It will be noted that the two symmetrical peaks of curve 1, with the flat valley between them, are produced by the addition to the sine curve of the fundamental, of the sine curves of the first and second odd harmonics, while the dissymmetry of the peaks of curve 2 and the slight shift in phase of that curve is due, almost wholly, to the addition of the cosine curve of the fundamental. The diagram of Figure 4 illustrates well, therefore, how a resonant harmonic analysis of the oscillations of a circuit may be used to shed light on the degree to which, if at all, the resistance of the spark depends upon the instantaneous values of the current and therefore differs from the law of simple inverse proportionality to the amplitude of the current. It

shows, moreover, that the degree to which the hysteresis of the spark or arc resistance is effective is not so easily determined.

Coming now to one of the practical applications of some of the considerations contained in my two papers on the resistance of the spark and its effect on the oscillations of electric circuits, it is to be noted that the quenching of a spark or arc is the same thing as the development of an infinite or practically infinite gap resistance. In the case of an alternating arc or spark, there are only two reasons why the resistance of the gap does not reach an infinite value every time the current passes thru a zero value, i.e., once for each alternation or reversal. The first reason is the resistance hysteresis and the second is that the current does not remain long enough at or near its zero value to overcome the effect of the resistance hysteresis and permit the gap to assume its "static resistance" in accordance with the expression (20).

The hysteretic lag of the resistance of an arc or spark behind the instantaneous value of current change may be minimized by anything that tends to cool the arc or spark gap quickly, or otherwise to extinguish the arc or spark. For this purpose air blasts and powerful magnetic fields across the gap have been successfully used. The characteristic structure now so well-known as the quenching gap, giving as it does the maximum mechanical opportunity for the conduction and radiation of the heat away from the spark as well as the sub-division of the spark into a number of small series sparks, is much used. The use of electrodes of such metals as silver and copper, which minimized the hysteretic lag of the resistance of the spark, and the use of hydrogenous vapor at the gap are common. But besides minimizing the hysteresis of the spark, it is customary in the quenched spark systems to cause two oscillations of approximately equal amplitude and differing from each other in frequency by a small percentage, to pass across the gap at each spark so that owing to the beats between these oscillations, the *amplitude* of the oscillating current across the gap will fall to zero after say about five oscillations or so have passed and the resistance of the gap thereby permitted to attain a practically infinite value in spite of such resistance hysteresis as the spark may have.

Up to the present time most effort has been exerted to the diminution of the resistance hysteresis of the spark, i. e., to the enhancing of the quenching action at the gap *per se*. But it is evident from the aspect of the subject as here presented that when great or rapid quenching is required as in the production of *impulsive excitation* or in the production of *sustained oscillations*,

it is quite as important to provide time for the hysteresis of the arc or spark to expend itself and allow the gap to attain its "static resistance" as it is to minimize the resistance hysteresis at the gap. This can only be done by providing intervals of time when the current across the gap is zero or practically zero, these intervals being made long enough to permit the resistance at the gap to become sensibly infinite.

To accomplish this result we may have recourse to a combination of alternating currents or oscillations simultaneously flowing across the gap, these alternating currents or oscillations being so chosen that they will combine, together and with the supply current, to form intervals of zero current or substantially zero current across the gap.

By Fourier's theorem, it is evident that we can produce a combination of alternating currents, which will satisfy this requirement with as great a degree of approximation as we may wish. Thus the series:

$$i=I_o\left\{1+\frac{4}{\pi}\left(sin\frac{2\pi}{T}t+\frac{1}{3}sin\frac{6\pi}{T}t+\frac{1}{5}sin\frac{10\pi}{T}t+\quad . \quad . \quad .\right)\right\} \quad (21)$$

is a function which alternately has the values $2I_o$ and 0, holding each of these values for successive intervals of time $\frac{T}{2}$. The degree of approximation to the desired result which may be obtained by this series when using four of the sine terms, is illustrated in Figure 5. In this diagram curve 1 is the sum of all the dotted sine curves, 2, 3, 4, and 5, taken about axial line I_o. These dotted curves may be taken to represent alternating or oscillating currents and the straight line I_o may be taken to represent a steady unidirectional charging current. It will be noted that for the first half of the period the resultant current represented by curve 1 is approximately $2\,I_o$ in value, and for the second half it is approximately zero. Such a combination of currents passing across a spark gap would cause it to quench once in each half period even were the fundamental alternating or oscillating current of radio frequency.

If it is desired to make the interval of time during which the current is approximately zero long compared to that during which it is at full strength, another series of alternating currents may be deduced by Fourier's theorem, but for practical purposes it would not be desirable or necessary to have resort to a Fourier's series, which would lead to the necessity of using too great a number of different alternating or oscillating component currents. One who

is reasonably familiar with the practical working of Fourier's series can work out much simpler combinations of alternating or oscillating currents, which will give the desired intervals of approximately zero current with a sufficient degree of precision for

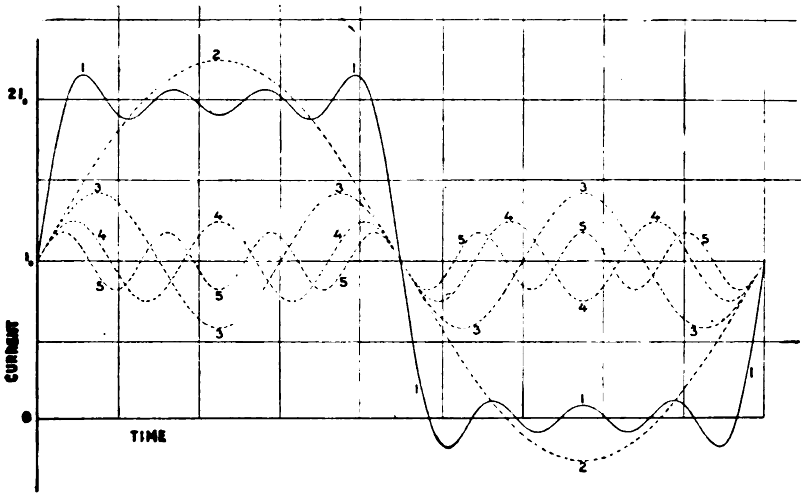

FIGURE 5

practical purposes. As an example, Figure 6 shows a case where the gap current illustrated by curve 3 is approximately zero 5/9*ths* of the half period. This result is accomplished by the

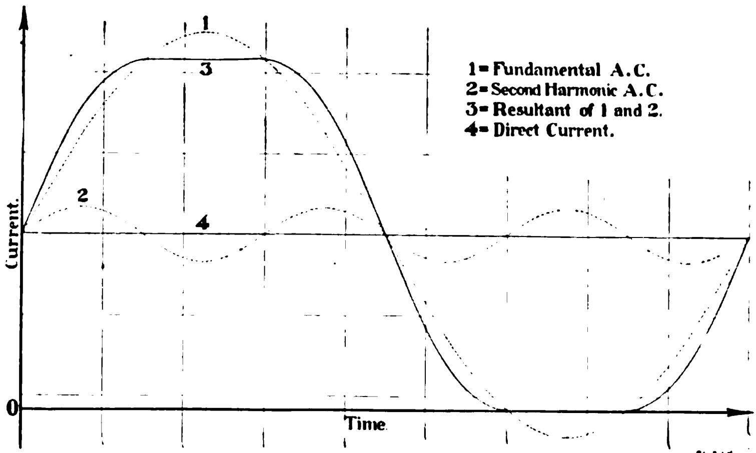

FIGURE 6

combination of a unidirectional charging current and two alternating or oscillating currents, the equation for the gap current being:

$$i = 0.86 + \sin \omega t + 0.135 \sin 3 \omega t \tag{22}$$

Perhaps a better arrangement might be secured for some purposes by the use of three frequencies, the equation for the gap current then being approximately

$$i = 0.80 + \sin \omega t + 0.26 \sin 3 \omega t + 0.064 \sin 5 \omega t \tag{23}$$

This case is illustrated in Figure 7 and it is seen that the gap current is approximately zero for two-thirds of the half period. Probably the easiest way to secure impulsive excitation, how-

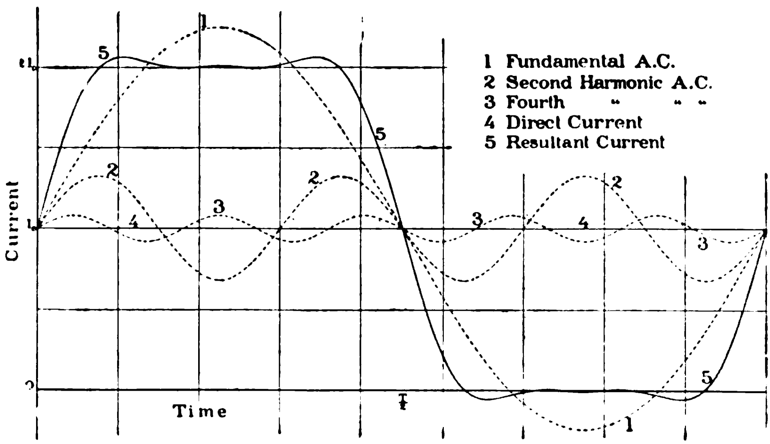

FIGURE 7

ever, would be to employ a supply current not great enough, *per se*, to maintain an arc across the gap and then use a combination of oscillating or alternating currents whose sum would be zero over a considerable fraction of the time period of the fundamental. Such a combination is given approximately by the series:

$$\sin \omega t + \sin 2 \omega t + \frac{1}{3} \sin 3 \omega t \tag{24}$$

This series is approximately zero for about 4/11*ths* of the total period.

Several ways in which these combinations of gap currents may possibly be secured in the case where oscillating circuits are

used and some suggestive modes of associating the oscillating circuits with the aerial are shown in Figures 8, 9, 10, 11 and 12, where S represents a quench gap which may have its quenching power enhanced, if necessary, by the use of hydrogenous vapor between its plates and by the use of a powerful magnetic field acting axially along the gap.

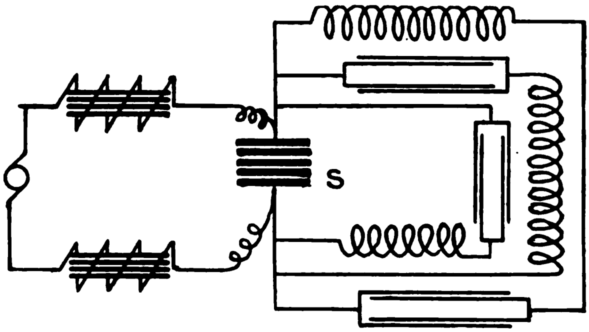

FIGURE 8

No attempt can be made here to go into a detailed discussion of the theory of operation of such circuits. Suffice it to point out that when there is more than one oscillating circuit shunted about a gap, the quenching of the spark after the initial surge has passed leaves the remaining circuits free to oscillate without the damping effect of the spark. If there be two oscillation circuits, these residual oscillations will have but a single frequency. If there be three oscillation circuits these oscillations will have three frequencies. If there are four there will be six frequencies, etc., the number of frequencies being given by the expression

$$N_1 \frac{N_1 - 1}{2} \tag{25}$$

where N_1 is the number of branch oscillation circuits. Of course, if the aerial or radiating circuits be one of the oscillating circuits shunted about the gap, it tends to develop additional higher frequency component currents which are not, strictly speaking, harmonics, tho always of higher frequency than the main or fundamental natural frequency of the aerial. This phenomena may, in some instances, be of sufficient importance to modify, somewhat, the proportion of the other circuit or circuits shunted about the gap for the purpose of producing impulsive excitation of the aerial. These high frequencies can be suppressed to a

very considerable degree by working the aerial at a frequency much below its fundamental natural frequency. Moreover, when the aerial is one of the oscillating circuits shunted about

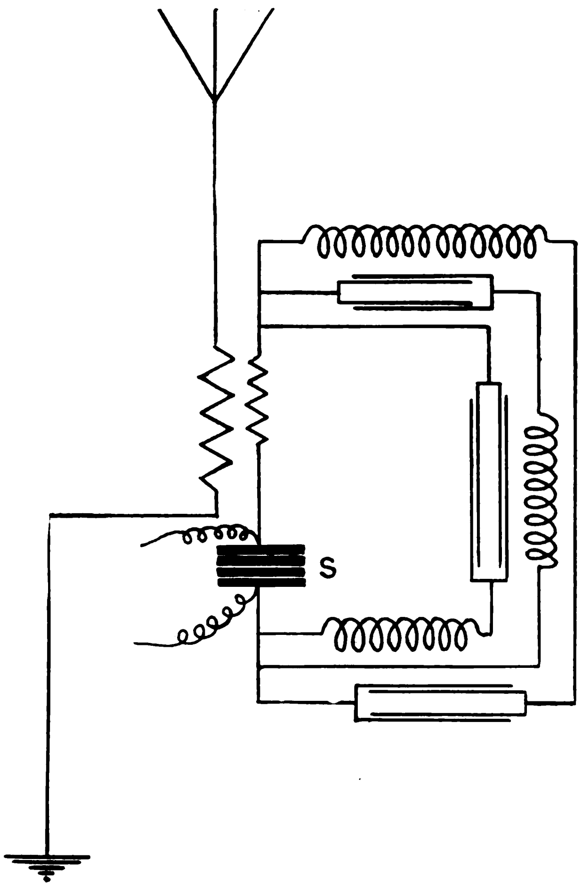

FIGURE 9

the gap, and a single wave length of radiation is desired, it will, in general, be necessary to employ but a single additional circuit shunted across the gap.

When two spark gaps are used in parallel as shown in Figure 12, the sparks occur alternately in the two gaps and if the gaps be shunted by similar oscillators, the oscillations developed in them will be opposite in phase. By this means, two trains of oscillations having any desired phase relation to each other

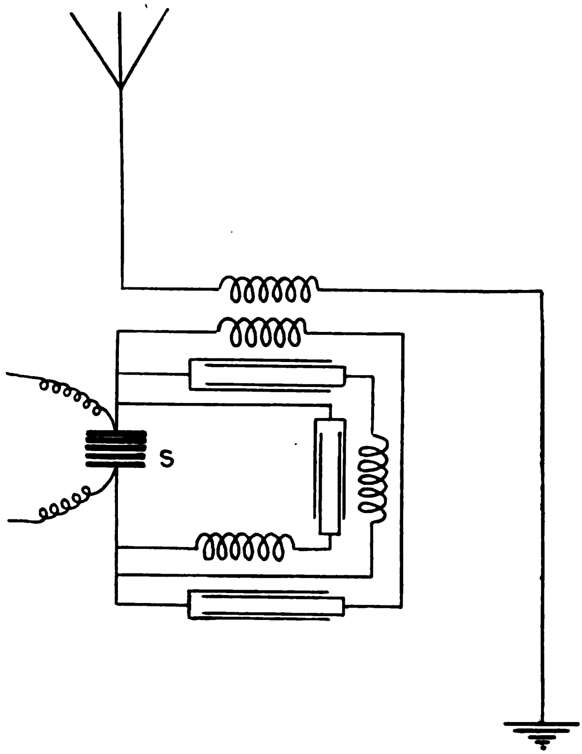

FIGURE 10

should be attainable. Furthermore, in this arrangement of parallel spark gaps, it should be possible to maintain the supply current extremely constant and to secure for a given power much higher frequencies of oscillation than with a single gap.

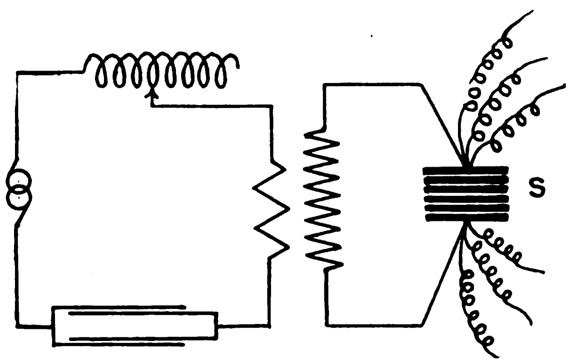

FIGURE 11

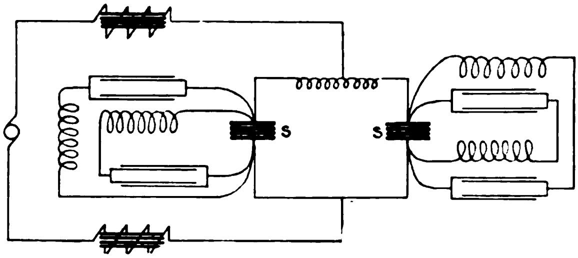

FIGURE 12

SUMMARY: The Thompson or "logarithmic decrement " theory of oscillatory circuits is contrasted with the linear decrement theory. Both are limiting cases of the more general theory which considers circuits containing both conductor and spark resistance. It is this last and most general theory which is fully developed in the present paper.

It is found that the effect of the spark is to introduce a simple harmonic counter E. M. F. of constant amplitude during the oscillations and having the same frequency as these oscillations. The mathematical expressions for the current and voltage, under practical working conditions, are found to be simple, and to reduce to the older expressions in the limiting cases.

The new theory gives the value of the spark resistance at any time and determines the quenching time. A new resonance analysis method of studying the law governing the dependence of spark or arc resistance on current is given.

A novel method of producing "quenching" in gaps is the following. There are shunted around the gaps several circuits of different frequencies so proportioned that the total current remains appreciably zero for a considerable portion of each cycle (of the fundamental frequency). The practical applications of this method to radio telegraphy are described in detail. Finally, a parallel spark gap method for obtaining oscillations of any desired phase difference is given.

DISCUSSION

J. Zenneck: We can speak of a "spark resistance" R only if we take the absorbed energy in the gap in the time dt as being $R\,I^2\,dt$. Here I is the instantaneous value of the current. The actual energy consumption in the gap can be studied by ascertaining the change of V, the terminal potential difference, with time. The energy consumption has the well-known value $I\,V\,dt$. Since we know by actual researches that the current in a condenser circuit is practically sinusoidal, a knowledge of the time-variation of V will readily give the energy absorbed.

Researches as to V have given the following results. There are two groups of metals (used for gaps) which are to be distinguished: (1) the copper group, consisting principally of copper and silver and (2) the magnesium group, which contains also calcium. The remaining metals lie between these extremes indicated by these groups, and, zinc and antimony lie near the magnesium group. With metals of the copper group, the gap potential follows a curve like that of Figure 1, while with the

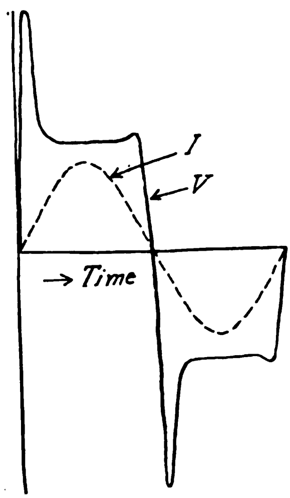

FIGURE 1

magnesium group it follows a curve like that of Figure 2. (D. Roschansky, "Annalen der Physik," 36, page 281, 1911.) The curves given hold even for the frequencies used in radio telegraphy. Whereas with the magnesium group the voltage is at

least approximately proportional to the current, with the copper group the gap voltage is similar to that of an arc at ordinary audio frequencies.

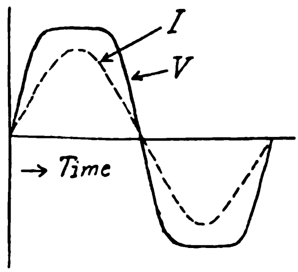

FIGURE 2

The foregoing principles lead to quite different types of decay of current amplitude in the two groups. For the magnesium group, the amplitude curve is one lying between an exponential curve (which corresponds to a constant spark resistance wherefor V would be proportional to I) and a straight line. According to conditions, the curve actually obtained will approach one or the other extreme. (See Figure 4). With the

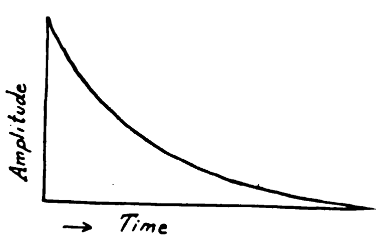

FIGURE 3

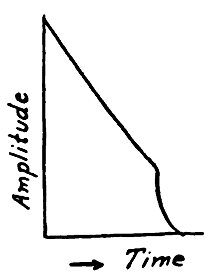

FIGURE 4

copper group, on the other hand, we found (carrying on a research with Braun tubes at the Physical Institute of Danzig) the amplitude curve to consist first, of a practically linear portion. which then suddenly fell off past a definite point. The oscillations are not actually quenched, but, as a careful examination of the photographs shows, simply fall off very rapidly.

Ralph H. Langley: It was my privilege to perform those calculations which were necessary for the plotting of the curves shown in Figures 1, 2, and 3. In all, over 170 numeric solutions of the final equation were made. It is at once apparent that this work would have been very onerous, had the substitutions been made in the complete equation (8). It was found however that the work could be greatly simplified by a suitable simplification of the equation. Mr. Stone has given the form:

$$i=\left\{Q_o\,\omega_o\,\varepsilon^{-a_1 t}-\frac{a_o}{a_1}\left(1-\varepsilon^{-a_1 t}\right)\right\}\sin\omega t \qquad (1)$$

but in order to justify this simplification, it was first necessary to compute the values of

$$\frac{\omega_o}{\omega}, \qquad \tan^{-1}\frac{a_1}{2\omega}, \qquad \text{and } \tan^{-1}\frac{a_1\,\omega}{a_1^2+2\,\omega^2}$$

in order to demonstrate that these values might be neglected without seriously affecting the accuracy of the results. From the relations

$$a_1=\frac{\delta\,\omega}{2\,\pi} \quad \text{and} \quad \omega^2=\omega_o^2-a_1^2$$

and assuming $\omega=10^6$, and taking $\delta=0.1$, 0.2, 0.3, 0.4, and 0.5 successively, the values which Mr. Stone has given on the fifth page of his paper were obtained.

The coefficient of ($\sin\omega t$) in equation (1) gives the envelope curves which it was desired to plot. This expression was further simplified and thrown into a form suitable for numeric substitution as follows:

The amplitude
$$I=Q_o\,\omega_o\,\varepsilon^{-a_1 t}-\frac{a_o}{a_1}\left(1-\varepsilon^{-a_1 t}\right)$$

$$=\varepsilon^{-a_1 t}\left(Q_o\,\omega_o+\frac{a_o}{a_1}\right)-\frac{a_o}{a_1}$$

$$=\log^{-1}\left\{\log_\varepsilon\left\{Q_o\,\omega_o+\frac{a_o}{a_1}\right)-a_1 t\right\}-\frac{a_o}{a_1}$$

since $-a_1 t$ is the natural logarithm of the quantity ($\varepsilon^{-a_1 t}$).

One set of curves were obtained holding Q_o constant at 10^{-5}, and taking the ratio $\frac{a_o}{a_1}$, which expresses the relative resistance of the spark gap at 0.1, 0.5, 1, 2, 3, 4, 5, 6, 7, 8, and 9 successively. Another set of curves was obtained holding $\frac{a_o}{a_1}$ constant at 1

and taking the initial charge Q_o at 10^{-5}, 2×10^{-5}, 3×10^{-5} 4×10^{-5}, and 5×10^{-5} successively. In each case between 5 and 20 values of I were computed for increasing values of t. The logarithmic decrement δ was considered constant at 0.2 and the periodicity ω constant at 10^6. The values of t were obtained from $T=\frac{2\pi}{\omega}$ where T is the time in seconds elapsed at the end of one complete oscillation, and from $a_1=\frac{10^5}{\pi}$ (from the assumptions). Whence $a_1 t=\frac{2\pi}{\omega}\times\frac{10^5}{\pi}\times n=0.2$ at the end of the first oscillation, 0.4 at the end of the second, and so on, n being the number of complete oscillations considered.

The actual method then, was as follows: a table of Naperian or natural logarithms being used.* Go into the table with $\left(Q_o\omega_o+\frac{a_o}{a_1}\right)$, take out $log_\varepsilon\left(Q_o\omega_o+\frac{a_o}{a_1}\right)$. Subtract $a_1 t$. Go into the tables with $\left\{log_\varepsilon\left(Q_o\omega_o+\frac{a_o}{a_1}\right)-a_1 t\right\}$ and take out $log_\varepsilon^{-1}\left\{log_\varepsilon\left(Q_o\omega_o+\frac{a_o}{a_1}\right)-a_1 t\right\}$. Subtract $\frac{a_o}{a_1}$, and the result is I.

In order to determine how many oscillations would occur before I became equal to zero, in each of the various cases, the following equation was obtained:

$$a_1 t=log_\varepsilon\left(1+\frac{Q_o\omega_o}{\frac{a_o}{a_1}}\right)$$

from which we may calculate the value of t for $I=0$. These values have also been tabulated by Mr. Stone (seventh page of the paper).

In connection with Figure 6 which shows a method of producing shock excitation of an untuned antenna, by the use of a fundamental alternating current and a third harmonic alternating current superimposed on a direct current, the following observations may be made. Let the maximum value of the fundamental frequency current be unity. Then the maximum value of the third harmonic current should be 0.13397, which is the difference between the sin of 60° and the sin of 90°. It has

*There are many equations used in radio calculations which may be very easily handled by means of a suitable table of Naperian or natural logarithms. For such a table the reader is referred to "Logarithmic Tables," by Professor George William Jones of Cornell University (Macmillan & Co., London), or to the "Mathematical Tables" published by the Smithsonian Institute, Washington, D. C.

been stated that this will not give a flat top to the resultant wave form, and that a second approximation for the amplitude of the third harmonic current should be made, in order that the current may be more nearly equal to zero during that part of the oscillation in which it is desired to have no activity.

The question here is, how great is the difference between the middle third of the fundamental frequency current, and the half oscillation of the third harmonic current? Computation shows that at no point is this difference greater than 0.00668, which is entirely negligible. No second approximation of the third harmonic current therefore seems necessary. Its amplitude should be 0.13397.

John Stone Stone: The question as to what may be strictly called the "resistance" of the arc or spark was not considered in my two papers. The inquiry therein made was merely as to the form of what might perhaps best be termed the effective resistance at the gap and its effect upon the oscillations.

Because of the purely utilitarian purpose of these papers, it was sufficient, as I pointed out in my first paper (see first paragraph, page 320, volume 2), to style the ratio $\frac{V}{i}$, of the instantaneous values of the voltage across the gap to the current traversing it, the resistance of the arc or spark. From this paragraph, to which I have referred, it will be seen that it was not the intention in these two papers to contend that all of this ratio $\frac{V}{i}$ is, strictly speaking, and in the last analysis, a true dissipative resistance. A consideration of what component of this ratio is an actual dissipative resistance, and what component may be regarded as a thermo-electric counter e. m. f., is interesting but was not considered germane.

Coming now to the law of decay of the oscillations between spark terminals of the two classes of metals mentioned by Dr. Zenneck, it is a fact that in the case of electrodes of the copper, silver group, if the frequency of the oscillations be not too high and if the conditions be such as to minimize the resistance hysteresis of the arc or spark, the gap voltage may cease to be proportional to the gap currents. Thus the gap voltage may depart from the shape of curve 4, Figure 8, of my first paper on this subject where I discussed this subject, and approach more nearly in shape to say curve 3, of that Figure. But the conditions which

bring this phenomenon into prominence are perhaps somewhat exceptional; and even when this phenomenon is present, it is not likely sensibly to modify the law of subsidance of the oscillations in such cases as may be met with in radio practice for reasons which I hope to make clear.

The presence of such a deviation from the normal constancy in the ratio of the gap voltage to gap current, when it exists to any marked degree, is as I have shown (see Figure 4 of my present paper and the text relating thereto) easily detected; as, under such circumstances, the first odd harmonic of the fundamental will make its appearance and may be detected by a suitable resonant harmonic analysis of the oscillations in the oscillator. It is important to note that the magnitude of this phenomenon will be determined by the ratio of the amplitude of this first harmonic in the gap voltage to the amplitude of the fundamental in that voltage (see Figure 4). In any event, however, the effect of this phenomenon on the character of the oscillations is smaller than would at first seem possible because the power of this first odd harmonic component of the gap voltage to modify the character of the oscillations is proportional to its power of producing a current in the system and this is usually very small. Evidently, if it could produce no current in the system it would not modify the oscillations at all, and if the current which this component of the gap voltage can produce in the oscillator is a small fraction of that which the fundamental component of the gap voltage could produce in a circuit, the effect of the odd harmonic on the mode of oscillation of the circuit must be correspondingly small.

It may be shown that, in the case of an arc sustaining oscillations in an oscillator, the ratio of the amplitude or strength of the current which can be produced by the first odd harmonic in the gap voltage to that which can be produced in it by the fundamental of the gap voltage, is approximately $\frac{3\,a}{8\,S}$ where a is the ratio of the amplitude of the first odd harmonic in the gap voltage to the amplitude of the fundamental in that voltage and S is the selectivity of the oscillator or $\sqrt{\frac{L}{C\,R^2}}$, L, C, and R being respectively the inductance, capacity, and resistance of the oscillator as a whole. Thus, if, in a given case of an arc sustaining oscillations in an oscillating circuit, the ratio of the amplitude of the first odd harmonic in the gap voltage to the amplitude of the fundamental in that voltage be one-third and the selectivity

of the oscillator be 12.5, then the current which the first odd harmonic in the gap voltage will produce will be but 1 per cent. of that which can be produced by the fundamental of the gap voltage. In this example, however, the selectivity chosen was rather low and in radio practice the effect of the odd harmonic might be expected to be even much less than 1 per cent.

In the case where the oscillations are not sustained but consist of damped trains, this general proportion of effects must still obtain, but the resistance at the gap always becomes the dominant factor towards the extreme end of the train of oscillations as shown in curves 2 and 3 of Figure 4 of my first paper on this subject (see pages 312 and 313, volume 2) and in Figure 3 of my present paper. At the extreme end of the train, therefore, the odd harmonic in the gap voltage, when it exists to a marked degree has a chance to assert itself and to produce a modification of the mode of subsidance of the oscillation train. This is undoubtedly the reason for the sudden and interesting change in the law of the damping near the end of the oscillation train cited by Dr. Zenneck as having been observed at Danzig.

In this connection it is interesting to note that, as stated in my first paper on this subject (see first paragraph, page 313, volume 2), the experimental data upon which the present determination of the law of subsidance of the oscillations is based, is ambiguous as to the rate of subsidance at the extreme end of the oscillation train. But, as I there remarked, "the question of the extreme end of the train of oscillations is of minor importance, since by the time the amplitude of the oscillations has reached one-fifth its initial value, the energy of the remaining oscillations represents but 4 per cent. of the total energy of the train."

To sum up, the proportionality of the instantaneous values of the gap voltage to the gap current, may notably depart from constancy (thru the use of electrodes of the silver, copper group of metals and thru the minimization of the resistance hysteresis of the spark or arc), in any particular circuit, without sensibly modifying the frequency or mode of subsidance of the oscillations except perhaps near the end of a train of oscillations. Furthermore, the end of the train of oscillations is of relatively small importance because of the relatively small proportion of energy represented by these train ends. Tho the law of decrescence or subsidance which I have deduced depends for its rigid mathematical justification on the resistance at the gap being inversely proportional to the amplitude of the oscillatory current flowing

across the gap, a notable departure from this proportionality in the instantaneous value of this resistance, due to the presence of a fairly strong third harmonic in the gap voltage curve, produces but slight deviation from the law except at the extreme end of the oscillation train and in the case of slightly damped oscillators, it is doubtful if an appreciable modification of the law can be produced.

VOLUME 4 DECEMBER, 1916 NUMBER 6

PROCEEDINGS

of

The Institute of Radio Engineers

(INCORPORATED)

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EDITED BY
ALFRED N. GOLDSMITH, Ph.D.

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ONE HUNDRED AND ELEVEN BROADWAY

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PROCEEDINGS OF THE SECTIONS

WASHINGTON SECTION

On the evening of Wednesday, June 21, 1916, a meeting of the Washington Section of The Institute of Radio Engineers was held at the University Club, Washington, D. C. A dinner was tendered Captain William H. G. Bullard, Superintendent of the Naval Radio Service, and Member of the Board of Direction of the Institute, on the occasion of his departure for duty at sea.

BOSTON SECTION

A meeting of the Boston Section of the Institute was held on the evening of Thursday, May 25, 1916, at the Cruft High Tension Laboratory, Harvard University, Cambridge, Massachusetts. Two papers were presented. These were: "On the Energy Losses in Radio Telegraph Transmitters" by Messrs. Bowden Washington and P. H. Royster, and "Notes on Tone Circuits" by Mr. Fulton Cutting. The papers were followed by discussion. Professor G. W. Pierce of Harvard University presided, and the attendance was fifty-nine.

SEATTLE SECTION

On the evening of April 8, 1916, a meeting of the Seattle Section of the Institute was held at Denny Hall, University of Washington. A paper on "Safety Thru Radio" was presented by Mr. V. Ford Greaves, Radio Engineer, United States Department of Commerce. A general discussion followed. Mr. Robert H. Marriott, Expert Radio Aide of the United States Navy, presided; and the attendance was thirty-five. Mr. T. M. Libby was elected Secretary-Treasurer of the Section, and a future place of meeting chosen.

A meeting of the Seattle Section of the Institute took place on the evening of May 6, 1916, at the Chamber of Commerce, Central Building, Seattle. A paper on "The Mechanism of Radiation and Propagation in Radio Communication" by Mr. Fritz Lowenstein was presented. A discussion followed. Mr. Robert H. Marriott presided, and the attendance was fifteen.

On the evening of June 10, 1916, a meeting of the Seattle Section was held in the Chamber of Commerce, Seattle. A paper on "Sustained Wave Range Chart" was presented by Mr. Tyng M. Libby of the Bremerton Navy Yard. This was followed by a discussion. Mr. Robert H. Marriott presided, and the attendance was twenty.

The Institute of Radio Engineers announces
with regret the death of

Mr. Frank B. McSoley

(Engineer of the Narragansett Electric Lighting Company, of Providence, R. I., and Associate-Member of the Institute since June 25, 1913).

ON A DETERMINATION OF THE ENERGY LOSSES IN A RADIO TELEGRAPH TRANSMITTER*

By

BOWDEN WASHINGTON

(RADIO ENGINEER, CUTTING & WASHINGTON)

AND

P. H. ROYSTER

(RESEARCH STUDENT, HARVARD UNIVERSITY)

The research herein described was originally taken up with the idea of determining the efficiency of the spark gap of a radio telegraph transmitter. Calorimetry immediately suggests itself as the simplest, if not the only available, method. This method was found so exceedingly satisfactory and convenient at the desired accuracy that it was decided to extend the research to cover the entire transmitter circuit. The radio apparatus used comprised a later modification of the Chaffee system, consisting of a symmetrical copper gap in alcohol vapor, operating on 250 volts, 500 cycle A. C. The action of the gap may be considered as identical with that of the Chaffee gap when used with a tone circuit, the feed current being in that case unidirectional pulsating current; i. e., A. C. with a displaced zero axis. Pure impact excitation still exists with the later type of gap, the condenser discharging thru the gap in a large number of discrete loops or half cycles during each cycle of the A. C. feed.

This has been proven in several ways:

(1) By examining the gap current and antenna current with a special high-speed Duddell oscillograph, using circuits of high audio frequencies;

(2) By the peculiar period relations existing between the closed and radiating circuits;

(3) When the circuit is adjusted for maximum radiation, a careful search with an exceptionally good wave meter discloses but one wave, whose period is the natural period of the radiating circuit, and whose decrement is apparently the decrement of that circuit alone.

* Presented before the Boston Section of THE INSTITUTE OF RADIO ENGINEERS, May 25, 1916.

This operation was performed with great care; and determination having been made of where the "coupling waves" should occur if the gap behaved as a "quenched gap," particular care was taken to search at these points.

Under the best conditions, when the closed circuit consists of an exceedingly large condenser and a very small inductance, and the least obtainable uncoupled or free inductance, maximum radiation is obtained when the natural period of the primary is from 0.5 to 0.8 that of the secondary. Adjustment between circuits is far from critical, as shown by the following table taken on a ¼ kilowatt set with a 10.7 ohm phantom antenna. The secondary current and secondary wave length are given. Nothing whatever was changed but the capacity of the secondary condenser. The natural period of the primary was 440 meters.

λ_s	I_s	λ_s	I_s
460	3.8	815	4.0
575	4.0	1000	3.9
640	4.0	...	...

It will be noticed that the amplitude of the secondary current decreases as "resonance" is approached.

The most widely used forms of calorimeter consist essentially of a liquid bath carefully heat-insulated and properly stirred; and some device to measure the rise in temperature from which is determined the quantity of heat liberated. This determination can be made in two ways: viz., by knowing the heat capacity of the calorimeter and its contents, or by comparison with a known quantity of electrical energy introduced by means of a suitable heating coil. Both these methods, tho accurate, have not appealed to the engineer, on account of the laboriousness and attention to detail required. The most serious obstacles in the way of applying this method of calorimetry to commercial apparatus are the facts that it is impracticable to submerge certain parts of the apparatus in an oil bath, and that a calorimeter of sufficient size to contain some of the larger parts of the apparatus, in which the loss is small, would show an almost inappreciable temperature rise occasioned by this small loss. We were faced, therefore, by the engineering problem of devising some form of calorimeter which, tho giving the desired accuracy, would be quick, convenient in operation, and of suf-

ficient size to contain the largest single unit in the apparatus, and at the same time be appropriate for the measurement of heat losses of a widely divergent magnitude. The method which we have evolved has proved so satisfactory that we feel that a short note on this method may be of use, if not of interest, to engineers engaged in a similar line of work.

If the interior of a closed vessel is at a uniform temperature, T_1, the temperature of the surrounding medium being T_o, then h, the heat lost per second by conduction, radiation, convection and the leakage of air, is some function F (T_1, T_o), which for small values of T_o may be written as an explicit function of the temperature difference, ϕ $(T_1 - T_o)$. The value of h, expressed in watts, may be ascertained for a given vessel by liberating suitable variable quantities of electrical energy, and observing the temperature when thermal equilibrium is attained. This thermal equilibrium will obtain when the heat generated is equal to the heat lost to the surrounding medium. This temperature will be a measure of heat evolved in the apparatus, and can be read off immediately from the previously computed calibration curve.

This method does not require the heat capacity of the calorimeter and its contents to be known, nor does it require knowledge of how the heat is dissipated into the surrounding medium; and the result is not affected by the leakage of air into or out of the calorimeter, or by changing the contents during the experiment. If the container is made of a substance of good thermal conductivity, the changes in room temperature will affect the result. But if the walls of the vessel are made of a material of low heat conductivity and high specific heat, the effect of any change in room temperature will take quite a long while to affect the interior; so where results of greater precision are required, calibration points should be taken within an hour or so of the experiment; but for ordinary engineering purposes one calibration curve will be adequate. There is an objection to using a calorimeter with a low time period on account of the length of time before equilibrium to be attained. It is found, however, that the temperature within the calorimeter when a constant amount of heat is being generated rises exponentially with the time; i. e., $T = a\ (1 - \varepsilon^{bt})$. The value of the constants a and b will vary with the contents of the calorimeter and its previous thermal state. If, however, observations are made of the temperature at successive intervals of time, and the temperature computed against the anti-logarithm of time, its

intersection with the axis of $log^{-1}(-b\,t)=0$ will give the final temperature.

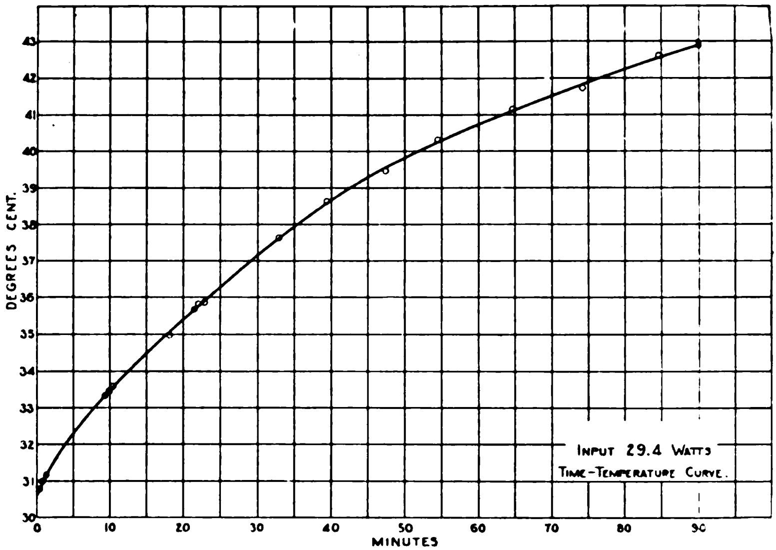

FIGURE 1

Figure 2 shows the points computed in a run which was carried out until the final temperature was maintained, in this case after 145 minutes; but it will be seen that observations on the first 10 minutes would have given this final temperature within 4 per cent. Making use of this device greatly shortens the time without any marked loss of accuracy; also where a calorimeter of too high sensitivity is used, the final temperature might be so great as to prove injurious to the apparatus. A certain amount of judgment is necessary in using this method of extrapolation if the calorimeter has been recently used for another experiment, as the presence of heat waves and loops sometimes leads one astray. Since, however, the cost of a calorimeter is negligible, it is no great matter always to have a cold calorimeter at hand.

It will be seen that the use of air as a calorimetric fluid obviates the necessity of stirring. If the apparatus is not too near the radiating walls, the temperature within will be sensibly constant. We have found it possible with a 30 degree rise of temperature to maintain the final temperature constant within

0.01 of a degree during a period of fifteen or twenty minutes. This temperature being constant to one part in 3,000 requires the heat evolved to remain constant to the same order of magnitude. The result in accuracy is not only greatly beyond engineering needs, but extremely difficult to attain with the highest form of precision instruments. It is impossible to conceive of the temperature remaining constant thru any such period of time, were any temperature gradients to exist in the large and unobstructed air spaces.

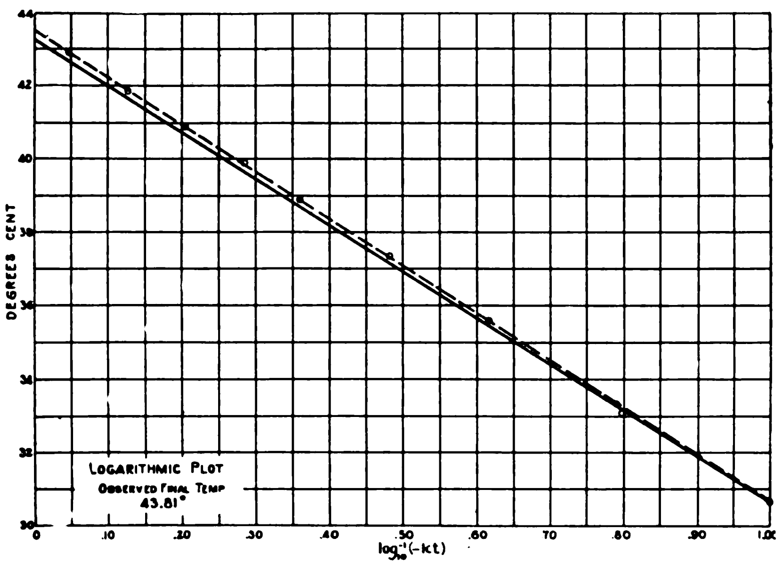

FIGURE 2

For the purpose of measuring energies up to 100 watts, we have found a satisfactory container in a corrugated double-walled cardboard packing case or carton, resting upon the top of an inverted and similar packing case, the junction between the two being rendered reasonably air-tight by strips of felt permanently attached to the lower box. The apparatus to be tested is placed on the lower case, and the upper case placed over it. A thermometer is introduced thru a hole in the top. It will be seen that since the calibration and the measurements are taken with the same thermometer, it is not necessary for the thermometer to be accurate. We have used a Fritz Köhler thermometer, graduated in tenths, which can be read with great

ease to 0.01 of a degree. This thermometer happened to have been carefully calibrated against the Bureau of Standards platinum thermometer, but there seems to be no reason why the worst thermometer would not serve equally well. The calibration curve of this calorimeter is shown in Figure 3.

This curve approaches the equation

$$h = 0.60\ (T - 25) + 0.124\ (T - 25)^2$$

hence if T_2 is the final temperature in an experiment, and T_1 is the final temperature in a calibration test, then

$$h_2 = h_1 + D_T\, h\ (T_2 - T_1).$$

If $(T_2 - T_1)$ is small, this obviates the necessity of adjusting the calibration energy by successive approximations for absolute coincidence of temperature. Where the apparatus under test

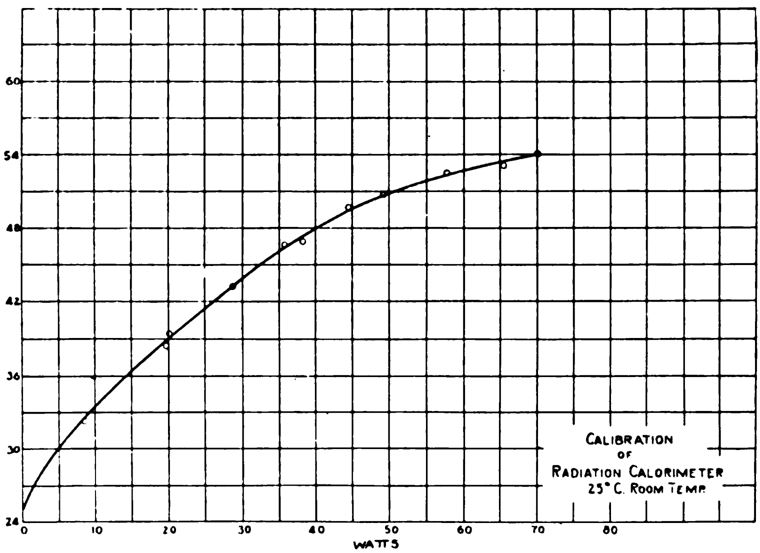

FIGURE 3

operates at a perceptibly higher temperature than the calorimeter, such as the gap shown in Figure 4, two sources of error may prove troublesome: loss of heat directly thru the walls of the calorimeter, and radiation to the bulb of the thermometer. In the arrangement shown in Figure 4, the gap rests on a piece of heavy felt, thus greatly minimizing the heat transmitted to the lower wall. Further, the heat transferred thru this lower

wall is a very small percentage of the total heat lost, on account of the insulating effect of the air space in the lower carton. Radiation to the bulb of the thermometer is prevented by the shield shown in the figure, a piece of cardboard secured to a wooden base.

The transmitter circuit under test consisted of a 500-cycle transformer charging a mica dielectric primary condenser of 0.07 μf. capacity; and a symmetrical copper gap, coupling coil and a phantom antenna composed of a standard copper-coated jar of 0.002 μf. and a 10-ohm resistance. This very excellent

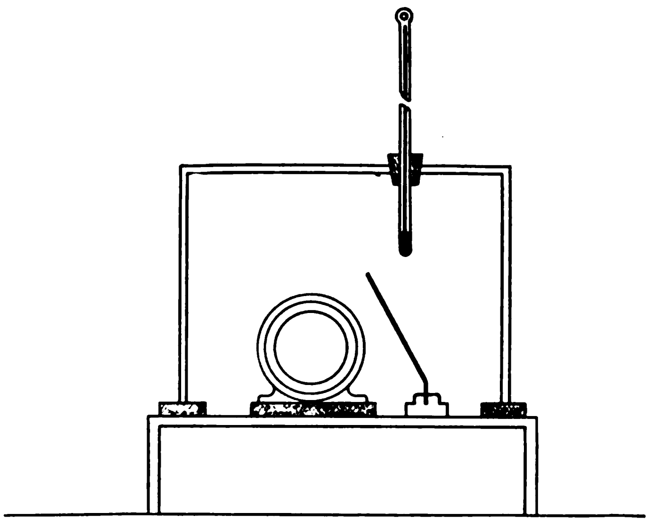

FIGURE 4—CALORIMETER

instrument, made for us by Mr. Melville Eastham, consisted of a 0.125 inch by 0.005 inch (0.32 cm. x 0.013 cm.) manganin strip wound on a flat asbestos holder. This was compared by Mr. Eastham with a number 38 manganin wire* at 3,000,000 cycles, and no discrepancy of resistance was noted.

Table 1 gives the resultant loss when the set was operated at its best adjustment.

* Diameter of number 38 wire = 0.004 inch = 0.0101 cm.

TABLE I

	Final Temperature	Watts Loss
Transformer	42.85	27.35
Primary Condenser	37.82	17.1
Gap	52.21	55.8
Coupling Coil	31.73	7.1
Secondary Condenser	33.25	8.3

The input in this case was 225 volt-amperes. The total energy in this set, as we can show later, was 207.2 watts, indicating a power factor of 0.92, and from the above table a transformer efficiency of 86.9 per cent. The gap loss given in the above table has been shown by repeated trials to be that yielded when the gap remains relatively cold. Under such conditions the closed circuit current is 14.7 amperes. However, during a test of some hours' continuous operation, due either to the increase of temperature of the primary condenser or of the gap, or both, the radiated energy decreases and the gap and primary condenser losses increase, resulting in a lowering of set efficiency.

Table 2 gives the result of an attempt to follow out to some extent the mutual dependence of these phenomena.

TABLE II

Test	Final Temperature	Watts Loss	Closed Circuit Current	Apparent Gap Resistance	Antenna Watts
1	52.21	55.78	14.7	0.258	100.2
2	53.45	62.55	15.2	0.211	93.1
3	54.15	68.18	15.9	0.260	87.6
4	54.75	74.05	16.5	0.272	82.0

We have carried out in the above table the gap loss to the second place of decimals; for altho the probable error in absolute magnitude of the energy measurements is greater than 0.01 of a watt, yet for changes of energy the figures carried are significant. The increase of gap loss appears to account for the loss of radiated energy between tests 1 and 2. The radiation decreases 7.1 watts; the gap loss increases 6.73 watts. For tests 2 and 3 these figures are 5.5 watts for the radiation, and 5.68 watts for the gap; while for 3 and 4 the energy changes are 5.6 against 5.83.

The value of the gap resistance given in the fifth column of the above table seems to indicate that the average gap resistance is somewhat independent of the current, certainly for such small variations, and within the limit of experimental error of the ammeter measuring the closed circuit current. The primary condenser during these tests we attempted to keep as cool as possible with a fan. Changing the impressed frequency on this condenser from 60 to 500 cycles, and varying the closed circuit current from the range as given in Table II, caused a maximum variation of condenser loss of 2 watts. We have as yet found no satisfactory explanation of this change of energy distribution from radiation to gap loss, and the accompanying falling off in set efficiency.

Since the value of the radiated energy was a matter of extreme importance, our measurements upon this quantity were made with greater care than in any other single test. The manganin-strip phantom antenna placed in the calorimeter was connected to a double-throw switch so arranged that the antenna current and a suitably chosen direct current could be applied alternately, the switch being thrown over very rapidly, and the direct current being adjusted until thermal conditions within the calorimeter were unaffected by the change.

Three interpretations of our results are possible. Since $W = I^2 R$, we may assume; first, that the phantom resistance R had the same resistance at 300,000 cycles that it showed on direct current, giving the true value of the current $I = \frac{\sqrt{W}}{R}$; second, we may select any one of the eight hot band ammeters which were placed in series as indicating the correct radio frequency current I, giving a measure then of the high-frequency resistance of the phantom $R = \frac{W}{I^2}$; finally, we may assume that both the observed R and I are correct, and that "e" is the measure of our calorimetric error where $e = W - I^2 R$. We think we may safely accept the value of the antenna resistance as the closest approach to the truth. The readings of the eight ammeters, as shown in the following table, being so discordant, it is impossible to select any one as a standard.

TABLE III

Ammeter	$W=I^2R$		
	Test I	Test II	Difference I–II
1	92.3	92.3	0.0
2	95.7	90.4	−5.3
3	89.2	92.0	+2.8
4	91.6	92.9	+1.3
5	91.0	91.0	0.0
6	89.2	88.2	−1.0
7	94.5	92.5	−2.0
8	95.7	96.6	+0.9
Mean	92.4	92.0	

The heat generated in the calorimeter was in Test I 91.6 watts, in Test II 92.2 watts, indicating an increase of 0.6 in the second test, whereas the mean of the ammeter readings shows a decrease of 0.4—an outstanding discrepancy of 1.0 watts between the two methods. But the variation noted between the individual meters is so much greater that the actual calorimetric error unfortunately cannot be determined.

Accepting then our antenna resistance as 10 ohms, and the heat generated corresponding to the ammeter reading in test 1 of Table II as 91.9, we arrived at the value of the radio frequency current, of 3.03 amperes. The energy loss of the secondary condenser being 8.3 watts, the resistance is 0.90 ohms. Our total antenna resistance is then 10.90 ohms, and the antenna energy 100.1 watts. This gives a set efficiency of 48.4 per cent.

The conclusions drawn from these experiments led us to construct a small set of about the same capacity but of the panel type, care being taken to shorten the closed circuit leads as much as possible, and to provide sufficient radio frequency conductance. The primary inductance in this case consisted of two turns of a cable composed of seven strands of 5-ampere "Litzendraht," each strand containing twenty-four number 30 wires.* The condenser was mounted directly on the back of the gap, the coupling coil immediately above; the primary wiring then consisting of practically nothing but these two coupled turns. It was found that the lead foil used in the primary condenser

* Diameter of number 30 wire = 0.010 inch = 0.025 cm.

was carrying approximately 0.25 ampere per sheet, and had not sufficient conductance. Heavier copper foil was substituted. The secondary of the coupling coil was wound with a single strand of 5-ampere "Litzendraht."

The generator was wound to a suitable voltage, namely, 500 volts no load, 250 volts full load, thus doing away with the transformer losses. The results obtained with this set thoroly justified the small amount of labor expended in calorimetry.

With the same 10.7 ohm phantom antenna, an antenna current of 4.5 amperes was obtained (216 watts) with an input of 308 watts—an efficiency of 70 per cent.

SUMMARY: A simple method of calorimetry is devised to measure the total losses of a modified Chaffee gap 0.25 K. W. transmitter. The equilibrium temperature of the air in a slightly heat-conductive box containing the transmitter under test enables the direct determination of the heat evolved. A supplementary calibration with known heat evolution renders the method simple and free from certain previously common calorimetric complications. The thermometer used must be shielded from direct radiation from the transmitter.

Guided by results thus obtained, the authors increased the over-all efficiency of the set under test from 48 to 70 per cent.

THE HEAVISIDE LAYER*

By

E. W. Marchant, D. Sc., M. I. E. E.

(David Jardine Professor of Electrical Engineering, University of Liverpool)

The formulas which have been obtained for the strength of signals received at a station when a measured amount of high frequency power is sent into a distant transmitting antenna (as the result of investigations and measurements by Austin and others) are well known, and have been dealt with fully in Mr. Fuller's paper before the American Institute of Electrical Engineers.[1]

It is within the knowledge of all radio operators that signals vary widely in strength, often in the course of a few minutes; and such variations can most easily be explained by reflection and refraction from moving masses of "cloud" or ionic fog. The surface wave theory developed by Sommerfeld, while explaining transmission over long distances, round the curvature of the earth, does not explain these sudden changes. The fact that these changes occur more by night than by day provides further evidence that the reflection and refraction theory of which Dr. Eccles has been, in this country, the chief exponent, is the most likely one to explain observed phenomena.

The experiments described by Balsillie[2] in which he found that dust storms occurring along the line of transmission affect signal strength, when the transmission is in the direction in which the wind is blowing, are of interest, as they indicate that the *atmosphere* immediately adjacent to the earth is a factor in the absorption of waves. The chief phenomena, however, which require further explanation are (*a*) the sudden variations in signal strength at night, and (*b*) that comparatively small changes in wave length may make relatively enormous changes in the strength of received signals. The experiments recently described by Mr. Fuller[3] have added much exact information

*Received by the Editor, April 18, 1916.

[1] "Proceedings of the A. I. E. E." Volume 34, p. 567.

[2] "Proceedings of British Association (Australia)," p. 514. See also the "Electrician."

[3] "Proc. A. I. E. E.," *loc. cit.*

to that already available for the discussion of this subject, and it may be useful, therefore, to consider them in their bearing on the existence and probable nature of what, in this country, is generally called the "Heaviside layer."

Tho it is usually called by this name, Professor Fleming[4] observed recently that it was Sir James Dewar, who was one of the first to draw attention to its existence. In a lecture to the Royal Institution in 1902[5], when discussing the constitution of the atmosphere Dewar pointed out that there were really two parts to it: the lower part in which atmospheric currents circulated, and in which the constituents were similar to those of the atmosphere at lower levels, and the upper part in which the distribution of gases was governed by their density. In two lectures delivered recently to the Royal Institution[6], Professor Fleming has discussed the formation of this upper ionized layer and the causes which produce it. He points out that, in order to produce ionization in such a gas as oxygen, by light radiation, it is necessary to have a wave length of the order of 1,500 to 1,800 ångstrom units (10^{-7} mm.), that is, light which is far beyond the ultra violet end of the spectrum. If such light really produces any ionization, then it is to be expected that the ionization would be reduced at night; and, therefore, that signals might be expected to vary in strength at night, if these ionized gases are the cause of signal variation. Professor Fleming suggests, however, that at heights of the order of 60 miles (100 km.), where the ordinary constituents of the atmosphere disappear and are replaced by hydrogen and helium and possibly other lighter gases, the most likely agency in producing ionization is the solar dust projected from the sun and transmitted to the earth thru the agency of light pressure. This explanation of the production of an upper ionized layer of gas is verified by the fact that the time interval elapsing between the passage of a sun spot across the solar meridian and the corresponding magnetic storm, as shown by Arrhenius, is about 45 hours, a figure which agrees fairly closely with the time Professor Fleming calculates that a particle of 1,200 ångstrom units diameter would take to pass from the sun to the earth. Whatever the cause which produces this layer, there is little doubt that such a layer exists, not necessarily in the form of a shell concentric with the earth, with fairly flat surfaces, but more likely in the form of large masses of gases in the upper regions

[4] "Electrician," Volume 75, p. 348.
[5] "Proc. Royal Institution," Volume 17, p. 223.
[6] *Loc. cit.*

of the atmosphere which act as reflectors and refractors for the waves that are used for the transmission of radio signals.

Other facts bearing on the presence of this layer have been dealt with by Dr. Eccles in a paper published by the Royal Society.[7] It will not be necessary to reproduce the argument he uses to prove its existence, that may be assumed. It is the object of this paper to discuss to what extent the Heaviside layer can explain the phenomena described by Mr. Fuller.

THEORY OF INTERFERENCE BANDS ON A SPHERICAL REFLECTOR

In the first place, the conditions governing the production of interference bands on a spherical surface with a surrounding envelope, also of spherical form, whose internal surface acts as a reflector, may be discussed. It is usually assumed that the height at which the Heaviside layer becomes sufficiently conducting to act as a reflector is about 50 miles (80 km.). This is shown to scale in Figure 1, the points A and B corresponding respectively with San Francisco and Honolulu. It is clear that the passage between this layer and the earth is very much in the nature of a narrow crevasse between two parallel surfaces or reflecting mirrors.

FIGURE 1

It is sufficient, as a first approximation, to consider, therefore the formation of interference bands by reflection from a pair of flat parallel surfaces. The distance between the bands of electromagnetic lightness and darkness is determined by the fact that the difference between the paths of the rays reaching these points by two alternative routes is half a wave length. If the difference in distance along the two paths is a multiple of a wave length the point is one of brightness; if it is an odd number of half wave lengths the point is dark.

It may be assumed in the first instance that one ray travels along the surface of the earth, as supposed in Sommerfeld's

[7] Eccles, "Proc. Roy. Soc.," A, Volume 87, pp. 79–99.

theory, and that the other is reflected by the Heaviside layer and by the earth's surface.

Altho it is hardly to be expected that reflection will be regular, in the first instance it may be advisable to consider the conditions which govern the width of interference bands formed by regular reflections between plane surfaces. Let the two surfaces be represented by $O\,P$ and $Q\,R$ distant h kilometers from each other.

Let n be total number of successively reflected rays, and θ the angle at which the reflected ray strikes the Heaviside layer. It is easily seen that

$$tan\,\theta = \frac{n\,h}{D} \qquad \text{(See Figure 2)}$$

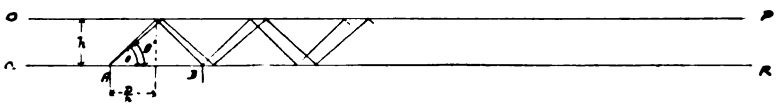

FIGURE 2

The difference in length of path for two rays going from A to B, one directly along the surface of the earth and the other reflected by the surface $O\,P$ is given by

$$2\left(\frac{D}{n}\,sec\,\theta - \frac{D}{n}\right)$$

For the whole series of n reflections, this equals $D\,(sec\,\theta - 1)$, and it is seen that this is dependent only on the value of θ and D. If one assumes a ray impinging on the reflecting layer at a slightly different angle, θ', the difference in length of path to a point distant $D+a$ from the sending station will be $(D+a)\,(sec\,\theta' - 1)$. To get the distance between interference bands, differentiate $D\,(sec\,\theta - 1)$ with respect to D, making the necessary substitution for $sec\,\theta$,

$$\frac{d\,[D\,(sec\,\theta - 1)]}{d\,(D)} = \frac{d\left(D\left\{\sqrt{1+\left(\frac{n\,h}{D}\right)^2} - 1\right\}\right)}{d\,(D)}$$

$$= \left(\sqrt{1+\left(\frac{n\,h}{D}\right)^2} - 1\right) - \left(\frac{n^2\,h^2}{D^2\sqrt{1+\left(\frac{n\,h}{D}\right)^2}}\right)$$

If a be half the width of an interference band, i. e., the distance from a dark to a light patch, the difference between the dif-

ferences in length of path for the two rays at angles θ and θ' which interfere at the points distant D and $(D+a)$ from the sending station respectively must equal $\pm\frac{\lambda}{2}$.

Hence
$$a\left(\frac{d\left[D\left(\sec\theta-1\right)\right]}{d\left(D\right)}\right)=\pm\frac{\lambda}{2}.$$

Reducing this quantity we arrive at the condition

$$\left(1-\sqrt{1+\left(\frac{n\,h}{D}\right)^2}\right)=\pm\frac{\lambda}{2\,a}\sqrt{1+\left(\frac{n\,h}{D}\right)^2}$$

But
$$\sec\theta=\sqrt{1+\left(\frac{n\,h}{D}\right)^2},$$

then
$$\sec\,\theta=\frac{1}{1\pm\frac{\lambda}{2\,a}}.$$

Putting now $\lambda=6$ km., $a=14.4$ km. (the values given in Mr. Fuller's paper), $\sec\theta=1.26$ and $\theta=37.6°$, since it is evident that the $-$ sign must be taken.

Substituting now for $\sec\,\theta$, its value found above,

$$\frac{n\,h}{D}=0.76.$$

If $D=3{,}700$ km. and $h=80$ km., D being the distance between San Francisco and Honolulu, and h the height usually assumed for the Heaviside layer, it follows that

$$n=\frac{0.76\times3{,}700}{80}=35\text{ times,}$$

or the rays reflected by the Heaviside layer must go up and down 35 times between it and the earth to give an interference band of this width. This is certainly not likely to happen without considerable loss of energy of the reflected ray, which would prevent sharpness of the interference band. Moreover, such reflections as these would give a difference in path for the two rays of

$$D\left(\sec\theta-1\right)=3{,}700\times0.26=960\text{ km.}$$

THE HEAVISIDE CLOUD THEORY

Now the very interesting curves given by Mr. Fuller in which he shows that weakening and strengthening of the signal occurs as the wave length at the sending station is altered, indicate that the difference in length of path of the two interfering rays is much less than that calculated above.

In Figure 13 of his paper he obtains the following results—

Wave lengths for which minimum values of signal strength are found	Wave lengths for which maximum values of signal strength are found
5 km.	6 km.
7 "	8 "
10 "	—

If the two interfering rays travel along their two different paths in the same direction independently of wave length, it is evident that the difference in length of path must be an odd number of half wave lengths for 5 kilometers, 7 kilometers, and 10 kilometers waves, and an even number of half wave lengths for the 6 kilometers and 8 kilometers waves.

Let δ be the difference in length of path,

then $$\delta = \frac{5}{2} m = \frac{7}{2}(m-2) = \frac{10}{2}(m-4)$$

where m is any odd number.

In the same way $\delta = \frac{6}{2}(m-1) = \frac{8}{2}(m-3)$.

This gives a series of equations from which m and δ can be found, which give fairly consistent results. The average value obtained by solving them gives $m=7$ or 9 (the values of m found are 7.5, 8 and 9) and $\delta = 17.5$ or 22.5 kilometers, a result very different from that found by the simple reflection theory.

From the other curves given in Mr. Fuller's paper, in which maximum and minimum points are shown with varying wave length, the difference in length of path of the two sets of interfering waves may be readily calculated. The results of these calculations are given on the following page. (All records are for transmission from San Francisco to Honolulu.)

There is no relation observable between the difference in length of path of the interfering rays and the state of the sky, i. e., whether it is all light or all dark, but the variation in the difference of path is evidence that the observed phenomena are due to reflections from irregularly placed surfaces and lends support to the "Heaviside Cloud" theory.

Curve, Figure, Number	Date, 1914	San Francisco Time	Difference in length of path of interfering waves.	Condition of intervening space
5	Mar. 8	3 p.m.— 3.54 p.m.	28 km.	all light
6	" 15	10.30 a.m.— 12 p.m.	21 "	" "
7	" 15	3 p.m.— 4 p.m.	35 "	" "
8	Apr. 1	3 a.m.— 4 a.m.	28 "	all dark
10	May 17	11.30 a.m.— 12 a.m.	15 "	all light
11	" 24	10.25 a.m.—11.25 a.m.	36 "	" "
12	" 24	11.25 a.m.— 12 a.m.	14 "	" "
13	June 13	2.45 a.m.— 3.30 a.m.	20 "	all dark
			average	
14	" 22	10.30 a.m.—11.15 a.m.	15 km.	all light
15	" 28	10.45 a.m.—11.30 a.m.	36 "	" "

There is, of course, a difference between the day and night records in that the variations at night, due to altering wave length, are usually greater than they are in the day time, thus showing that the interfering waves are more nearly equal in intensity at night than they are by day. This is clearly shown in Figure 13 of Mr. Fuller's paper. The morning records give large variations also, and it would appear that altho the intervening space is fully lit, night conditions of semi-transparency still supervene. One of the most interesting results mentioned at the end of Mr. Fuller's paper is that he finds that the character of the land or water between the stations appears to make very little difference in long distance transmission, i. e., that the signals over land are as strong as they are over water. This seems almost conclusive proof that the observed signals are due to rays which are almost entirely refracted and reflected. The result, of course, is at variance with observations with shorter wave lengths over smaller distances. In dealing with interference phenomena it may be assumed therefore that the two rays travel round the earth thru the narrow passage formed by the earth and the Heaviside layer, being refracted as they pass, as was explained by Dr. Eccles, and being also reflected from the lower surface of the Heaviside layer.

If one assumes a ray refracted so as to follow nearly the curvature of the earth, which strikes a reflecting surface as shown

in Figure 3, interference might take place between the waves reflected from two regions $C E$ and $D F$. If one assumes a reflecting surface of such a nature[8] that the angle of incidence of a ray on it changes considerably for slightly differing altitudes as indicated in the figure, i. e., if the reflecting surface is irregular, a series of rays may be reflected at different angles by such a cloud at such points as $C E$ (closely adjacent) in directions $C A$

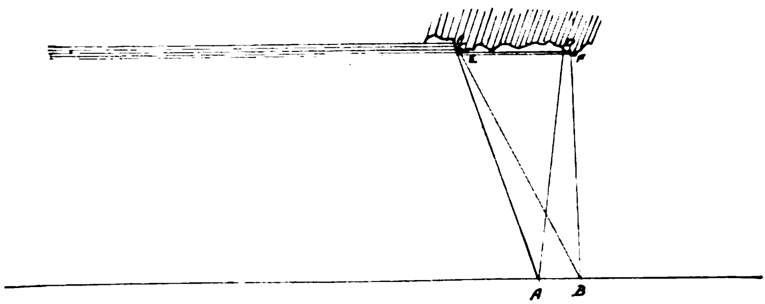

FIGURE 3

and $E B$, and it is easy to see that the difference in length of the path from C to A direct, and by the path $C D A$ may be an odd number of half wave lengths, while the difference in length of path from E to B direct, and by the path $E F B$ may be an even number of half wave lengths.[9] If this is the case, the distance between the two positions A and B at which the two waves would reinforce and neutralize each other respectively, may vary within wide limits. The diagram has been drawn to correspond, as nearly as possible, with the conditions given in Mr. Fuller's paper in which he has two stations 9 miles apart. The difference in length of the two paths to A is about 30 kilometers and to B about 26 kilometers. For a wave length of 7,500 meters, the signals at A would be strong, since the difference in path is 8 half wave lengths, at B the signals would be weak, since the difference in path is nearly 7 half wave lengths. For a wave length of about 8,500 meters the difference in path to A would be just under 7 half wave lengths so that signals

[8]It is perhaps misleading to speak of angle of incidence when the dimensions of the reflecting surface are of the same order of magnitude as the wave length, the incident radiation is really scattered.

[9]Altho the diagram has been drawn with the two interfering rays in one plane, it is clear that the rays may be reflected from other directions not coplanar.

would be weak, whereas at B the difference in path would be just over 6 half wave lengths and the signals would therefore be strong. These phenomena can hardly be described as due to the production of an interference band in the sense in which it is usually understood, when speaking of interference phenomena in light. The interference between the rays traveling along the different paths is entirely tortuitous, and the regions of electromagnetic lightness and darkness are probably scattered in a most irregular way. The explanation here given, tho only one of an infinite number of possible explanations, is consistent with all the observations made by Mr. Fuller, and it is one which corresponds very closely with what may be anticipated from our knowledge of the upper atmosphere. The interesting calculation made recently by Mr. Cohen[10] that Austin's results could be represented by a formula of the form:—

$$I_R = \frac{K}{D}(1+N\,D)\,\varepsilon^{-0.0019\frac{D}{\sqrt{\lambda}}}$$

also has a bearing on the subject, as it would seem to point to the strength of scattered waves, reflected from the lower face of the Heaviside layer. Mr. Fuller's results therefore, lend weight to the theory that the reflecting surface formed by the Heaviside layer is quite irregular, or rather they point to the existence of an irregular mass of reflecting clouds which form the lower surface of the layer. Combined with this, there must be a certain amount of refraction to enable the rays to get round the arc of nearly 30° of the earth's surface which they have to cover in going from San Francisco to Honolulu, as already explained by Eccles.[11]

Altho one can do no more than speculate on the causes of phenomena occurring in media of whose properties one can have no direct experimental knowledge, the facts seem to be adequately explained by some such conception. The Heaviside cloud theory may therefore be considered as further established. The author's own experiments which, unfortunately, have been completely interrupted by the war, pointed in the same direction, tho the distance over which he was working, London to Paris (650 km.) was comparatively short. In the account of his tests published by the Institution of Electrical Engineers[12] he emphasized the "cloud" theory first suggested by Fessenden as the most likely explanation of the phenomena observed, and Mr.

[10] Cohen. "Electrician," Volume 76, p. 743.
[11] Eccles, "Proc. Roy. Soc.", *loc. cit.*
[12] "Proc. I. E. E.", Volume 53, p. 329.

Fuller's results now confirm this explanation, tho it is necessary in addition, in the San Francisco-Honolulu tests, to assume refraction. It is to be hoped that experiments may be continued, since, besides giving a great deal of information on radio telegraphic technology, such tests may enable us to gain further knowledge of the nature of the upper atmosphere, at altitudes higher than those at which balloon observations are possible. The author wishes to express his indebtedness to Professor Wilberforce for his co-operation.

SUMMARY: Variations in received signal strength are ascribed partly to the existence of a "Heaviside cloud" layer consisting of masses of ionized gas at considerable heights. A partial bibliography of the subject is given.

The theory of interference caused by a spherical reflector is given, and results of receiving experiments due to Mr. Leonard Fuller, are studied in the light of the derived theory. It is shown that the Heaviside layer is probably quite irregular, and that refraction of the traveling waves is probably existent

DISCUSSION

Leonard F. Fuller (by letter): Operators using continuous waves have frequently observed that altho the received night signals may be stronger on the shorter waves, fading is so much more frequent on these that they are often commercially inferior to longer waves even for night work.

This may be explained by the following:

If the mean of the observations on curve Figure 13 of the American Institute of Electrical Engineers' paper mentioned by Professor Marchant are plotted, a curve of received watts is obtained following approximately the curve of energy radiated from the transmitting antenna.

This is as it should be and is the reason for the first of the practical observations of operators. The second may be explained by the following consideration of the theory of Fgure 3 of Professor Marchant's paper.

As he points out, interference bands, in the sense in which they are usually understood in light phenomena, do not exist if we consider the path of transmission parallel to the earth surface as a whole or even locally at A and B. Inasmuch as the possible number of Heaviside cloud arrangements are infinite, there are possibilities of an infinite number of points similar to A and B. Thus regions of weak or strong signals may be entirely irregular as to dimensions and spacing.

These regions are produced by the superposition of two waves of the same frequency, *out of phase*, rather than by the combination of two waves of different frequency. The relative amount of signal fading or amplification at a given point is therefore dependent upon the angular phase displacement of the two interfering waves. This displacement is dependent upon the actual mechanical dimensions of the convolutions on the under side of the Heaviside layer. Assuming the contours of this layer constantly changing, it is obvious that the longer the wave length the less the phase displacement of the interfering waves for a given change in the Heaviside layer. Thus fading may be less troublesome on the longer waves and their commercial value enhanced thereby.

SKIN-EFFECT RESISTANCE MEASUREMENTS OF CONDUCTORS

At Radio-Frequencies up to 100,000 Cycles Per Second*

By

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The research here reported was conducted in the Research Division, of the Electrical Engineering Department of the Massachusetts Institute of Technology, during 1915–16, under an appropriation from the American Telephone and Telegraph Company. The object of the research was to obtain experimental data on the alternating-current resistances of various electrical conductors at frequencies up to 100,000 ∾.

OUTLINE OF PRECEDING HIGH-FREQUENCY ALTERNATING-CURRENT RESISTANCE MEASUREMENTS

Publications of skin-effect measurements at high frequencies are very rare. The Bibliography of the Kennelly-Laws-Pierce A. I. E. E. paper of 1915, and that appended to this paper, contain references to the work of Fleming, Lindemann, Dolezalek and others, in this direction. Fleming's measurements were published in 1910, and he was able to obtain confirmation of Kelvin's formula for the skin-effect resistance ratio of certain straight wires using oscillatory currents. The measurements here reported seem to differ from those previously published, in that they have been obtained with steadily sustained alternating currents, furnished by an a. c. generator, and using a null method, with a somewhat sensitive induction bridge. The research

*Presented as the Presidential Address before The Institute of Radio Engineers, New York, May 3, 1916. Manuscript received April 4, 1916. A table of the symbols used is given at the end of the paper.

constitutes a continuation, at higher frequencies, of that reported in the A. I. E. E. 1915 paper above mentioned.

In that paper it was reported that the standard skin-effect formulas for round wires had been observed to hold, within the limits of experimental error, up to 5,000~, the highest frequency used. It was also noted that the skin-effect resistance ratio of conductors formed of 7 round bare strands, with normal spirality, was slightly greater than that of the equisectional solid wire. Straight flat-strip conductors were found to have a skin-effect resistance ratio far in excess of that predicated from existing available formulas.

The ratio Z/R of the linear a. c. internal impedance of a conductor (Z ohms ∠ per centimeter), to the linear d. c. resistance (R ohms per centimeter), is a complex numeric called the *skin-effect impedance ratio.*[1] Its real component R'/R, is called the *skin-effect resistance ratio*, and is the usual factor which expresses the magnitude of skin-effect. The imaginary component X/R, is similarly called the *skin-effect reactance ratio.* This paper deals almost exclusively with the skin-effect resistance ratio R'/R. When subdivided wires are cabled, they are spiralled for mechanical reasons. This usually increases the resistance ratio, owing to what is called the *spirality effect.* When wires carrying alternating currents are placed in such a degree of proximity as to affect appreciably the distribution of alternating-current density over their cross section, owing to their mutually interacting magnetic fields, the resistance ratio R'/R is ordinarily increased, owing to *proximity effect.* The ordinary skin-effect resistance ratio of a long straight conductor is therefore to be understood as occasioned by the distortion of alternating-current density over its cross section, due entirely to the magnetic field of that conductor. This research has been directed to the measurement of the resistance ratio in straight conductors only, in view of the fundamental character of such measurements, and the sparsity of published observations in this field.

GENERATING APPARATUS

The source of the alternating currents used in all the tests was an Alexanderson radio-frequency[2] alternator made by the General Electric Co., and kindly loaned to our Department, by the National Electric Signaling Co. and Professor R. A. Fes-

[1]The angle sign ∠ indicates a complex quantity or "plane vector."

[2]Radio frequencies are, for convenience, arbitrarily chosen as those over 10,000 cycles per second.

senden. A picture of the machine appears in Figure 1. The direct-current driving motor M, on the right-hand side, is a 4-pole 125-volt 82-ampere machine, operating, when at full speed, at 2,000 r. p. m., and driving the alternator on the left,

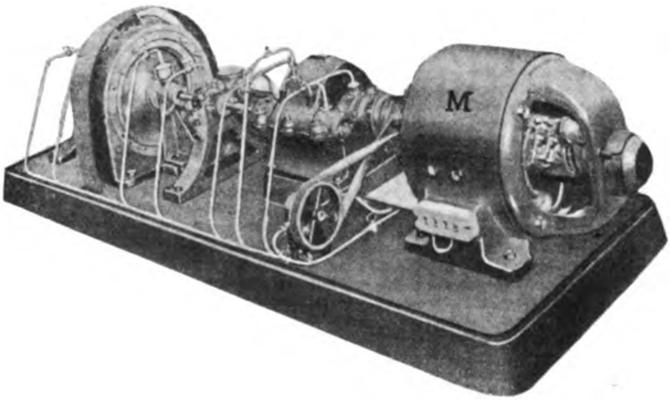


FIGURE 1—Motor-Driven High-Frequency Generator

thru a 10:1 step-up gearing, at a speed of 20,000 r. p. m. The alternator, of the inductor type, has a steel disk rotor, represented in Figure 2, with 300 radial slots. The stationary armature

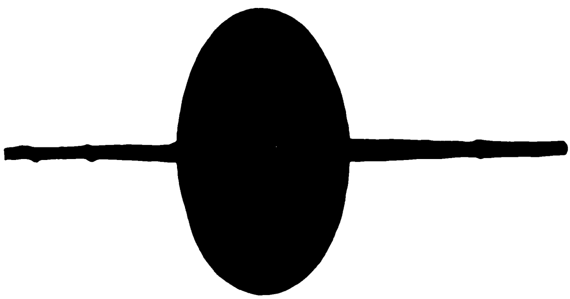


FIGURE 2—Rotor and Shaft of Standard 100,000-Cycle Alternator

windings are supported on polar projections, on each side of the rotor, so that there is one complete cycle of induced e. m. f. in the armature winding at the passage of each tooth on the rotor. The full-speed frequency is therefore $20,000 \times 300 = 6,000,000$ cycles per minute, or 100,000 cycles per second. The

alternator is so designed that the clearance between the rotor and the stator pole faces can be closely adjusted. With a small air gap, power up to 2 kilowatts, at about 15 amperes and 130 volts, can be obtained. In the tests described, the current taken from the machine rarely exceeded 5 amperes, thus enabling a less troublesome mechanical clearance adjustment to be used. The machine, which is self-oiling by forced lubrication, could be run at constant full speed for an hour or more without particular attention.

The wave form of the generated e. m. f. was not investigated, but no evidence has presented itself of any appreciable irregularities, to which moreover the additional reactance automatically developed would be enormous.

The generator set was placed in a room about 20 meters away from the testing laboratory, and controlled from the latter, thru a set of armature and field resistances. A calibrated little magneto generator, geared to the motor shaft, but not shown in the picture, actuates a calibrated d. c. voltmeter on the testing table, so as to indicate continuously both speed and frequency.

METHODS OF MEASUREMENT

The first means for measurement tried, consisted of a simple Ohm's Law circuit, illustrated in Figure 3, using a hot-wire voltmeter V, and ammeter A. A series circuit was made up, comprising the alternator G, a hot-wire ammeter A, the test conductor X, and the adjustable condenser C. This condenser was needed in measuring the resistance of the test-wire X, in order to compensate for the relatively high reactance, that even a simple grid distribution of a few meters of test conductor will offer at so high a frequency. The operation consists in causing the radio-frequency current to flow in the circuit, and so adjusting the variable condenser C, that the minimum e. m. f. is indicated by the hot-wire voltmeter V, connected as shown. Under these circumstances, the ratio E/I of the branch circuit $a\ b\ c$, will be the effective a. c. resistance of the test conductor and condenser, the latter being relatively small.

The method has the advantages of simplicity and directness of measurement. It is swift and convenient. It has the disadvantages that it is not a null method, and that the final resonance adjustment is very troublesome; also, that at least two ranges of voltmeter reading are required, in order to complete the adjustment. At resonance; i. e., when the inductive reactance of the branch circuit $a\ b\ c$, is equal and opposite to

the reactance of the condenser C, the current thru the ammeter A is relatively large; while the e. m. f. indicated on the voltmeter V is relatively small. The capacitance C is such that for low-resistance test wires, at frequencies not exceeding 100,000~, an air condenser cannot be used, and hence the apparent resistance of the test wire is increased, owing to the

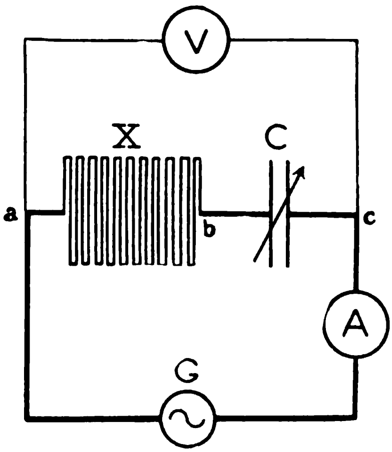


FIGURE 3—Radio-Frequency Resistance Measurement by Ohm's Law Circuit

presence of power loss in condensers with solid dielectrics. For these reasons, the method was later discarded, in favor of an inductive-bridge method; but it is nevertheless recommended as a suitable method in commencing the study of high-frequency resistances; besides serving as a convenient check upon such null methods as may later be employed.

The test wires employed had apparent radio-frequency resistances of less than half an ohm. The condenser C was adjustable up to a maximum of 26 μf. (microfarads), by steps of 0.001 μf., and a final adjustment, with the aid of a little rotary oil-condenser, continuously variable to a total of about 4 mμf. (millimicrofarads).

There were two multiple-scale Hartmann and Braun voltmeters available, covering a range from 0 to 200 volts, and a multiple-scale ammeter, covering a total range from 0 to 5 amperes without shunt. Altho calibrated with continuous currents, the design and construction of these instruments warrants the belief that their calibration is substantially correct for alternating currents over the range of frequency employed.

INDUCTIVE BRIDGE METHOD

The bridge method later employed corresponds, in essential features, to that described by A. Hund* in 1915. The diagrammatic connections are indicated in Figure 4. A is the radio-frequency alternator, and X is the conductor under test. It is balanced, as to resistance and inductance, by the adjustable resistor R, and the adjustable inductor L. The equal arms of

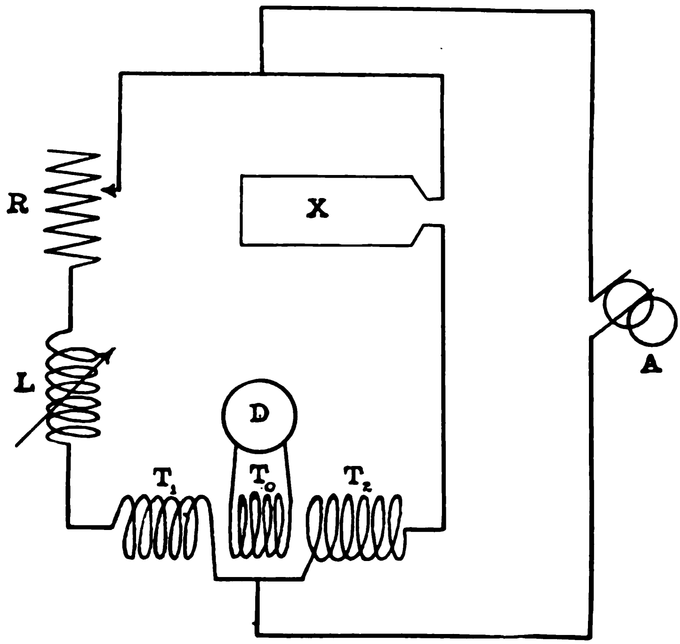


FIGURE 4—Differential Transformer Bridge System

the bridge are the two opposing primary coils T_1 and T_2 of a small special testing transformer, the secondary winding T_0 of which is connected to the detector D. When zero balance is obtained in the detector D, for any measured impressed frequency within the range of the alternator A, the values of R and L are respectively the apparent resistance and inductance of the test conductor X, corresponding to that frequency.

The details of the apparatus, as used in practice, are shown in Figures 27 and 28, the description of which is given in Appendix I, in such detail as should enable the apparatus to be duplicated if desired.

*Bibliography, Number 14.

Straight Solid Wires.—A round solid annealed copper wire number 12 A. W. G. (diameter 0.205 cm. = 0.0808 inch) was tested by both of the above methods. The results are given in Figure 5. The curve is the locus of the skin-effect resistance

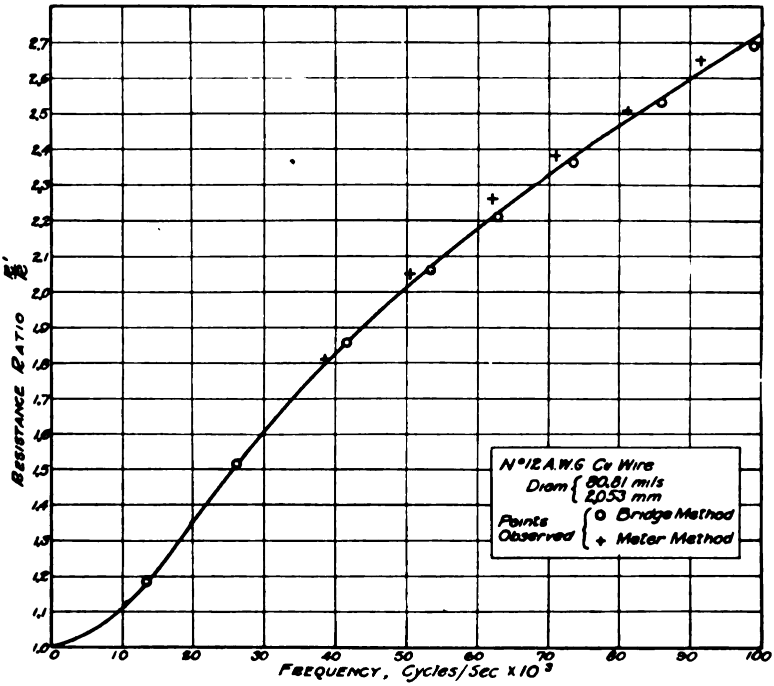


FIGURE 5—Skin Effect Ratio for Number 12 A.W.G. Copper Wire as Observed and Computed

ratio R'/R, as computed from the real part of the Bessel-function formula:

$$\frac{Z}{R} = \frac{a_o X}{2} \cdot \frac{J_o(a_o X)}{J_1(a_o X)} \qquad \text{numeric} \angle \quad (1)$$

where Z is the internal linear impedance of the wire, of which R' is the linear resistance component, X is the radius of the wire in centimeters, and $a_o = \sqrt{-j\,4\,\pi\gamma\mu\,\omega}$ (see Appendix II). The crosses represent observations by the Ohm's law method, and the circles by differential method. It will be seen that the latter observations are in substantial accordance with the computed curve. The wire was mounted over insulators, on a flat frame

illustrated in Figure 6, which was made standard in these tests. The advantage of this particular mounting is that it holds the wires in one plane at a uniform spacing (about 5 cm. or 2 inches), sufficiently great to make the proximity effect negligibly small.

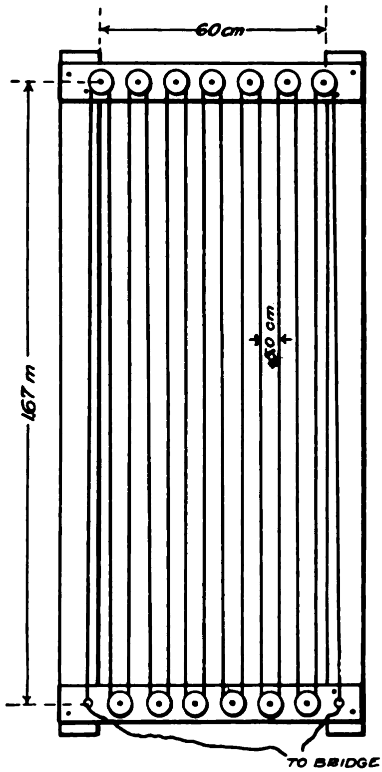


FIGURE 6—Test Wire on Frame

The external inductance of the wire, thus mounted, is also relatively small. It was ascertained that altering the spacing of the conductor on the frame; even to the extent of placing the 25 meters of wire in a single horizontal loop, 20 centimeters wide, on the wall, did not appreciably affect the skin-effect resistance ratio R'/R, to the degree of precision used in these tests. The mounting of the test wire on such a frame has the additional advantage that it brings the wire into a small compass, and minimizes its external electromagnetic disturbing fields on the measuring apparatus. Moreover, it enables a framed test wire to be kept indefinitely for reference, when desired.

A second test wire, formed of 25.3 meters of number 14 A. W. G. solid round copper wire (diameter 1.625 mm. = 0.064 inch) was tested by differential bridge at 22° C., at a later period, and with greater precision. Its linear resistance was found to be 0.00825 ohm per meter. The skin-effect resistance ratio of this wire was found to agree with the value computed from the standard formula, within the limits of observational error, the following results being obtained:

Frequency Cycles/second	Skin-effect Resistance Ratio R'/R Observed	Skin-effect Resistance Ratio R'/R Computed	Ratio $\frac{\text{Observed}}{\text{Computed}}$
96,100	2.192	2.189	1.002
82,200	2.051	2.049	1.001
59,300	1.775	1.774	1.001
38,000	1.453	1.458	0.997
20,000	1.162	1.170	0.993

The entire series of observations are presented by the circles in Figure 7, the curve showing the Bessel-function computed values, according to formula (1). It will be seen that there is a satisfactory agreement between the observations and this curve. Altho no reason can perhaps be assigned why the skin-effect resistance ratio of a round wire should depart, at greater frequencies, from the theoretical formula, already found to hold up to 5000 ∾; yet it is satisfying to find the agreement maintained. It should be noted that when attempts are made to measure with precision the skin-effect resistance ratio of such a straight round wire, the temperature of the wire must be carefully noted.*

STRANDED WIRES

Stranded Copper Wires.—After ascertaining that round solid copper wires, at radio frequencies, conformed to standard formulas, measurements were extended to subdivided or stranded copper wires with insulated strands. A test was made of stranded wire, of 20 strands, each number 26 A. W. G. (0.405 mm.) and enamel insulated to a diameter of 0.42 millimeter. The strands had a usual amount of spiralling or lay (4 cm. pitch). The unbraided diameter, i. e., the diameter of the bundle of strands,

*The result is also in conformity with that obtained by Fleming, thru another method, to frequencies up to 900,000 ∾. See Bibliography, Number 3.

under the braiding, was about 25 per cent. greater than that of an equisectional solid or unstranded wire. The skin-effect resistance ratio R'/R, for this stranded wire, between 50,000 ∾ and 100,000 ∾, was about 10 per cent. less than that computed for the equisectional solid wire.

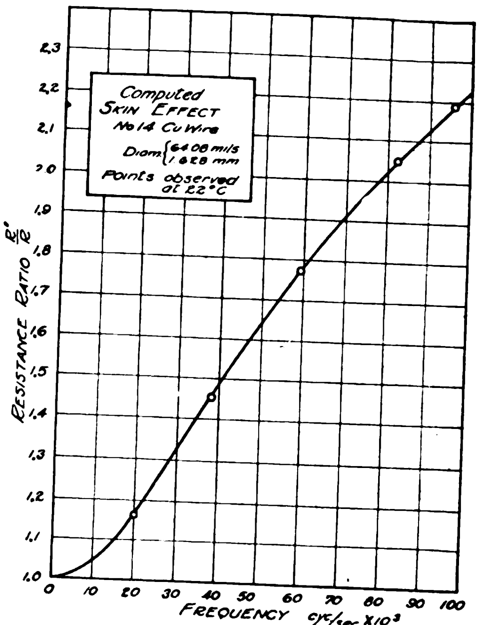


FIGURE 7—Agreement between Measured and Computed Skin-Effect Ratios for Round Wire

Another stranded wire had 50 strands of number 28 A. W. G. copper wire (0.32 mm., silk covered to 0.45 mm.) the strands being made up with a very long lay (about 1 turn per meter) and covered with braiding. The unbraided diameter was about 80 per cent. greater than that of the equisectional solid wire. This conductor had a skin-effect resistance ratio, at 100,000 ∾, about 33 per cent. less than that computed for the equisectional solid wire. Since measurements of the skin-effect resistance ratio of seven-strand copper cables, last year, showed a slight increase over the equisectional solid conductor, at frequencies up to 5,000 ∾, as stated in the above-mentioned A. I. E. E. paper of August, 1915, it seemed desirable to separate the effects of subdivision thru insulation, from those of spirality. By the courtesy of the Simplex Wire and Cable Co. of Boston, we were

able to secure a sample of 26 meters of a stranded and braided wire, containing 48 strands, each of number 30 A. W. G. copper wire, diameter 0.255 millimeter, silk insulated to about 0.33 millimeter. These strands were braided up without twist, by being carefully drawn thru a die, whipped with silk thread by hand and then laid on the supporting wooden frame for testing. By this treatment, it was hoped and believed that each individual strand would preserve its proper position substantially unchanged in the cross-section thruout the sample. The over-all unbraided diameter was about 50 per cent. greater than that of the equisectional solid wire. Another sample of stranded wire was made up at the same time, except that it had the usual amount of spirality, i. e., 1 turn in 4 centimeters about the central axis, and was machine braided.

Figure 8 shows the results of the skin-effect measurements on these two stranded conductors, and gives the resistance ratio R'/R against impressed frequency, up to 100,000 ∾. The curve A refers to the stranded untwisted sample, B to the stranded twisted sample, and C to the computed solid equisectional conductor. It will be seen that the subdivision of the wire into separate parallel untwisted strands, with this amount o silk separation, reduced the resistance ratio, at 100,000 ∾, from 2.38 to 1.86; but that the spirality effect of the lay in sample B, brought the ratio nearly half way back, or to 2.11. The B curve lies roughly midway between the A and C curves thruout. The reasons for the greater skin effect in the B sample over that of the A sample are probably twofold; namely (1), the additional copper losses in B due to the spirality effect; i. e., the longitudinal component of alternating magnetic field in the interior of the wire B, due to the action of the lay as a solenoid, and (2) the greater tightness under the braiding of the spiralled B sample, than that which could be safely given to the unspiralled A sample by hand, with a corresponding diameter enlargement ratio of only 1.4 for B, as against 1.5 for A.

If there were no insulation separating the individual parallel strands in the A sample, then it is believed that the skin-effect resistance ratio would rise into coincidence with C, or would become the same as in the equisectional solid wire. In other words, the theory and tests, to date, of skin effect in round wires, seem to show that the mere subdivision of the wire into a round bundle of parallel round filaments, which make contact with each other, but entrain channeled air spaces, has no appreciable effect on the skin-effect ratio; because the magnetic

flux maintains a cylindrical distribution. When, however, even a thin layer of insulation separates the individual strands, some magnetic flux circulates radially to the mass, around each strand, and diminishes the skin-effect resistance ratio of the whole, while raising somewhat the skin-effect reactance ratio of the whole.

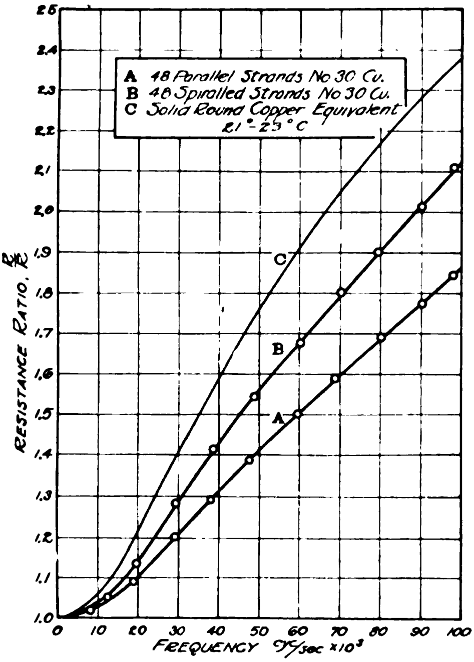


FIGURE 8—Effects of Stranding with Insulated Strands

Measurements with Definitely Spaced Strands.—In order to study, in greater detail, the effect of separating the strands of an unspiralled stranded conductor, a test loop was constructed, about 10 meters long, of a copper conductor comprising 7 strands of number 22 A. W. G. wire (diameter 0.644 mm.). These strands were supported parallel to each other in air, at the positions they would occupy in any 7-strand cable; i. e., one wire at the center, and the six others forming a hexagon about this, as indicated in Figure 9. The spacings were, however, changed in the different tests, by means of specially prepared insulating frames, strung on the wires at suitable distances. In this way, the interaxial spacing distance could be varied from

1 strand diameter, to nearly 10 strand diameters, in 6 successive steps.

The results of the skin-effect resistance-ratio measurements

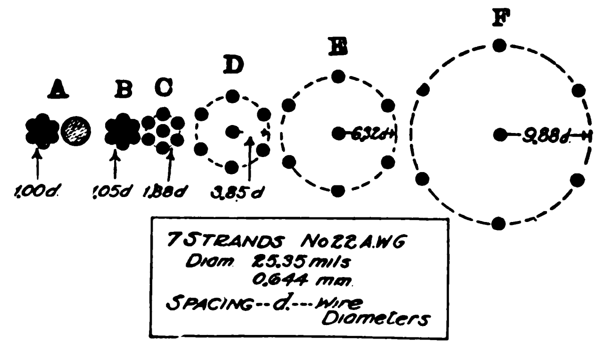


FIGURE 9—Wire Spacing

in these successive geometrical arrangements are presented in the six curves of Figure 10. Curve A is for the 7 wires bound

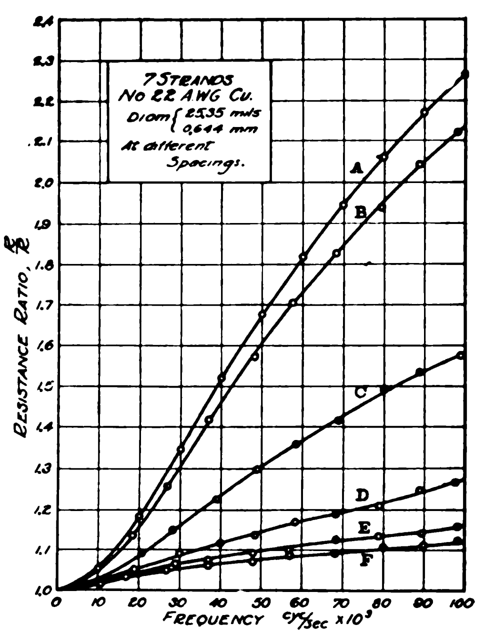


FIGURE 10—Effects of Strand Spacing on the Skin Effect of a Stranded Conductor

together in close parallel contact. It agrees with the computed resistance ratios of the equisectional solid conductor, within the limits of observational error. Curve B gives the corresponding ratios when all of the 7 wires were insulated with an enamel thickness of approximately 0.015 millimeter, thus increasing the interaxial spacing distance to 1.05 times the strand diameter. It will be seen that the ratio R'/R has thus been lowered about 10 per cent., with respect to number 1, or the equisectional solid wire. The other curves, which will be self explanatory, show how rapidly the spacing distance diminishes the skin-effect resistance ratio. At 100,000 ∾, this ratio is only 1.12, for a spacing distance of 9.88 diameters, as against 2.26 for either the solid wire, or a spacing distance of 1.00 diameter. At great separating distances, it appears that the resistance ratio of the 7-strand conductor would substantially coincide with that of any one of its components.

First-approximation formulas have been worked out (see Appendix IV) for dealing with the geometrical relations of such variable "strands," as are indicated in Figure 9. It thus far appears that these formulas represent the observed facts fairly well for the larger spacings, but give lower ratios than are measured at the smaller spacings; perhaps because there is extra skin effect in the component strands at close spacings, due to proximity effect, or current-density distortion due to the magnetic fields of neighboring strands.

The approximate proportionality maintained between the curves in Figure 10 permits of using suitable approximate reduction factors for all frequencies between 40,000 and 100,000 ∾. This relation is presented more clearly in Figures 11 and 12. The upper curves of Figure 11 here approach simple hyperbolas; so that doubling the spacing distance approximately halves the extra resistance of skin effect.

Very Finely Stranded Wire.—Two 10-meter samples of highly subdivided wire, with enamel-insulated strands, were obtained thru the courtesy of Dr. A. N. Goldsmith of New York. The first (Figure 13) contained 169 wires, arranged in 13 twisted bundles, each of 13 twisted strands, of number 40 A. W. G. diameter 0.080 millimeter, enamelled to about 0.085 millimeter. The total diameter over all wires, and under the braiding, was approximately 1.47 millimeters, representing a diameter enlargement ratio of about 1.4 with respect to the equisectional solid copper wire. It is obviously difficult to remove the enamel successfully from all of these little wires, and to solder them to-

gether into a common terminal with the certainty that none have failed to make good electrical contact therewith. Moreover, it would be very difficult to make sure that each of the individual insulated strands is intact, without discontinuity, thruout the test piece. In fact, the linear resistance of the

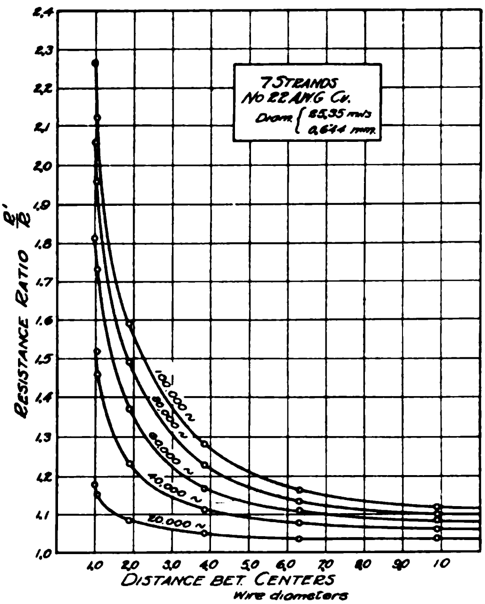


FIGURE 11—Curves Showing Drop in Skin Effect with Increased Spacing Distance between Strands of Conductor

sample was observed to be 0.0221 ohm per meter at 20° C.; whereas, without allowance for spirals, the computed linear resistance was 0.0204 ohm per meter, if all the wires were active, indicating a conductance defect from all causes of 8.4 per cent. However, treating the sample as a stranded wire of, say, 162 strands, having this observed linear resistance, the measured skin-effect resistance-ratio curve is given in Figure 13. It will be observed that, at 100,000 ∿, the ratio R'/R is 1.17, and that the extra resistance of skin effect is roughly 60 per cent. less than that computed for the round solid copper wire of equal d. c. linear resistance. The reduction may be attributed partly to the insulation separation of the strands, and partly to transposition effects; since the twisting of the component strands in each

bundle tends to interchange their relative positions in the cross-section of the whole.

The second sample was made up, in a similar manner, of 49 strands, each number 38 A. W. G. (0.1007 mm. enamelled),

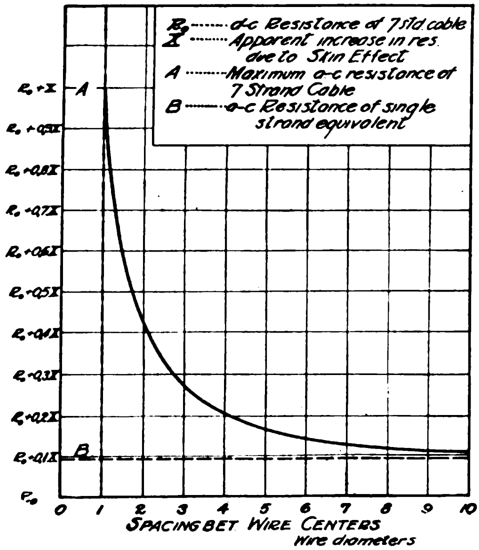
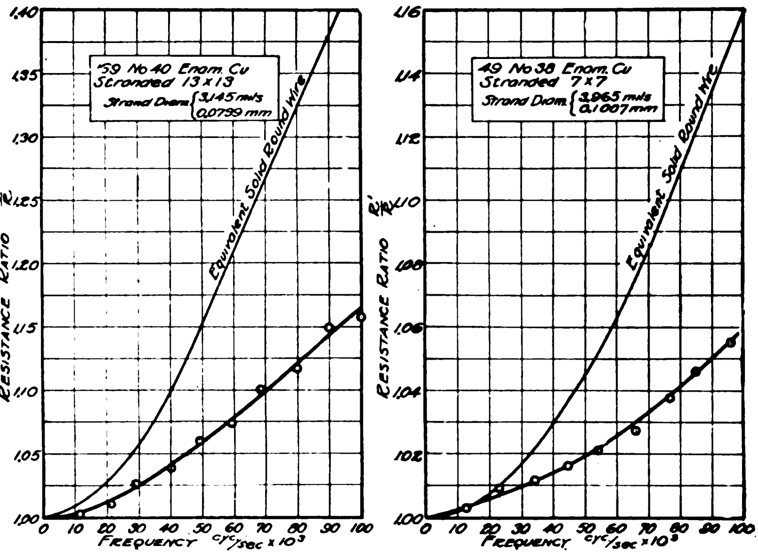


FIGURE 12—Average Decrease in Skin Effect of 7-Strand Cable with Increased Wire Spacing

arranged in 7 bundles of 7 strands each. The observed linear resistance indicated that all of the 49 strands were probably intact and connected in parallel between the soldered extremities. The total cross-section of conductor is approximately 0.39 square millimeter. The unbraided diameter was 0.83 millimeter, representing a diameter enlargement ratio of about 1.5. The skin effect of this strand at various frequencies is shown in Figure 14, and is similar in regard to equivalent round wire to that shown in Figure 13 for the other and more finely subdivided stranded wire.

Litzendraht Wire.—Thru the courtesy of Dr. Goldsmith, a sample of "Litzendraht" wire, 13 millimeters long, was secured for test. This wire is constructed by braiding enamelled copper wires, stocking fashion, in the form of a continuous long cylinder. This sample contained 48 number 38 copper strands, in 16 groups of 3 each. Each strand had a diameter of approximately 0.1

millimeter, enamelled to about 0.12 millimeter. Such a basket braiding necessarily provides a certain central air space, of diameter depending on the braiding and tension. The external diameter, under a heavy braiding of silk, was approximately 1.15 millimeter (0.045 in.), representing a diameter enlargement ratio of nearly 1.7. Such a construction involves a trans-



FIGURES 13 and 14—Skin-Effect Resistance Ratios of Finely Divided and Interwoven Wires

position of strands at regular intervals, between the inner and outer layers, also a hollow cylindrical cross-section. The results of the test of this sample are given in Figure 15. The total cross-section of conductor is approximately 0.38 square millimeter, or nearly the same as in the previous case, but the skin-effect resistance ratio is only 1.016 at 100,000 ~.

Spirality Effect in Stranded Wires.—Thru the courtesy of the Simplex Wire and Cable Co., six samples of stranded bare copper wires were obtained, and of the same particulars and construction, except in regard to the degree of spiralling or lay. It was considered desirable to ascertain, with the aid of these conductors, the influence of spiralling upon their skin-effect resistance ratio. It had already been ascertained that the resistance of a 7-strand conductor, with the strands uninsulated,

and in contact without spiralling, is the same as that of the solid equisectional round wire of the same metal and temperature. Consequently, any increase in the resistance ratio of 7-strand uninsulated and spiralled conductors could be attributed to spirality effect, assuming that the extra length of the external strands in the spiralled conductors was taken into account.

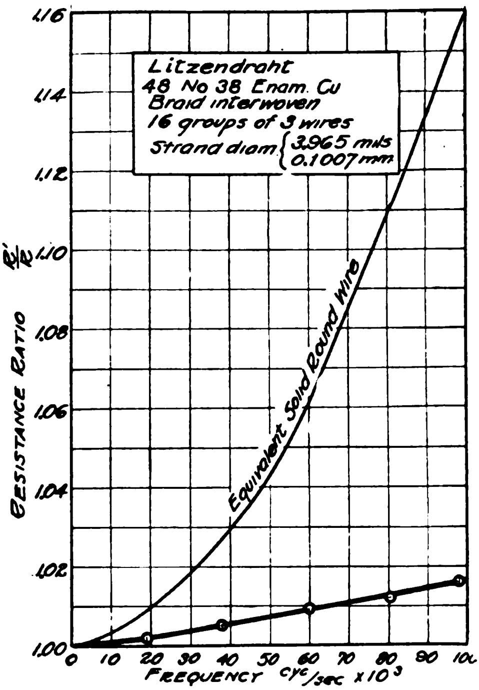


FIGURE 15—Skin-Effect Resistance Ratio for Litzendraht

Each of the six special test conductors was 25 meters long, and consisted of seven strands of bare tinned copper wires, each number 22 A. W. G., of diameter 0.668 millimeter (0.0263 in.), made up in the usual manner by a stranding machine, except in regard to the lay. The lay varied between 1 turn in 7.62 centimeters, to 1 turn in 1.52 centimeter; i. e., a spirality of 0.131 to 0.656 turns per centimeter. To assist in maintaining insulation between these bare copper strands, they were passed thru oil before entering the stranding machine in each case. They were then braided and thus held closely in contact.

As might be expected, it was found that the skin-effect resistance ratio, at any one frequency, such as 100,000 ∾, increased with the spirality. Figure 16 shows the results obtained with the conductor of greatest spirality, (0.656 turn per centimeter). At 100,000 ∾, the observed resistance ratio was 2.51; whereas the ratio computed for an unspiralled conductor of the

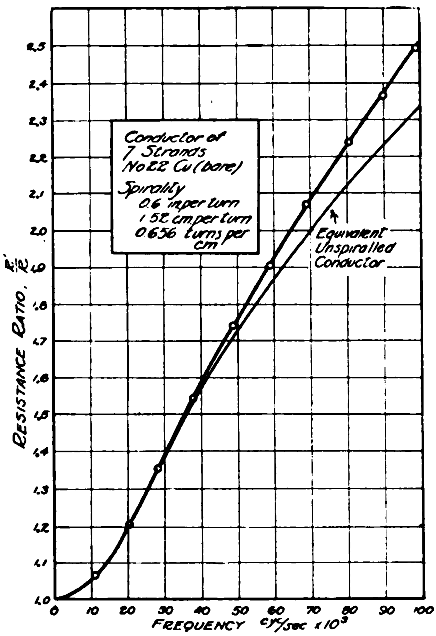


Figure 16—Increase in Resistance Ratio of 7-Strand Conductor Due to Spirality

same linear resistance was 2.34. This is the maximum deviation observed among the six samples tested. The corresponding curves for the other samples are omitted, partly because they would lie very close together, and partly because they are not strictly comparable on the same curve sheet, owing to small differences in d. c. linear resistance. In the case of the sample with least spirality, the resistance ratios were 2.41 observed, and 2.38 computed for the solid wire.

The extra linear resistance of spirality may perhaps, as already mentioned, be attributed to loss of power in the substance of the conductor accompanying the longitudinal magnetic field within the spiral. It may therefore be regarded as an extra

percentage of the d. c. linear resistance at normal temperature. On this basis, Figures 17 and 18 have been drawn. Figure 17 shows the extra percentage resistance of spirality in four different samples plotted against different frequencies; while Figure 18 shows the same percentage resistance at six different frequencies, plotted against spirality. The Figures magnify the spirality

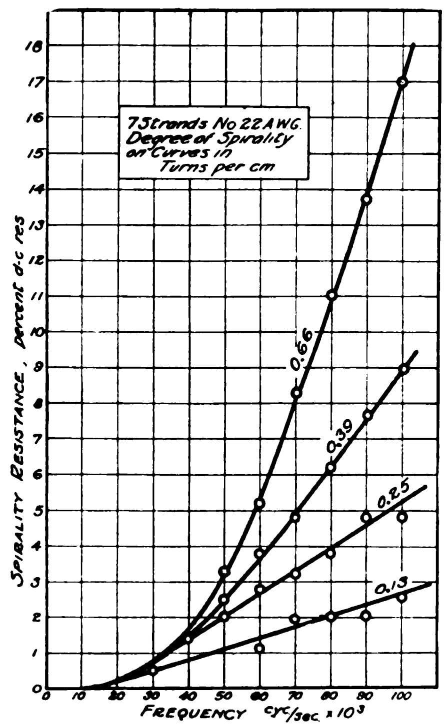


FIGURE 17—Loss Due to Strand Spiralling for Different Degrees of Spirality

effect at the expense of some apparent loss in precision. It will be seen that the spirality effect is almost negligible up to 30,000 ∾. Above 50,000 ∾, the spirality extra resistance is nearly proportional to the frequency. Moreover, the spirality resistance is roughly proportional to the spirality, especially in the neighborhood of 70,000 ∾.

Summing up the results for stranded wires, it appears that the mere subdivision of a solid round wire into seven uninsulated parallel round strands, has no appreciable effect upon the skin-

effect resistance ratio. The separation of the strands, as by their insulation, rapidly diminishes the resistance ratio, but increases the reactance ratio of the conductor. Spiralling the strands, to the extent ordinarily employed, slightly increases the resistance ratio.

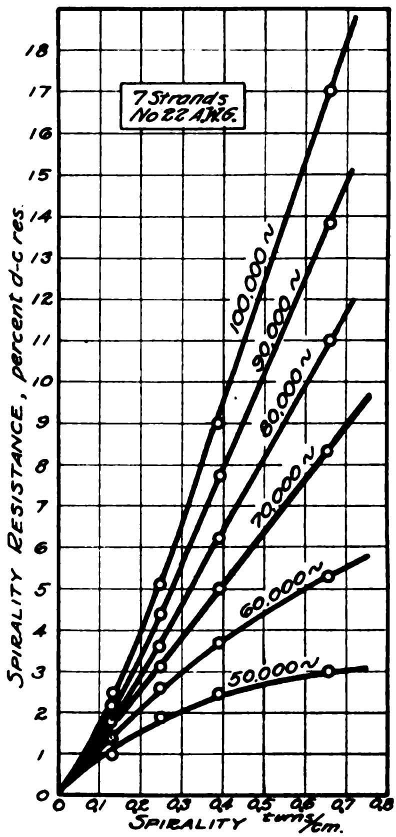


FIGURE 18—Variation in Loss with Degree of Spiralling

Copper Strips—Several samples of copper strip 0.003 inch (0.0076 cm.) were supplied by the courtesy of the Western Electric Co. These were tested in various widths from 0.356 centimeter to 3.8 centimeters (0.14 inch to 1.5 inch). The observed skin-effect resistance ratios are indicated in Figure 19, for five different widths of strip, the abscissas being impressed frequencies, and the ordinates R'/R. Each tested strip was

mounted in a horizontal loop, or loops, on the testing-room wall. The length of each tested loop varied from about 3 meters, for the narrowest strip, to 25 meters for the widest strip. The spacing between the sides of the loop or loops, varied from 10 to 20 centimeters, and the proximity effect was found to be negligible under these conditions. It will be observed that at

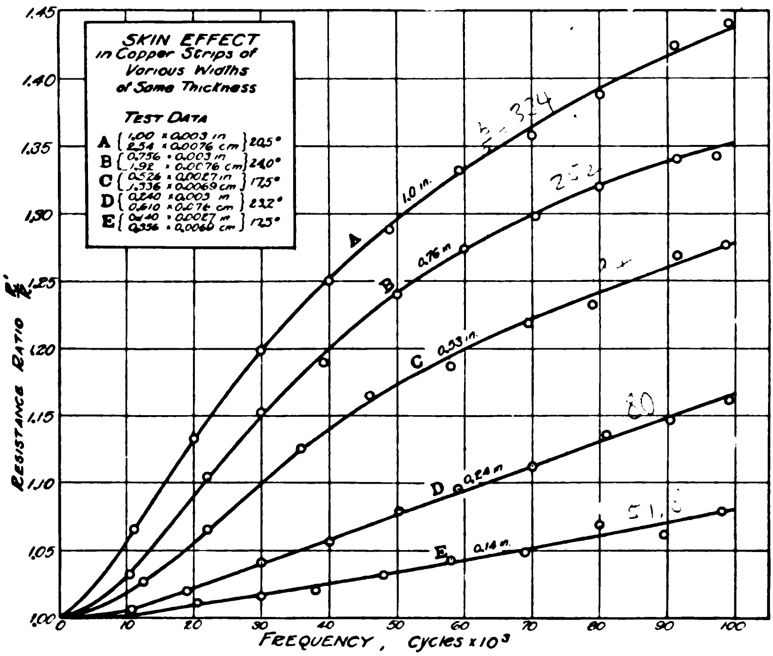


FIGURE 19—Skin Effect for Strips of Different Width but the same Thickness

100,000 ~, the resistance ratio varies from 1.08 for the 0.356-centimeter strip width, to 1.44 for the 3.8-centimeter width. Owing to slight differences of thickness and of test temperatures the curves are not precisely comparable; but they resemble the curves of resistance ratio for wires of increasing diameter, and the extra resistance is, to a first approximation, proportional to the width of the strip. This relation is presented more clearly in the series of curves of Figure 20, which are taken from the last Figure, for selected frequencies. The abscissas are strip widths, in inches and cm., and the ordinates are resistance ratios. The curves are seen to be, within the limits of frequency presented, rough approximations to straight lines, particularly in the neigh-

borhood of 30,000 ∾. It is evident that for 0.76-millimeter strip, the effect of increasing width is to increase the resistance ratio in somewhat the same manner as increasing diameter in a round wire, so that a 4-centimeter strip at 100,000 ∾, is by no means twice as good a conductor as a 2-centimeter strip. A strip at radio frequencies is, however, always a better conductor than

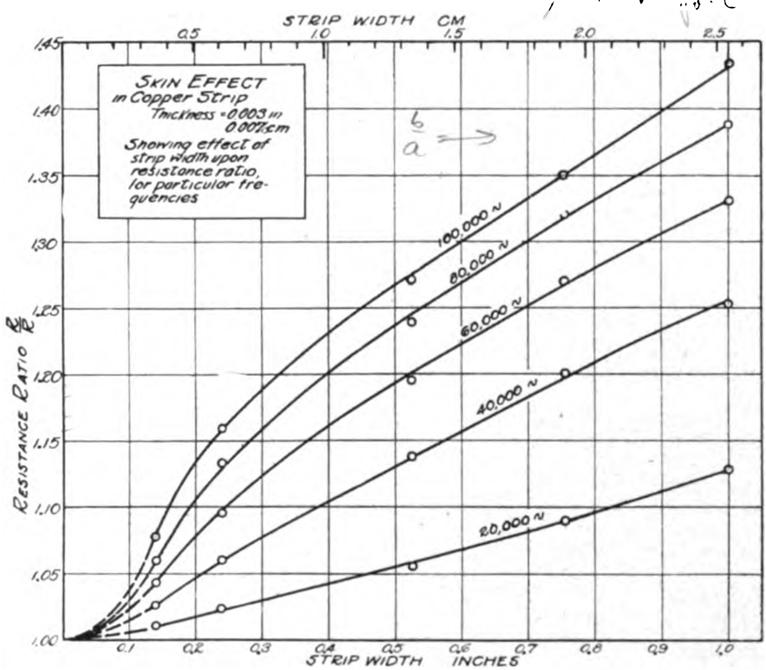


Figure 20—Skin Effect Variations with Strip Width, for Particular Frequencies

an equisectional round wire of the same material. Thus, Figure 21 shows the resistance ratio of the 3.8-centimeter strip, by comparison with that of an equisectional round wire 0.192 centimeter in diameter. Up to the frequency of 16,000 ∾, the round wire is the better conductor; but for all higher frequencies, the strip shows distinctly increasing advantages.

Similar tests were made upon samples of strip 0.16 millimeter thick (0.0063 in.) and of widths from 0.64 centimeter to 3.81 centimeters. The comparison between these strips and the solid round copper wire, equisectional to the widest, is shown in Figure 22. It will be observed that the curves are of the same general

appearance as those for the 0.76 millimeter strips, yet they rise somewhat higher, owing to greater skin-effect in thickness.

Formulas for Skin-Effect in Strips—It has been already pointed out* that the Rayleigh skin-effect formula for indefinitely wide strip is quite inapplicable to single strips of ordinary widths. For particular ranges of dimensions and frequency,

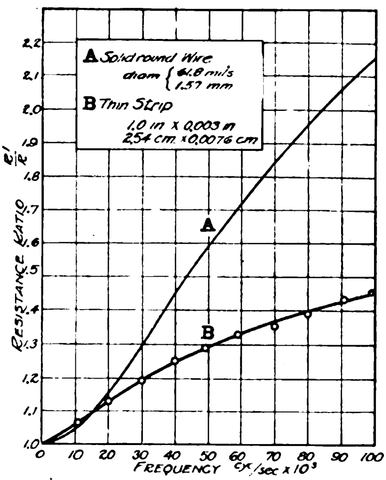


FIGURE 21—Comparative Skin Effects of Thin Copper Strip and Equisectional Round Copper Wire

empirical formulas have recently been suggested.** It is evident experimentally that a long flat strip, carrying an alternating current, sets up a more or less cylindrical magnetic flux distribution about the center, which flux intersects the strip itself. This must give rise to eddy-current e. m. fs. in the strip. If these e. m. fs. produce local eddy-currents in the strip, they would tend to increase the power loss and the effective a. c. resistance. If, however, they produce along the entire length of the strip, an organized e. m. f. distribution, the effect will be to crowd the current toward the edges of the strip, and away from the center; thus producing what may be called an *edge-effect,* perpendicular to that plane.

In order to ascertain which eddy-current action predominated, the local eddy-current effect or the edge-effect, a sample of copper

* Bibliography, Kennelly, Laws & Pierce, number 16.
** Bibliography, Dwight, number 17.

strip 2.22 centimeters wide (7/8 inch) and 13 meters long, was slit longitudinally near the edges with a sharp knife, in the manner indicated in Figure 23. These slits were from 6 to 8 centimeters

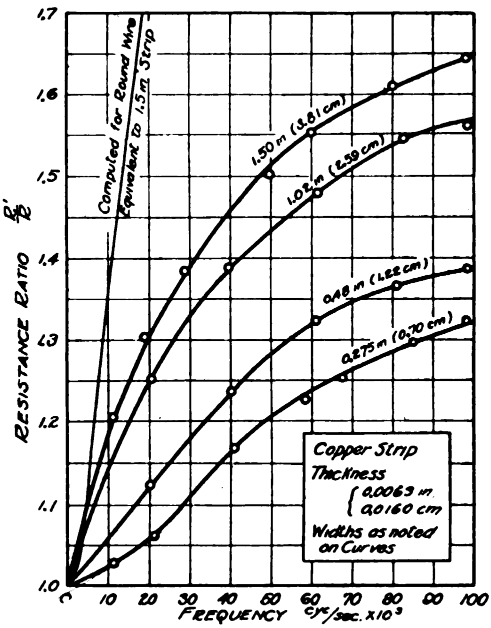


FIGURE 22—Skin-Effect for Strips of Different Width but the Same Thickness.

long, and then ceased for a distance of about 0.5 centimeter, the gaps in the slits overlapping one another; so as to maintain the structural rigidity of the conductor, while at the same time greatly hampering the circulation of eddy-currents across the strip. A strip of similar dimensions was also mounted without any slits. Up to 100,000 ∾, no appreciable difference could be detected in the skin-effect resistance ratios of the slitted and unslitted strips. The inference may, therefore, be drawn that the large increase in skin-effect resistance ratio, observed in copper strips, with reference to what should be found in infinitely wide strip, is attributable, at least in large measure, to edge-effect.

We may, therefore, consider, for purposes of discussion, that any long straight conductor of rectangular cross-section, such as a bar, prism, strap or strip, has a skin-effect in two directions,

namely across its thickness and along its width. The former is commonly regarded as a skin-effect, and the latter may be called, in contradistinction thereto, an edge-effect; but actually, the two effects are essentially manifestations of one and the same phenomenon; that is, a tendency of alternating current to leave the center, and to crowd into outlying positions of the cross-

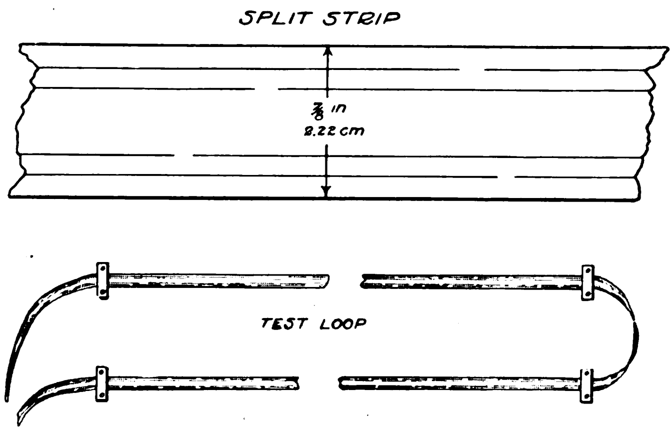


FIGURE 23—Strip Split to Reduce Eddy Currents

section. This distortion of current density may be regarded in either of two ways; namely (1) as due to imperfect penetration of the current from the outlying positions towards the center, (2) as due to the opposing mutual inductive influence, (see Appendix IV) of parallel filaments, whereby the retarding and diminishing effects of mutual inductance are greatest at the center, and least at most remote points. According to either of these ways of viewing the matter, it must appear that the tubular form of conductor can be made to offer the minimum skin-effect at any given frequency, by reducing the wall thickness to a minimum. In the case of a rectangular section, however, the advantage of thinness may be overbalanced by the disadvantage of width, the edge-effect increasing as the skin-effect is reduced. There will thus be a best form of cross-section for each particular cross-sectional area, and frequency.

Parallel Opposed Strips in Close Proximity—In view of the interesting fact that in the case of a pair of parallel going and returning flat strip conductors, in close juxtaposition, one above the other, and separated only by a strip of thin insulator, the

skin-effect resistance ratio at frequencies below 5,000 ∾ is much lower than when the strips are widely separated, it seemed important to check this result at frequencies up to 100,000 ∾. It has been pointed out by Mr. H. B. Dwight, that the data for such surface-juxtaposed parallel strips, published in the A. I. E. E. paper of 1915,* follow a certain formula which he developed, and which, as pointed out in Appendix IV, coincides with the formula for infinite strips given in that paper, when X is taken as the full thickness, instead of the half thickness of the strip, as is necessary for infinite widths:

$$\frac{Z}{R} = \frac{a X}{\tanh (a X)} \qquad \text{numeric} \angle \qquad (2)$$

This case constitutes the first reliable skin-effect formula for flat strips of finite width known to the writers.† Available formulas for finite strips seem to be all based on Rayleigh's infinite-strip formulas, and are therefore open to very large correction for edge-effect.

The test was made with a loop 6.1 meters long, of two flat strips, each 2.54×0.0076 centimeters (1.0×0.003 inch) laid on a flat board, the lower strip being entirely enveloped in tissue paper, 0.0038 centimeter (0.0015 inch) thick, and the upper strip laid carefully over this, so as to keep the edges parallel. Little bindings of paper were then applied to the two insulated strips, after which weights were applied uniformly to the system. Great care had to be taken to secure good alignment of the edges in the opposing strips, since it was found, in preliminary trials, that a relatively small amount of overprojecting at the edges, added materially to the losses and apparent resistance of the loop.

The d. c. resistance of the loop at 20° C. was 0.102 ohm, and the a. c. resistance at 93,000 ∾, was the same, within the limits of observational error; so that the skin-effect resistance ratio up to this frequency was sensibly 1.00. In such a disposition of conductors, virtually forming a paper condenser, the susceptance of the dielectric may have an appreciable effect on the measurement. In this case, however, the loop was so short that the error due to capacity susceptance was negligible.

Copper tubes are of especial interest in relation to skin-effect; since, as already mentioned, their resistance ratio should be capable of being made lower than that of any other form of

* Bibliography, number 16.

† A somewhat similar case is, however, discussed by Lord Rayleigh, Bibliography, number 1.

conductor. A tube of large diameter and very thin wall should presumably have a skin-effect resistance-ratio calculable on the basis of infinitely wide strip with X, see formula (2), taken as equal to the wall thickness.

Two copper tubes were tested. One was a slit tube of 2.22 centimeter (⅞ inch) perimeter, made by rolling a length of the thin flat strip previously tested (2.22 cm. wide × 0.076 mm. thick) over a cylindrical wooden form. This produced a slit copper tube 12 meters long, and of an external diameter approximately 7.2 millimeters. The calculated skin-effect resistance ratio of 0.076 millimeter copper strip of indefinitely great width is, by (2) 1.005 at 100,000 ∾. When the tube conductor made of this strip was tested at 100,000 ∾, its skin-effect resistance ratio was found to agree with this value, within the limits of observational error. The same strip, when tested flat, before being rolled into tube, was found to give 1.38 at 100,000 ∾. This experiment gave a striking demonstration of the influence of edge-effect in one and the same single loop of copper conductor. With the edge-effect present in the flat strip, the a. c. resistance at 100,000 ∾ was nearly 40 per cent. greater than the d. c. resistance. With the edge-effect removed, by bending the same strip into a simple slit tube, the a. c. resistance, at the same frequency, was hardly greater than the d. c. resistance.

The second tube was a soft drawn continuous copper tube of external diameter 3.18 millimeters and of wall thickness 0.65 millimeter (number 22 A. W. G.). Its length was approximately 25 meters, in two horizontal loops, or four horizontal passages, along the wall, with an interaxial distance of about 20 centimeters between adjacent conductors.

Figure 24 shows the observed skin-effect resistance ratios of this tube at various frequencies up to 100,000 ∾. It will be observed, that at 100,000 ∾, this ratio is 2.46. The round wire of equivalent cross-section, shown in dotted lines, would have, at the same frequency, a ratio of 3.17. The curve C shows the computed ratios for infinitely wide strip, of the same thickness (0.65 mm.), that is, a tube of infinite diameter and this wall thickness; while curve D shows the corresponding ratios for a tube of zero internal diameter; that is, a solid wire of radius 0.65 millimeter. It will be seen that the curve OA of observations lies nearly midway between the curve of infinite internal diameter, and of zero internal diameter, having the same wall thickness, on the assumption that the magnetic flux density vanishes within a uniform straight hollow cylinder carrying a

current. The curve OA corresponds very closely to formula (2) when X is taken as 82 per cent. of the actual wall thickness.

Comparisons of Skin-Effect Resistance Ratios for Different Types of Conductor—When an alternating current of radio-frequency has to be carried with the least loss for a given amount of copper, it is evident that the skin-effect resistance ratio must be made as near unity as possible. It is therefore important to consider what form a cross-section of given number of square

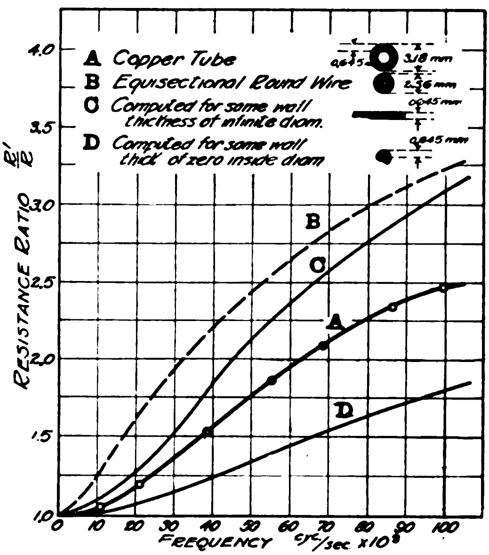


FIGURE 24—Skin-Effect in Copper Tube

millimeters of copper should have, in order to make the conductor most efficient. The best form of the conductor will depend to some extent upon the frequency.

For the particular frequency of 100,000$\sim$, Figure 25 shows the skin-effect resistance ratios of various kinds of geometrical section. The ordinates are in R'/R. The abscissas are in square millimeters, and also in square mils (10^{-6} sq. inch) of copper cross-section. It will be seen that round solid wire has the greatest resistance ratio thruout almost the entire range of cross-section shown, altho spiralled 7-strand conductor, devoid of internal insulation, would have a resistance ratio in excess of this by an amount depending on the degree of spirality. All other forms of conductor, for a given cross-section, have less resistance ratio.

The few tests thus far made of stranded conductors are plotted on the diagram. The diminution in resistance ratio depends upon the number of strands, their internal transposition and separation. Below most of these come three-mil (0.076 mm.) copper strip, which is not so low as six-mil (0.15 mm.) strip. As already explained, the edge-effect of 3-mil strip, which

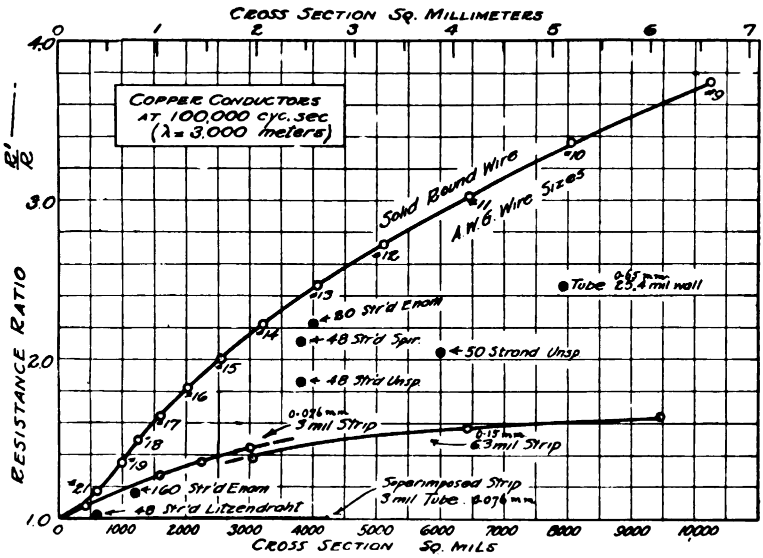


Figure 25—Comparative Skin-Effects for Different Types of Conductors

must be twice as wide as 6-mil strip for the same cross-section, predominates over skin-effect at 100,000~, so that there is a disadvantage, within the limits of cross-section shown, in rolling 0.15-millimeter strip into 0.075-millimeter strip for 100,000~. The limit of thickness to which flat strip should be rolled, for a given cross-section, is evidently a function of the frequency, which determines the equivalent penetration skin-depth δ. The tests have not yet been carried sufficiently far to assign the economical thickness limit; except that, at 100,000~, it is above 3 mils (0.076 mm.) for cross-sections less than 6 square millimeters.

The only apparent means of avoiding the edge-effect, so as to permit of using very thin sheets of conductor, is either to use two opposing parallel strips in close proximity, as described in

Appendix III; or to roll a single strip up into circular tube form. In either case, as Figure 25 shows, the resistance skin-effect ratio of 0.075-millimeter strip fell to almost 1.0. The use of opposing juxtaposed parallel strips, as going and returning conductors, has limitations, owing to the large linear capacitance involved. No matter how high the frequency, there seems to be no electrical limit to the reduction of extra skin-effect resistance, by using tubular conductors of sufficiently thin wall. Mechanical considerations suggest that this might best be accomplished by using a flexible insulating core, with fine copper wires braided over the surface. In such a flexible conductor, by taking thin enough wire, and avoiding spirality effects, the skin-effect should be reduced indefinitely, since there could be no edge-effect.

The data contained in Figure 25 are presented in a somewhat different form in Figure 26, which shows the total linear resistance of copper conductors of different types of cross-section at 100,000~ and 20° C, for different cross-sectional areas as abscissas. By reference to this diagram, the most suitable conductor can be chosen for this particular frequency, so far as the experimental data, thus far obtained, will permit.

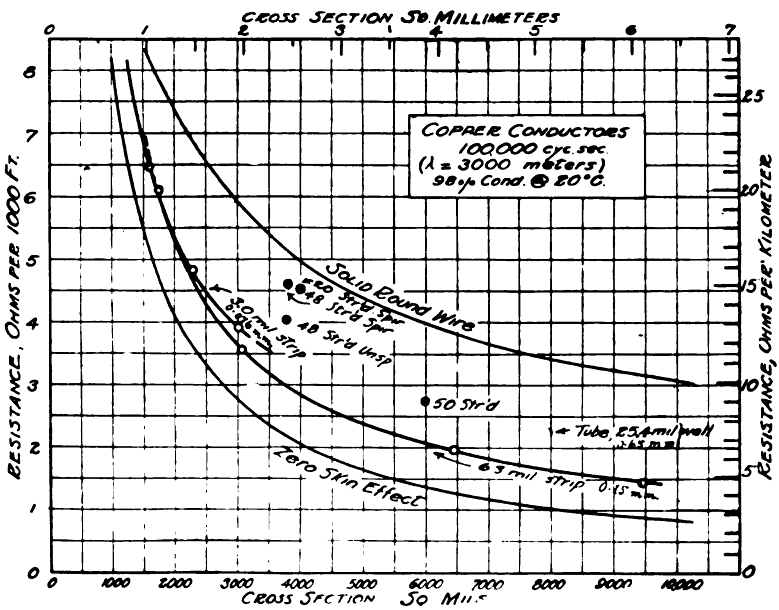


FIGURE 26—Comparative Linear Resistances of Conductors of Different Types at 100,000 Cycles

SUMMARY

The following conclusions have been reached from tests at frequencies up to 100,000$\sim$, on straight conductors, shielded from proximity effects, and of cross-sections up to 6 square millimeters.

(1) The skin-effect resistance ratio of round copper wires has been found to conform to the standard Heaviside-Kelvin Bessel-function formulas, within the limits of experimental error.

(2) The skin-effect resistance-ratio of a copper conductor formed of seven equal and parallel round bare strands, (six surrounding a central seventh), was found to be the same as that of the equisectional solid round wire up to 100,000$\sim$, within the limits of observational error. It is therefore inferred that the subdivision of a round wire into a round cable of uninsulated contacting strands does not alter the skin-effect.

(3) The effect of simple spiralling of strands in the same direction, increased the resistance ratio.

(4) The resistance ratio of a subdivided wire, with parallel insulated strands, fell below that of the equisectional solid wire, and diminished rapidly with the spacing, or degree of strand separation.

(5) The braiding of strands, so as to effect their transposition in the cross-section at frequent intervals, was found to diminish the skin-effect.

(6) The skin-effect in copper strips was found to be usually, but not invariably, less than that of equisectional round wires.

(7) Increasing the width of a copper strip within the limits reported, was found to increase the skin-effect resistance ratio; owing to what may be called "edge-effect."

(8) A pair of parallel going and returning flat strips, separated by a thin insulating layer, was found to have a skin-effect depending only on the strip thickness; that is, without perceptible edge-effect.

(9) Rolling a flat strip into the form of a slit tube destroyed the edge-effect, and reduced the resistance ratio to a minimum, dependent only on the strip thickness.

(10) Cutting longitudinal slits with a sharp knife in a thin flat copper strip, was found not appreciably to affect its skin-effect resistance ratio.

(11) In order to employ a stranded wire of minimum skin-effect at radio-frequency, it seems desirable to employ thick insulation on the strands, such as double cotton, to transpose

the strands by braiding, and to avoid spirality in one and the same direction.

(12) In order to employ a flat strip conductor most effectively, it is necessary to stop rolling it out laterally when the increasing edge-effect more than offsets the reduction in skin-effect.

(13) The hollow tubular form seems to be the most efficient type of radio-frequency metallic conductor, since, with proper mechanical internal support, the skin-effect can be indefinitely diminished by diminishing the wall thickness, and the edge-effect is absent.

APPENDIX I

DETAILS OF APPARATUS AND PROCEDURE FOR MEASURING SKIN-EFFECT RESISTANCE RATIOS, AT FREQUENCIES BETWEEN 10,000 ∾ AND 100,000 ∾

The detailed electrical connections of the induction bridge referred to above, under the title "Inductive Bridge Method," are presented in Figure 27. The alternator *A* supplies a current of from 4 to 5 amperes r. m. s. thru the hot-wire ammeter *M* to the bridge, with the aid of the adjustable condenser C_3 of

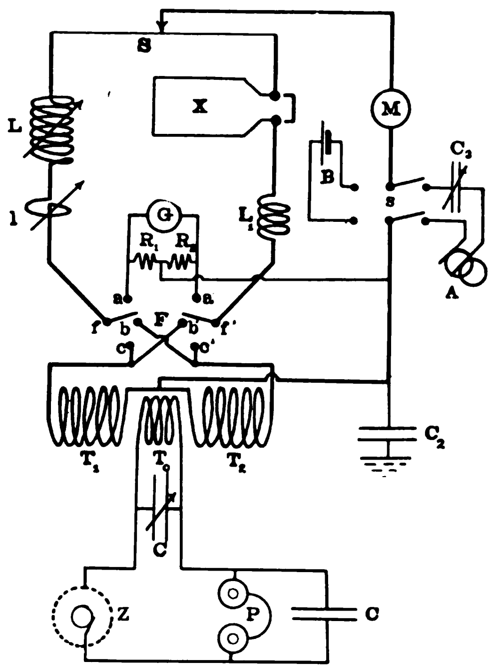


FIGURE 27—Detailed Diagram of Differential Bridge Circuit as Employed in Tests.

about 1 μf. (microfarad), which partially compensates the inductive reactance of the bridge circuit. The adjustable bridge resistance is a single slide-wire meter of Ia-Ia high-resistivity wire, number 18 A. W. G. (diameter 1.02 mm.) offering 0.0058 ohm per linear centimeter, or 0.58 ohm in all. The resistance measurements are all taken in terms of lengths of this slide wire, which possesses negligible skin-effect.

The test conductor *X* is connected between two stout ter-

minals containing mercury, that can be shorted by a heavy copper link. A bridge balance is obtained by adjusting L and S, first with X shorted, and then with X unshorted. The difference between the readings of S in these two tests, measures the resistance of X at the impressed frequency.

The switch s changes from the test dry battery B, of about 2 volts, for d. c. measurements, to the alternator A, for a. c. measurements. The switches ff' are thrown to aa', for d. c. measurements, and to bb' for a. c. measurements. The contacts cc' provide for the reversal of bb', that is, for the exchange of the two similar and equal inductive arms T_1 and T_2. Each of these is wound in two layers of 8 turns, or 16 turns all told, in a groove 1.5 centimeter wide cut in a wooden cylinder, the mean winding diameter being 15 centimeters. The wire in T_1 and T_2 consists of 9 strands each of 9 enamel-insulated number 30 A. W. G. (0.25 mm.) wires, or 81 insulated number 30 wires in all. The mid-point between these two coils T_1 T_2 is connected to ground thru an adjustable condenser C_2 of from 0.5 to 4 μ f., which helps to minimize electrostatic disturbances in the detector P. The secondary winding T_0 consists of 180 turns of plain double cotton-covered number 18 A. W. G. copper wire (1.02 mm. diam.) equally distributed in two grooves in the wooden cylinder, one on each side of the primary winding.

The fixed inductance L_1 has 20 turns in 2 layers of 90 insulated strands number 30 A. W. G. wire. The mean winding diameter is 8 centimeters and winding breadth 3.8 centimeters. The object of this inductor is to assist the test wire X to make up an inductance within the range of the inductance variometer L. This variometer has an outer fixed coil and an inner frictionally movable coil, the angle between the axes of the two being adjustable by hand. Each of the two coils has 12 turns in 2 layers. The mean diameter of the pair is 15 centimeters. The wire has 90 enamelled strands of number 30 A. W. G., and the total maximum inductance is about 0.1 mh. After an approximate inductance balance has been reached in L, a yet finer adjustment to balance can be made with the single turn l, suspended beneath the testing table, and capable of being altered in its area, and therefore in its self-inductance, by a winding spindle controlled from a distance, so that the observer's hand does not have to reach over the table when making the final adjustment.

The tuning adjustable condenser C has about 4 m. μ f. (millimicrofarads) and C_1, the condenser shunting the detector tele-

phones, about 3 m. μ f. C is adjusted for best hearing with the impressed frequency. The frequency being ordinarily above the limit of audibility, the commutator interruptor Z, motor-driven at approximately 1,000 makes and breaks per second, produces an audible effect in the head telephones P, which have each a d. c. resistance of approximately 1,000 ohms.

For the d. c. measurements, the equal-resistance arms R_1 R_2 have each 0.25 ohm, and the portable galvanometer G has 250 ohms, with a sensitivity of 1 scale division for 1 microampere.

To make a test of a conductor X, it is connected to the shorted binding posts. The switch s is thrown to the left, f to a and f' to a'. A d. c. Wheatstone-bridge balance is then obtained on G, by adjusting the position of the slider S. The short is then removed from X, and the balance repeated, at a new position of S. The change in the slide-wire reading measures R, the d. c. resistance of X. Arrangements, not shown in Figure 27, also permit of reversing the arms R_1 and R_2, so as to reveal any inequality they may possess.

The switch s is then thrown to the right, the alternator A having first been brought to the desired speed and frequency. Switch f is thrown to b and f' to b'. The alternating current in the bridge circuit is observed at M, and adjusted to between 4 and 5 amperes r. m. s., with the aid of the condenser C_3. The conductor X being shorted, a balance for both resistance and inductance is obtained telephonically at P, by successive adjustments of S and L. It is important that the observer should take up a stationary position with respect to the apparatus, and that the final adjustments should be made at S and l, without changing his position. The short is now removed from the conductor X, and a new balance obtained at S and $L + l$. The change in the reading of S measures R', the a. c. resistance of X, including the skin-effect. Reversals of f and f' enable average values to be secured.

With careful manipulation, the skin-effect resistance ratio in specimens of wire X, whose total d. c. resistance lies preferably between 0.2 and 0.4 ohm, may be measured to within a deviation probable error, for a single observation, of about $\frac{1}{3}$ of 1 per cent.

A photograph of the testing table is shown in Figure 28, with the elements marked to correspond with Figure 27.

It has been found advisable to select a pair of head telephones with cases of insulating material instead of metal, since the capacitance disturbance thru the body of the observer on the bridge balance is thereby greatly reduced.

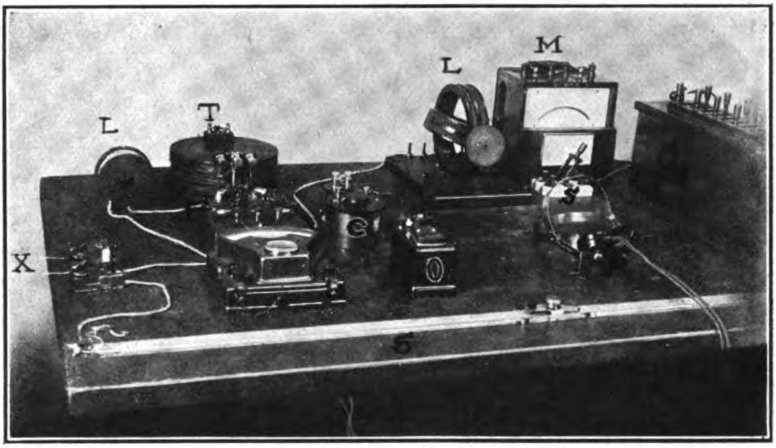


FIGURE 28—Photograph of Testing-Table Apparatus

APPENDIX II

COMPUTATION OF SKIN-EFFECT RESISTANCE RATIO IN ROUND WIRES

The theory of skin-effect in round wires has been developed and published by a number of investigators, commencing with the work of Clerk Maxwell in 1873. A bibliography of the subject appears in the paper by Kennelly, Laws and Pierce in the "Transactions of the American Institute of Electrical Engineers," September, 1915. The formula, as developed in that paper, following Jahnke and Emde is

$$\frac{Z}{R} = \frac{a_o X}{2} \cdot \frac{J_o(a_o X)}{J_1(a_o X)} \qquad \text{numeric} \angle \qquad (3)$$

where Z is the linear impedance of the wire at the impressed single frequency $f\sim$, R is its linear resistance to steady currents. X is the radius of the wire in centimeters, $J_o(a_o X)$ and $J_1(a_o X)$ are respectively Bessel functions of zero and first order, a_o is the semi-imaginary quantity

$$a_o = \sqrt{-j\,4\pi\gamma\mu\omega} = \sqrt{2\pi\gamma\mu\omega} - j\sqrt{2\pi\gamma\mu\omega} = a_2 - j\,a_2 \qquad \text{cm.}^{-1}\angle \qquad (4)$$

where $j = \sqrt{-1}$

$\pi = 3.14159\ \ldots$

γ = conductivity of wire at actual temperature $\left(\frac{\text{abmhos}}{\text{cm.}}\right)$

μ = permeability of wire at actual temperature $\left(\frac{\text{gausses}}{\text{gilberts per cm.}}\right)$

$\omega = 2\pi f$ the impressed angular velocity $\left(\frac{\text{radians}}{\text{sec.}}\right)$

This formula gives a vector skin-effect impedance ratio

$$\frac{Z}{R} = M \angle \beta^\circ \qquad \text{numeric} \angle \qquad (5)$$

the real component of which is the skin-effect resistance-ratio

$$\frac{R'}{R} = M \cos \beta \qquad \text{numeric} \qquad (6)$$

or

$$\frac{R'}{R} = \left| \frac{a_o X}{2} \cdot \frac{J_o (a_o X)}{J_1 (a_o X)} \right| \cdot \cos \beta \qquad \text{numeric} \qquad (7)$$

In the above mentioned paper, the values of a_o (at $\rho = 1724$ absohm-cm.), $J_o (Z \angle = 45^\circ)$ and $J_1 (Z \angle = 45^\circ)$, were tabulated and graphed over a certain range. Altho those tables assist computation, yet since for ordinary purposes the resistance ratio $\frac{R'}{R}$ only is required, a simpler basis of computation is obtained by tabulating and plotting the values of (3) assuming a certain representative value of γ.

Figure 29 gives the graph of $\frac{R'}{R}$ with reference to $\sqrt{\frac{f}{R'_o}}$, and also with reference to $|a_o X|$, as abscissas. It will be noticed that for values of $|a_o X|$ greater than 4.0, the curve approximates to the straight line

$$\frac{R'}{R} = \frac{|a_o X|}{2\sqrt{2}} + \frac{1}{4} = \frac{a_2 X}{2} + \frac{1}{4}.$$

Russell's formula* for the ratio $\frac{R'}{R}$ in a round wire, and using the notation of this paper, can be put in the form

$$\frac{R'}{R} = \frac{a_2 X}{2} + \frac{1}{4} + \frac{3}{32\, a_2 X} - \frac{1}{16\, (a_2 X)^3} + \cdots \qquad \text{numeric} \qquad (8)$$

It is evident that for large values of $a_2 X$, we may, for practical purposes, ignore all terms after the second, as is shown by the curve in Figure 29. We may also arrive at the same result by considering the depth of the equivalent skin δ centimeters in a plate conductor carrying rapid alternating currents; i. e., such a depth as, without skin effect, or with the full conductivity of continuous currents, would carry the same alternating-current

* Proc. Phys. Soc., London, Jan., 1909.

strength as the actual plate does in the presence of skin effect. It is shown in the Kennelly-Laws-Pierce paper of 1915 that, for a flat plate, this equivalent skin depth has the value

$$\delta = \left| \frac{\tanh \alpha X}{\alpha X} \right| \frac{X}{\cos \beta} \qquad \text{cm.} \quad (9)$$

where X is the half thickness of the plate, α is the semi-imaginary

$$\alpha = \sqrt{j 4 \pi \gamma \mu \omega} = \sqrt{2 \pi \gamma \mu \omega} + j \sqrt{2 \pi \gamma \mu \omega} = a_2 + j a_2 \quad \text{cm.}^{-1} \angle \quad (10)$$

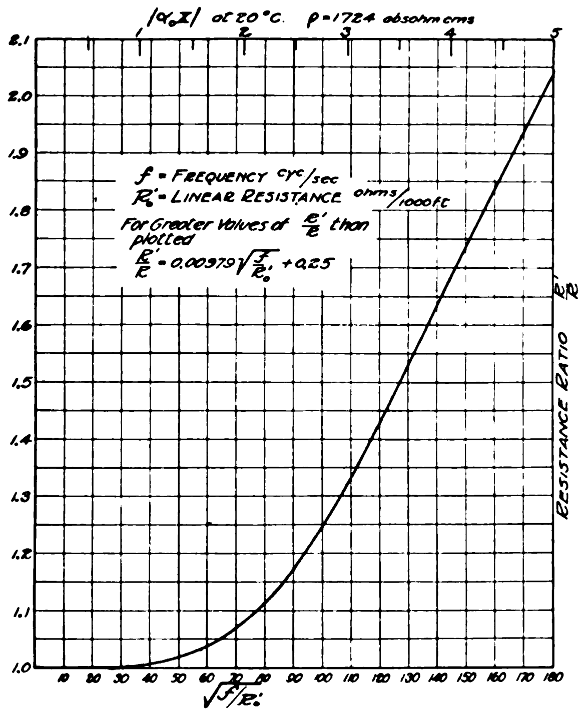


FIGURE 29—Chart for Skin-Effect Computations on Solid Round Wires

When $|\alpha X|$ exceeds 6.0, this approximates closely to

$$\delta = \frac{1}{a_2} = \frac{1}{\sqrt{2 \pi \gamma \mu \omega}} \qquad \text{cm.} \quad (11)$$

Applying this plate theory to a round wire of such dimensions that we may ignore the curvature of the surface, the linear resistance of the wire with continuous currents will be

$$R = \frac{\rho}{\pi X^2} \qquad \frac{\text{absohms}}{\text{cm.}} \quad (12)$$

where $\rho = \frac{1}{\gamma}$ is the resistivity. The linear resistance of the same wire to radio-frequency alternating currents will be

$$R' = \frac{\rho}{\delta \cdot 2\pi\left(X - \frac{\delta}{2}\right)} \qquad \frac{\text{absohms}}{\text{cm.}} \qquad (13)$$

since the current is now supposed to be carried entirely by a superficial cylinder of depth δ centimeter and mean radius $\left(X - \frac{\delta}{2}\right)$ centimeter. The skin-effect resistance ratio of the wire will therefore be

$$\frac{R'}{R} = \frac{X^2}{2\delta\left(X - \frac{\delta}{2}\right)} = \frac{X}{2\delta\left(1 - \frac{\delta}{2X}\right)} \qquad \text{numeric} \quad (14)$$

and since $\delta/(2X)$ is a small quantity with respect to unity, we may take

$$\frac{R'}{R} = \frac{X}{2\delta}\left(1 + \frac{\delta}{2X}\right) = \frac{X}{2\delta} + \frac{1}{4}$$

$$= \frac{a_2 X}{2} + \frac{1}{4} \qquad \text{numeric} \quad (15)$$

which is Russell's formula as far as two terms.*

For practical work with round non-magnetic wires, for which $\mu = 1$, it may be noted that if R_o be the resistance in ohms of a wire of length l centimeters; then for all frequencies

$$\frac{a_2 X}{2} = 3.162 \times 10^{-5} \sqrt{l\pi} \cdot \sqrt{\frac{f}{R_o}} \qquad \text{numeric} \quad (16)$$

for a wire of R_o, expressed in ohms per meter ($l = 100$)

$$\frac{a_2 X}{2} = 3.162 \times 10^{-4} \sqrt{\pi} \cdot \sqrt{\frac{f}{R_o}}$$

$$= 5.6 \times 10^{-4} \cdot \sqrt{\frac{f}{R_o}} \qquad \text{numeric} \quad (17)$$

If, however, we use R'_o as the resistance of 1,000 feet of wire as found in the ordinary wire tables ($l = 3.048 \times 10^4$)

$$\frac{a_2 X}{2} = 3.162 \times 10^{-5} \sqrt{3.048\pi \times 10^4} \cdot \sqrt{\frac{f}{R'_o}}$$

$$= 0.00979 \sqrt{\frac{f}{R'_o}} \qquad \text{numeric} \quad (18)$$

*A. Russell, *Phil. Mag.*, April, 1909; *Proc. Phys. Soc. London*, volume 21, 1909.

and $|a_o X| = 0.02768\sqrt{\frac{f}{R'_o}} = \frac{1}{36.1} \cdot \sqrt{\frac{f}{R'_o}}$ numeric (18a)

Consequently, in terms of wire-table resistances of a given size of wire at a certain temperature, when $a_2 X$ is greater than 3 say, formula (15) reduces to

$$\frac{R'}{R} = 5.6 \times 10^{-4}\sqrt{\frac{f}{R_o}} + 0.25 \qquad \text{numeric (19)}$$

or

$$= 9.79 \times 10^{-3}\sqrt{\frac{f}{R'_o}} + 0.25 \qquad \text{numeric (20)}$$

We may therefore plot a curve in terms of $\sqrt{\frac{f}{R_o}}$, or $\sqrt{\frac{f}{R'_o}}$, as abscissas, and read off the skin-effect resistance ratios as ordinates. For values of $\sqrt{\frac{f}{R'_o}}$ above say 200, the values of $\frac{R'}{R}$ fall near to the straight line of formula (15); see Figure 29.

APPENDIX III

ELEMENTARY THEORY OF A FLAT LOOP FORMED BY A PAIR OF FLAT PARALLEL COPPER STRIPS, ONE IMMEDIATELY OVER THE OTHER AND SEPARATED BY A UNIFORM THIN INSULATING LAYER

Let the width of each strip be b centimeters and d the thickness in centimeters of the thin insulating layer. The skin-effect resistance and inductance ratios of two such copper-strip loops of different sizes was recorded in Figures 14 and 15 of the paper by Kennelly, Laws and Pierce, in the A. I. E. E. Proceedings for August, 1915. It was discovered by Mr. H. B. Dwight,* that these curves correspond almost exactly (in the symbols here employed) with the formula:

$$\frac{Z}{R} = \frac{1 + \frac{(aX)^2}{2!} + \frac{(aX)^4}{4!} + \frac{(aX)^6}{6!} + \cdots}{1 + \frac{(aX)^2}{3!} + \frac{(aX)^4}{5!} + \frac{(aX)^6}{7!} + \cdots} \qquad \text{numeric}\angle \quad (21)$$

where $\frac{Z}{R}$ is the skin-effect impedance ratio of the loop, X is the thickness in centimeters of each strip, and a is a semi-imaginary quantity (affected by $\angle 45°$) defined below.

But this formula reduces to

$$\frac{Z}{R} = \frac{aX}{\tanh aX} \qquad \text{numeric}\angle \quad (22)$$

* Bibliography 20.

which is the skin-effect impedance ratio of an infinitely wide strip as given in formula (100) of that paper, differing only in form from Rayleigh's formula, except that, whereas for an infinitely wide single strip, X should be taken as the half-strip thickness, when used for Dwight's reduction it should be taken as the whole thickness. In other words, the formula for infinite strip applies to each of a pair of parallel finite strips, when their surfaces are juxtaposed, and X is taken as the full thickness instead of the half thickness of each strip.

Referring to Figure 30, if $A\ B\ C\ D$ and $E\ F\ G\ H$ are the cross-sections of a pair of similar and parallel copper strips forming respectively the going and returning conductors of a long flat

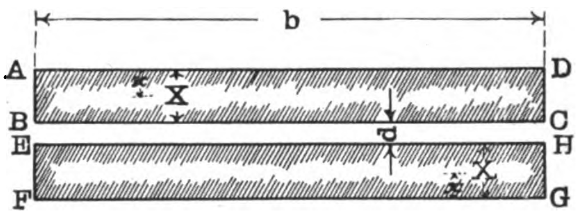


FIGURE 30—Superimposed Strips

loop, the strips being separated by an insulator of permeability unity and uniform thickness d centimeters; the symmetry of the geometrical relations permits of either strip being selected for obtaining the results applying to both. Then the magnetic flux inside this loop will be parallel to the strips except near the edges. Moreover, if $+I$ be the electric current strength (abs-amperes) flowing in the lower strip, and $-I$ that in the upper; then the total magnetomotive force in the magnetic circuit will be $4\pi I$ gilberts. The reluctance of the magnetic circuit will reside almost entirely in the narrow channel of insulator between the strips, and the residual reluctance in the external widely extending return path will be relatively negligible. This means that the $4\pi I$ gilberts will be expended almost wholly in the channel between the strips; where the magnetic field intensity will be very nearly

$$\mathcal{H} = \frac{4\pi I}{b} \qquad \text{gilberts per cm.} \qquad (23)$$

producing correspondingly a uniform flux density in the channel

$$\mathcal{B} = \frac{4\pi I}{b} \qquad \text{gausses} \qquad (24)$$

At the outer surfaces of the strips, the flux density will be substantially zero. The differential equation of magnetic instantaneous alternating flux density at any distance x centimeters inwards from the surfaces FG or $A\ D$ is then

$$\frac{d^2 \dot{\mathfrak{B}}_x}{d\,x^2} = a^2 \dot{\mathfrak{B}}_x \qquad \frac{\text{gausses}}{\text{cm.}^2} \angle \qquad (25)$$

where $a^2 = j\,4\,\pi\,\gamma\,\mu\,\omega$ $\qquad$ cm.$^2 \angle$

$j = \sqrt{-1}$

γ = conductivity of the strip in $\frac{\text{abmhos}}{\text{cm.}}$

μ = permeability of the strip taken as unity $\frac{\text{gausses}}{\text{gilberts per cm.}}$

$\omega = 2\,\pi f$ impressed angular velocity $\frac{\text{radians}}{\text{sec.}}$

f = impressed frequency $\frac{\text{cycles}}{\text{sec.}}$

Similarly, the differential equation for current density $\dot{i}_x$ absamperes per square centimeter at elevation x centimeters is

$$\frac{d^2 \dot{i}_x}{d\,x^2} = a^2 \,.\, \dot{i}_x \qquad \frac{\text{absamperes}}{\text{cm.}^2} \angle \qquad (26)$$

The solution of both (25) and (26) is of the same type:

$$\dot{i}_x = A_1 \cosh ax + B_1 \sinh ax \qquad \frac{\text{absamperes}}{\text{cm.}^2} \angle \qquad (27)$$

At $x=0$ or at the outer surface FG of the strip, $\cosh a\,x = 1$, $\sinh a\,x = 0$,

$$\dot{i}_o = A_1 \qquad \frac{\text{absamperes}}{\text{cm.}^2} \angle \qquad (27a)$$

and between (27) and (27a) B_1 can be shown to be zero; so that

$$\dot{i}_x = \dot{i}_o \cosh a\,x \qquad \frac{\text{absamperes}}{\text{cm.}^2} \angle \qquad (28)$$

$$\frac{\dot{i}_{x\,m}}{\dot{i}_{X\,m}} = \frac{\dot{i}_{x\,r}}{\dot{i}_{X\,r}} = \frac{\dot{i}_x}{\dot{i}_X} = \frac{\cosh ax}{\cosh a\,X} \qquad \text{numeric} \angle \qquad (29)$$

where the subscripts m and r represent respectively maximum cyclic and r. m. s. values of the current density. If $a\,x$, a semi-imaginary quantity, (of equal real and imaginary components) is defined as the *hyperbolic position angle* corresponding to the layer of distance x from the external surface, the current density varies directly as the hyperbolic cosine of the position angle. Such cosines have all been tabulated and charted. (Bibliography 18.)

The average r. m. s. current density over the cross-section is

$$i_{qr} = \frac{1}{X}\int_o^X i_{xr} \cdot dx = \frac{1}{X} \cdot \frac{i_{Xr}}{\cosh aX} \cdot \int_o^X \cosh ax \,.\, dx$$

$$= \frac{1}{aX} \cdot \frac{i_{Xr}}{\cosh aX} \cdot \sinh aX = i_{Xr}\frac{\tanh aX}{aX}$$

$$\frac{\text{absamperes}}{\text{sq. cm.}} \angle \qquad (30)$$

Hence the skin-effect impedance ratio $\frac{Z}{R}$ of the whole strip is

$$\frac{Z}{R} = \frac{i_{Xr}}{i_{qr}} = \frac{aX}{\tanh aX} \qquad \text{numeric} \angle \qquad (31)$$

The real component of this expression is the resistance ratio R′/R, the imaginary component is the reactance ratio $\frac{L'\omega}{R}$

The quantity $\frac{\tanh aX}{aX}$ where aX is a semi-imaginary, has been tabulated and charted as far as the modulus of $aX = 3.0$ and $\tanh aX$ of the same semi-imaginary to a modulus of 20.

The solution of (25) is also

$$\mathfrak{B}_x = A_2 \cosh ax + B_2 \sinh ax \qquad \text{gausses} \angle \qquad (32)$$

and at $ax = 0$ $\qquad \mathfrak{B}_o = A_2 = 0 \qquad$ gausses ∠ (33)

Consequently $\qquad \mathfrak{B}_x = B_2 \sinh ax \qquad$ gausses ∠ (34)

and at X $\qquad \mathfrak{B}_X = \frac{4\pi I}{b} \qquad$ gausses ∠ (35)

whence $\qquad B_2 = \frac{4\pi I}{b \sinh aX} \qquad$ gausses ∠ (36)

so that $\qquad \mathfrak{B}_x = \frac{4\pi I}{b} \cdot \frac{\sinh ax}{\sinh aX} \qquad$ gausses ∠ (37)

The magnetic flux density in the strip, substantially in horizontal-layer distribution, is thus directly proportional, at any distance x from the outside surface, to the hyperbolic sine of the position angle ax. Such sines have all been tabulated and charted.

When the position angle aX at the inner surface of the strip exceeds in modulus 6.0, tanh aX is very nearly unity, and the skin-effect impedance ratio $\frac{Z}{R}$ becomes aX simply, a semi-imaginary. The real component, or skin-effect resistance-ratio, then becomes $R'/R = |aX|/\sqrt{2} = a_2 X$, and is equal to the imaginary component or skin-effect reactance ratio $L'\omega/R$, while the equivalent skin depth δ is $1/a_2$ centimeter.

APPENDIX IV

ELEMENTARY THEORY OF THE LINEAR IMPEDANCE OF A LONG STRAIGHT CONDUCTOR FORMED OF SEVEN EQUALLY SPACED STRANDS UNSPIRALLED

Let a central round wire O Figure 31, of radius ρ centimeters be surrounded by six similar and parallel wires 1 to 6, forming a hexagon all at equal interaxial distances d centimeters. The linear resistance of each wire, with its own extra skin-effect resistance is r absohms per linear centimeter. The return conductor is a similar parallel open strand, at a distance D centimeters, which is large with respect to d. A simple alternating e. m. f. of angular velocity $\omega = 2\pi f$ radians per second, is impressed upon the loop. Required the linear impedance of the conductor.

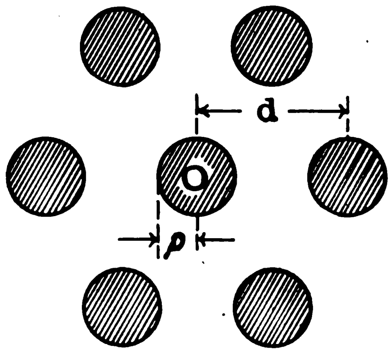


FIGURE 31—Spacing of 7-Strand Conductor.

Let i_o be the r. m. s. complex current strength in the middle wire (absamperes $\angle$)

i_1 be the r. m. s. complex current strength in any outside wire (absamperes $\angle$)

l_o be the linear inductance of the middle wire (abhenrys/cm.)

l_1 be the linear inductance of any outside wire (abhenrys/cm.)

z_o be the linear impedance of the middle wire (absohms/cm. $\angle$)

z_1 be the linear impedance of any outside wire (absohms/cm. $\angle$)

m be the linear mutual inductance between middle and an outside wire (abhenrys/cm. $\angle$)

m' be the linear mutual inductance of any five outer wires on the sixth (abhenrys/cm. $\angle$)

M be the linear mutual inductance of return strand on any wire (abhenrys/cm. $\angle$)

e be the linear r. m. s. e. m. f. impressed on any wire $\left(\frac{\text{abvolts}}{\text{cm.}}\angle\right)$

$\varepsilon = 2.71828 \cdots, \quad j = \sqrt{-1}.$

Then on any outside wire

$$e = i_1 r + j i_1 l_1 \omega + j i_o m \omega + j 5 i_1 m' \omega - j 6 i_1 M \omega - j i_o M \omega \quad \frac{\text{abvolts}}{\text{cm.}}\angle \qquad (38)$$

and on the inside wire

$$e = i_o r + j i_o l_o \omega + j 6 i_1 m \omega - j 6 i_1 M \omega - j i_o M \omega \quad \frac{\text{abvolts}}{\text{cm.}}\angle \qquad (39)$$

The values of the self-inductances are

$$l_o = l_1 = 2\left(logh\frac{2x}{\varepsilon\rho} + \frac{\mu}{4}\right) \quad \frac{\text{abhenrys}}{\text{cm.}}\angle \qquad (40)$$

where x is the length of the conductor, *logh.* represents hyperbolic or Napierian logarithms, and μ is the permeability of the wire which for copper may be taken as unity.

The values of the mutual inductances are

$$m = 2 logh\left(\frac{2x}{\varepsilon d}\right) \quad \frac{\text{abhenrys}}{\text{cm.}} \qquad (41)$$

$$M = 2 logh\left(\frac{2x}{\varepsilon D}\right) \quad \frac{\text{abhenrys}}{\text{cm.}} \qquad (42)$$

$$m' = 2 logh\left(\frac{2x}{\varepsilon d 6^{\frac{1}{5}}}\right) \quad \frac{\text{abhenrys}}{\text{cm.}} \qquad (43)$$

Since the geometrical mean distance of all five outer wires from the sixth is

$$\sqrt[5]{d \cdot \sqrt{3}d \cdot 2d \cdot \sqrt{3}d \cdot d} = d\sqrt[5]{6} \qquad (44)$$

Equating (38) and (39) we have

$$i_1\{r + j\omega(l_1 + 5m' - 6M)\} + i_o\{j\omega(m - M)\} =$$
$$i_1\{j\omega(6m - 6M)\} + i_o\{r + j\omega(l_o - M)\} \quad \text{abvolts}\angle \qquad (45)$$

or

$$i_1\left\{r + 2j\omega\left(logh\frac{2x}{\varepsilon\rho} + \frac{\mu}{4} + 5\,logh\frac{2x}{\varepsilon d 6^{\frac{1}{5}}} - 6\,logh\frac{2x}{\varepsilon d}\right)\right\} =$$
$$i_o\left\{r + 2j\omega\left(logh\frac{2x}{\varepsilon\rho} + \frac{\mu}{4} - logh\frac{2x}{\varepsilon d}\right)\right\} \quad \text{abvolts}\angle \qquad (46)$$

whence

$$i_o = k\,i_1 \quad \text{where } k = \frac{r+2j\,\omega\left\{logh\left(\dfrac{d}{6\rho}\right)+\dfrac{\mu}{4}\right\}}{r+2j\,\omega\left\{logh\left(\dfrac{d}{\rho}\right)+\dfrac{\mu}{4}\right\}} \quad \text{numeric} \angle \qquad (47)$$

$$z_1 = r+2j\,\omega\left\{logh\left(\frac{D^6}{6\,d^5\rho}\right)+\frac{\mu}{4}+k\,logh\left(\frac{D}{d}\right)\right\} \quad \frac{\text{absohms}}{\text{cm.}} \angle \qquad (48)$$

$$z_o = r+2j\,\omega\left\{logh\left(\frac{D}{\rho}\right)+\frac{\mu}{4}+\frac{6}{k}\,logh\left(\frac{D}{d}\right)\right\} \quad \frac{\text{absohms}}{\text{cm.}} \angle \qquad (49)$$

and

$$Z = \frac{1}{\dfrac{6}{z_1}+\dfrac{1}{z_o}} \quad \frac{\text{absohms}}{\text{cm.}} \angle \qquad (50)$$

Taking the case of the test represented in Figure 10, with $f=$ $50{,}000\sim$; or $\omega=314{,}159$, $\rho=0.03213$ centimeter, $d=20\rho$, or 10 diameters $=0.643$ centimeter, $D=30\,d=300\,\rho=9.64$ centimeters, $r=5.711\times10^5$ absohms per linear centimeter, which includes a skin-effect resistance ratio of 1.030 for this wire remote from other conductors, we find $k=0.50872\angle-(16°\,22')$. That is, the current strength in the middle wire is only a little more than half that on any one of the six outer wires, and lags in phase 16° 22′ relatively thereto. Substituting in (48), we find $z_1=11.981\times10^6\ \angle 86°\,05'\,58''$ absohms per centimeter and $z_o=23.551\times10^6\ \angle 102°\,27'\,58''$. The joint linear impedance of the 7 strands is then $Z=1.8461\times10^6\ \angle 87°\,21'\,55''$ absohms per centimeter $=$ $(0.084865+j\,1.8461)\,10^6=R'+jX'$. Since the d. c. linear resistance of the same strands would be $R=0.07920\times10^6$ absohms per centimeter, the skin-effect resistance ratio $R'/R=1.0715$. The observed resistance-ratio of this 7-strand conductor at 10 diameter spacing and $50{,}000\sim$ was observed to be 1.072, which is in satisfactory agreement. At closer spacings, however, the agreement does not appear to be so good, the observed resistance-ratios being greater than the computed. Thus, at $d=10\,\rho$, the computed ratio is 1.08, but the observed was 1.11. It may be that at the closer spacings the current over the cross-sections of the wires is further distorted by proximity effect, thereby increasing the power loss and resistance ratio.

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LIST OF SYMBOLS EMPLOYED

A_1, B_1 Arbitrary constants of current density (absamperes/sq. cm. $\angle$)

A_2, B_2 Arbitrary constants of flux density (gausses $\angle$)

$a_0 = \sqrt{-j 4\pi\gamma\mu\omega} = a_2 - j\,a_2$, semi-imaginary propagation constant in round wires (cm.$^{-1}$ $\angle$)

$a = \sqrt{j 4\pi\gamma\mu\omega} = a_2 + j\,a_2$, semi-imaginary propagation constant in sheets (cm.$^{-1}$ $\angle$)

a_2 Components of a_o or a. (cm.$^{-1}$)

$\mathfrak{B}$ Magnetic flux density (gausses)

$\mathfrak{B}_o$ Magnetic flux density at external surface of strip $x=0$ (gausses $\angle$)

$\mathfrak{B}_x$ Magnetic flux density at distance x (gausses $\angle$)

$\mathfrak{B}_X$ Magnetic flux density at distance X (gausses $\angle$)

b Width of a flat strip (cm.)

$\beta°$ Argument of a complex quantity (degrees)

$\gamma = 1/\rho$ Electric conductivity of a metal (abmhos/cm.)

D Distance of return conductor from going conductor (cm.)

d Thickness of an insulating layer between two strips (cm.)

Also interaxial distance between strands in a 7-strand conductor (cm.)

δ Depth of equivalent skin (cm.)

e Linear alternating e. m. f. impressed upon one wire of a strand (r. m. s. abvolts per cm. $\angle$)

$\varepsilon = 2.71828$. . . the Naperian base

f Impressed frequency (cycles per second)

$\mathfrak{H}$ Magnetic intensity (gilberts/cm.)

i_{qr} Root-mean-square mean vector current density in a conducting strip (absamperes/sq. cm. $\angle$)

i_x Instantaneous current strength at distance x from origin $\left(\frac{\text{absampere}}{\text{cm.}^2}\angle\right)$

i_{xm} Maximum cyclic current strength at distance x from origin $\left(\frac{\text{absampere}}{\text{cm.}^2}\angle\right)$

i_{xr} Root-mean-square current strength at distance x from origin $\left(\frac{\text{absampere}}{\text{cm.}^2}\angle\right)$

i_o Root-mean-square current strength in central wire of a strand $\left(\frac{\text{absampere}}{\text{cm.}^2}\angle\right)$

i_1 Root-mean-square current strength in external wire of a strand $\left(\frac{\text{absampere}}{\text{cm.}^2}\angle\right)$

$j = \sqrt{-1}$

$J_o(\quad)$ Bessel function of zero order (numeric $\angle$)

$J_1(\quad)$ Bessel function of first order (numeric $\angle$)

k Ratio of current strength in central strand to current strength in an external strand (numeric $\angle$)

l Length of a wire of resistance R_o or R'_o ohms (cm.)

l_o Linear self inductance of central wire in a strand (abhenrys/cm.)

l_1 Linear self inductance of external wire in a strand (abhenrys/cm.)

M The modulus of the impedance ratio Z/R (numeric) Also the linear mutual inductance of return conductor on going conductor (abhenrys/linear cm. $\angle$)

m_o Linear mutual inductance between middle and any outside wire (abhenrys/linear cm. $\angle$)

m' Linear mutual inductance of any five outer wires of 7-strand conductor on the sixth (abhenrys/linear cm.)

μ Permeability (gausses/gilberts per cm.)

$\pi = 3.14159\ldots$

R Linear resistance of a conductor to continuous currents (ohms or absohms/cm.)

R' Linear resistance of a conductor to alternating currents (ohms or absohms/cm.)

R_o Linear resistance of a conductor to continuous currents (ohms per meter)

R'_o Linear resistance of a conductor to continuous currents (ohms per 1,000 feet)

$\rho = 1/\gamma$ Resistivity of conductor (absohm-cm.) Also radius of central wire in a 7-strand conductor (cm.)

X' Linear reactance of a conductor in the presence of skin-effect $\left(\frac{\text{absohms}}{\text{cm.}}\angle\right)$

X Radius of a round wire. Also thickness or half-thickness of a strip (cm.)

x Radius of a point in the cross-section of a wire (cm.)

z_o Linear impedance of central strand in a 7-strand conductor (absohms/cm. $\angle$)

z_1 Linear impedance of any external strand in a 7-strand conductor (absohms/cm. $\angle$

Z Linear impedance of a straight conductor in the presence of skin-effect $\left(\frac{\text{absohms}}{\text{cm.}} \angle\right)$

Also linear impedance of a 7-strand conductor (absohms/cm. $\angle$)

$\omega = 2\pi f$ Impressed angular velocity (radians/sec.)

$\sim$ Sign for cycles per second

μ f. Sign for microfarad

m μ f. Sign for millimicrofarad

$\angle$ Sign of a complex quantity or plane vector

SUMMARY: After defining the skin-effect resistance and reactance ratios, the "spirality" and "proximity effects" for conductors at radio-frequencies, a differential bridge circuit supplied with current from 10,000 to 100,000 cycles by an Alexanderson alternator is described.

There are tested at various frequencies straight solid wires, stranded wires (of various spiralities, and also braided), wires with definitely spaced strands, very finely stranded wire (with insulated strands), braided "litzendraht" wire, stranded twisted wire (of various spiralities), strips (both singly and in opposed proximity), and tubes.

Important practical design data, quantitative as well as comparative, are given for the range of frequencies investigated.

In a number of appendices to the paper, the details of the apparatus and certain theoretical calculations of radio-frequency resistance are given.

DISCUSSION

J. Zenneck: I was especially interested in the way the authors obtained some information on what they call the spirality effect. As far as I know, this part of their investigation is quite new.

I was, however, somewhat astonished that the authors used a mechanically driven radio frequency alternator. Of course the ideal generator for measurements with radio frequency alternating currents is an oscillating electron relay such as the audion. The constancy of that can hardly be obtained by any mechanically driven alternator and the power is quite sufficient for most laboratory measurements.

From a practical point of view the resistance of a wire, stranded or not, is especially interesting when the wire is wound in the form of a coil. The coil is the practical case of the radio engineer, not the straight wire. I fully realize why the authors have first restricted their investigation to straight conductors. But I hope, that after having in this way acquired a good deal of experience, they will extend their measurements to coils of different forms.

E. F. Northrup: I have listened to the presentation of this paper with the very greatest interest. I have known for a number of years of the very important work which Dr. Kennelly and his associates have been doing on the subject of power transmission and I am fully aware, having given considerable thought and study to the matter myself, of the very great importance which attaches to this whole subject and, in particular, in reducing the losses in cables. Every one must feel, after listening to a paper like the one presented this evening, that an exceedingly complex problem has been handled in a most thoro and masterly way. The investigators necessarily had to define and limit the extent of the problems which they would attack, for the problems connected with this subject rapidly open up in number as one attempts to investigate them; and this is probably the reason why I have been able to look at the matter of power losses in cables and how they may be investigated from a somewhat different angle.

We have ordinarily been accustomed to think of the transmission of power by radio methods as involving more complexity than when power is transmitted from a place A where it is generated to a place B where it is utilized; and is guided, in going from A to B, by means of a conductor. The radio transmission

of power, however, can be shown, I think, to involve fewer complexities than the transmission of alternating-current power (especially if the frequency be high), by means of guiding conductors. When power is transmitted by means of a belt, or pneumatically, or along a tube by water in pipes, or by any mechanical system of transmission, our thoughts are concentrated upon the material links between the place A where the power is developed and the place B where it is utilized. When, however, the transmission of power is accomplished by electrical methods, our thoughts should be concentrated, not on the conductor or cable which joins the places A and B, but on the medium, the dielectric, which surrounds the conductor. In every case it may be readily shown by analysis that the transmitter of electrical power is the medium or dielectric and not the conductor at all. The conductor is not a power-transmitter but acts as a *guide* which directs power thru the medium along particular channels. When power lets go all guidance of conductors it propagates itself thru space in rectilinear lines (ordinarily) with a velocity which is the velocity of light and its course can be predicted and followed with the aid of differential equations.

We must not think of the conductor as a transmitter of electric power but only a means of guiding its passage. On the contrary, a conductor fritters power away as it goes from the place A to the place B. When it is found necessary in meeting requirements to guide power by means of a conductor, we add enormously to the number and complexity of the phenomena which are manifested over those manifested in the direct passage of power thru the ether of space.

Not the least important consideration is how we may guide high-frequency power by means of a conductor from one place to another with the least loss of power, en route, in the conductor itself. This has ceased to be an academic question. It is a question of large engineering importance; if one realizes the large investment of capital in transmission of power of every kind.

A study of the loss of power in cables at frequencies varying from 167,000 cycles to 250,000 cycles was taken up last year by two graduate students in Princeton, Mr. Thompson and Mr. Monteiro, under my guidance. Our object was to investigate the losses in cables using frequencies comparable to those used on a radio telegraph antenna. As a description of our apparatus, methods and our results will shortly appear in a forthcoming

issue of the "Journal of the Franklin Institute," I shall only briefly indicate the line along which the problem was attacked.

We considered that at the frequencies employed practically all the losses in a cable would manifest themselves in the form of heat; the loss of power by radiation, especially with the arrangement of apparatus employed being quite negligible. Our method consisted in comparing directly the heat-losses in a solid-wire copper coil with the heat losses in the cable to be tested. Arrangements were, however, effected so that the heat losses in the cable might be obtained in absolute measure. The standard copper coil and the cable coil to be tested were each inclosed in a separate Dewar bulb which was unsilvered. The bulbs were filled each with the same quantity of oil and arrangements were made whereby a rise in temperature of the oil in each flask could be determined with an electrical resistance-thermometer to better than 0.01° C. High-frequency current was passed in series thru the standard and test coil and the rise in temperature in each of the bulbs at the end of a given time gave, within 1 per cent. or better, the heat-loss in the solid-wire standard and in the cable. Furthermore, a high-frequency ammeter of special construction permitted the effective value of the high-frequency (radio frequency) current to be measured within one per cent. Both the standard and cable coils were wound in some of the tests inductively and in others non-inductively and the heat-losses were compared using these two types of windings and found not to be the same. Six different types of cable were studied.

It is not practical with the space at my disposal and without diagrams and not having the data before me to give the results of these tests or to describe any of the details of their execution. As nearly as I recollect, however, the results obtained were, except in one particular, in good agreement with those which Mr. Affel has described this evening. Our results showed that with the frequencies we employed, which were considerably higher than those used by Dr. Kennelly and Mr. Affel, a cable made up of a number of round wires lying parallel and untwisted showed a greater heat loss than a solid round conductor of identical cross-section. This fact may be accounted for, possibly, as follows: The circumferencial surface of a cable, made up as described, is not continuous, the surface of the cable having a washboard surface. The very high—or radio—frequency current, seeking as it does the extreme surface, finds, in the case

of the cable, less cross-section of metal thru which to flow than it does in the case of the solid round conductor.

The quite different results which we obtained when using the inductively and non-inductively wound coils I think can be explained by the difference which exists in the distributed capacity in the first and second cases. I should like to go into this matter further as it brings out some very interesting questions on the influence of distributed capacity in the production of heat losses, but space will not permit.

In addition to the many other problems connected with heat-losses at radio frequency and its dependence upon skin-effect, there is the very extensive question of the losses due to skin-effect which occur in iron wires. Mr. Carson and myself went quite extensively into this phase of the problem both theoretically and experimentally and our results are published in an article of over 40 pages in the February, 1914 number of the "Journal of the Franklin Institute." The most curious result and a quite important one from an engineering standpoint, is that the ratio of the alternating current resistance to the direct current resistance of an iron wire is, for any given frequency, a function of the current carried by the wire. This ratio is small with a very small current, increases to a maximum for a particular value of current and then continually decreases, with increase in current, to an asymptotic value which would be the ratio of the alternating current resistance to the direct current resistance of the iron if it were non-magnetic.

I wish to congratulate the speaker of the evening for the splendid attack that has been made on this very complex but very important problem—skin-effect resistance of conductors at radio frequencies. It is indeed a vast field and, with full appreciation of what has already been done, I believe there is ten times as much still to do.

H. A. Affel: It seems to me that the Ohm's law and bridge methods of measuring resistance are at least as good as the calorimetric method which Professor Northrup has described. The Ohm's law method is simplicity itself and when applied to coils can be made quite accurate, in view of the fact that coils can be balanced by air condensers. Hot wire meters as a means of measuring current and potential are usually available. For small currents calibrated thermo-couples might be employed.

The bridge method as described previously will also give results up to at least 100,000 cycles, which are even better with

coils than with straight wires. One has to stretch the isolated straight wires in spans across the room for testing. This introduces certain capacity unbalancing, and radiation effects, to parts of the bridge circuit, which tend to affect the accuracy of balance. The compactness of coils decreases this source of trouble.

The matter of coils of seven-strand wire having a higher resistance than those of solid round wire was astonishing, but the explanation offered to explain the fact seems quite reasonable.

The subject of iron wires is also very interesting. Professor Kennelly has, for some time past, been investigating the skin effect of steel contact rails, such as used for railway work at commercial frequencies of 25 and 60 cycles. It is quite evident that these are but a somewhat more complex instance of skin effect in iron wires. In attempting to determine formulas for the predetermination of skin effects in these conductors, solid round iron rods were tested. These showed the usual humped-curve skin-effect characteristics as Professor Northrup describes, in which the resistance ratio $\frac{R'}{R}$ is largely determined by the permeability of the conducting medium. This is covered in the usual theory of skin effect in wires; the skin effect increasing practically as the square root of the permeability. Having determined the permeability by the usual methods, Professor Kennelly was able to check by theory the measured skin effects of these rods to a degree which justifies the mathematical predictions in the case.

Professor Zenneck suggests the use of oscillating audions as a source of current. We all realize that the oscillating audion can be made, and is a most suitable generator of radio frequency currents. The main reason for using a rotary generator in this case, however, was the fact that it was immediately available, and delivered without difficulty the 4 to 5 amperes necessary for the successful operation of the bridge. With ordinary audions, this is hardly feasible. No trouble was experienced in the matter of keeping the frequency constant within the necessary limits. It is quite conceivable that with more sensitive methods of detection, involving perhaps other vacuum tubes, the bridge might be successfully operated with the audion as a generator, and the frequency limit extended considerably.

The matter of the measurement of losses in coils is being pursued at present, and I can assure Professor Zenneck that

this will be covered as thoroly as possible. The preliminary intentions are to investigate the losses in solid wire coils as a function of wire spacing, and then determine the efficiency of stranding, both as affected by the number of strands and the thickness of insulation between strands.

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All references to any individual, company, or radio station are fully listed in this index. Answers to points brought out in the Discussions will be listed under the name of the questioner. With the exception of the names of companies, all topics will be listed in general under the *noun* referred to. The numbers correspond to pages in the text. The following abbreviations are used: r. f.—radio frequency; a. f.—audio frequency; l. v.—low voltage; h. v.—high voltage.